Wave propagation in Porous Piezoelectric Layer immersed in fluid

P. Khurana[a] and C. Glorieux[b]

[a]Katholieke Universtiteit Leuven, Laboratorium voor Akoestiek en Thermische Fysica, Celestijnenlaan 200D, B-3001 Leuven, Belgium
[b]Slovak Technical University, Radlinskeho 11, 81368 Bratislava, Slovakia
poonam.aroraneekhurana@fys.kuleuven.be

In this article, we study the wave propagation in a anisotropic porous piezoelectric layer which is in contact with a fluid on both sides. The Christoffel equation for the wave propagation is derived and solved to give the roots that represent the slownesses of all the quasi-waves that may propagate along a given direction in such a medium. A scenario is modelled where a elastic wave coming, from the fluid, is incident on the layer, resulting into reflected and transmitted waves in the fluid. Analytical expressions for the reflection and transmission coefficients are obtained. Various effects on wave propagation in the porous piezoelectric are observed for a particular model.

1 Introduction

Piezoelectric materials with their unique electromechanical coupling characteristics have been widely recognised for their potential utility in a large number of sensor, actuator and transducer applications [1]. These materials have been applied in many important fields such as geophysics, electronics, communication, instrumentation, non-destructive evaluation and testing of materials [2-3]. Some piezoelectric layers are embedded in laminated composite structures to form smart structures. But due to their brittle nature failure of devices take place easily under mechanical or electrical loading[16].

Introduction of porosity in piezoelectric materials used in can improve the signal to noise ratio, the acoustical impedance matching with air, and the sensitivity characteristics make porous piezoelectric materials of great interest. They are of technological importance in ultrasonic applications such as hydrophones, actuators and underwater transducers [9-14]. In order to obtain a fundamental understanding of the effects of porosity on the effective electromechanical response of piezoelectric materials, several analytical models have been developed. Different experimental studies have been conducted on selected classes of porous piezoelectric materials related to synthesis, characterizations, fabrication and microstructure of these materials. Ceramic foams with good mechanical properties have been synthesized using piezoelectric materials[15].

In the present paper, we study the mechanical wave propagation in anisotropic porous piezoelectric material(APPM). The Christoffel equation is derived, and, solved for roots to get slownesses. Analytical expressions for the reflection and transmission coefficients for an APPM layer immersed in fluid are derived.

2 The Christoffel equation for anisotropic piezoelectric materials and its solution

The equations of motion for a fluid-saturated porous piezoelectric medium, in the absence of body forces and dissipation and free charge density are

$$\begin{aligned} &\sigma_{ij,j} = \rho_{11}\ddot{u}_i + \rho_{12}\ddot{U}_i, \\ &\sigma_{,i} = \rho_{12}\ddot{u}_i + \rho_{22}\ddot{U}_i, \\ &D_{i,i} = 0, \\ &D^*_{i,i} = 0, \qquad (i,j=1,2,3). \end{aligned} \tag{1}$$

where σ_{ij} and σ are the stress components acting on solid and fluid parts of porous aggregate. u_i and U_i are the components of displacements of solid and fluid particles. ρ_{11}, ρ_{12} and ρ_{22} are dynamical coefficients related to the porosity (f), the tortuosity (α_∞), the density of porous aggregate (ρ), the pore fluid density (ρ_f) and the inertial coupling parameters. The subscripted comma denotes partial differentiation.

The constitutive equations for porous piezoelectric medium are

$$\begin{aligned} &\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} + \mathbf{m}\boldsymbol{\varepsilon}^* - \mathbf{e}\mathbf{E} - \boldsymbol{\zeta}\mathbf{E}^*, \\ &\sigma^* = \mathbf{m}\boldsymbol{\varepsilon} + R\boldsymbol{\varepsilon}^* - \tilde{\boldsymbol{\zeta}}\mathbf{E} - \mathbf{e}\mathbf{E}^*, \\ &\mathbf{D} = \mathbf{e}\boldsymbol{\varepsilon} + \tilde{\boldsymbol{\zeta}}\boldsymbol{\varepsilon}^* + \boldsymbol{\xi}\mathbf{E} + \mathbf{A}\mathbf{E}^*, \\ &\mathbf{D}^* = \boldsymbol{\zeta}\boldsymbol{\varepsilon} + \mathbf{e}\boldsymbol{\varepsilon}^* + \mathbf{A}\mathbf{E} + \boldsymbol{\xi}^*\mathbf{E}^*. \end{aligned} \tag{2}$$

where $\boldsymbol{\sigma}, \boldsymbol{\varepsilon} / \boldsymbol{\sigma}^*, \boldsymbol{\varepsilon}^*$ are the stress, strain tensor on solid/ fluid phase of porous aggregate. $\mathbf{D}, \mathbf{E} / \mathbf{D}^*, \mathbf{E}^*$ are the electric displacement and electric field vectors for solid/ fluid phase of porous bulk material respectively. $\mathbf{c}, \mathbf{e}, \boldsymbol{\xi} / \mathbf{R}, \mathbf{e}^*, \boldsymbol{\xi}^*$ are the elastic, piezoelectric and dielectric tensors for solid/ fluid phase respectively. $\mathbf{m}; \boldsymbol{\zeta}, \tilde{\boldsymbol{\zeta}}; \mathbf{A}$ are the tensors that take into account the elastic; piezoelectric; dielectric coupling between the two phases of porous aggregate.

The elastic strain tensors ($\boldsymbol{\varepsilon} / \boldsymbol{\varepsilon}^*$) and the electric field vectors ($\mathbf{E} / \mathbf{E}^*$) corresponding to solid/fluid phase are

$$\boldsymbol{\varepsilon} = \frac{1}{2}\nabla(\mathbf{u} + \mathbf{u}^{\mathrm{T}}), \quad \boldsymbol{\varepsilon}^* = \nabla.\mathbf{U}^*, \quad \mathbf{E} = -\nabla\phi, \quad \mathbf{E}^* = -\nabla\phi^*,$$

where $\mathbf{u} / \mathbf{U}^*$ and ϕ / ϕ^* are mechanical displacement tensors and electric potentials corresponding to solid/ fluid phase of porous aggregate.

For plane wave propagation in the $x_1 - x_3$ plane, formal solutions of equations (1) can be sought in the form

$$\begin{aligned} &(u_i \quad U_i) = (S_i \quad F_i)\exp\left(i\omega\left(\frac{x_1}{c} + q\,x_3 - t\right)\right), \\ &(\phi \quad \phi^*) = (G \quad H)\exp\left(i\omega\left(\frac{x_1}{c} + q\,x_3 - t\right)\right), \quad (i=1,2,3) \end{aligned} \tag{5}$$

where $\imath = \sqrt{-1}$, ω is the angular frequency and q is the slowness vector. c is the apparent phase velocity given by $c = v/\sin\theta$, with v the phase velocity of a wave propagating in $x_1 - x_3$ plane, along an direction making an angle θ with $-x_3$ axis. S_1, S_3, F_1, F_3, G, H are wave amplitudes.

Substitution of solutions (5) into the equations (1) through equations (4), provides a homogeneous linear system of six equations in S_1, S_3, F_1, F_3, G, H, which yields a tenth degree polynomial equation relating q to c as

$$T_1q^{10}+T_2q^8+T_3q^6+T_4q^4+T_5q^2+T_6=0 \quad (6)$$

The coefficients $T_1,T_2,.......,T_6$ depend on c and on the material properties. Equation (6) gives ten distinct roots for q or more precisely, five roots for q^2, thereby describing five wave modes in porous piezoelectric material. These are called quasi-waves.

Hence the formal solutions for the propagation of plane waves in anisotropic piezoelectric porous layer are

$$u_1=\sum_{n=1}^{10}S_1(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right)$$

$$u_3=\sum_{n=1}^{10}S_3(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right)$$

$$U_1=\sum_{n=1}^{10}F_1(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right)$$

$$U_3=\sum_{n=1}^{10}F_3(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right)$$

$$\phi=\sum_{n=1}^{10}G(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right)$$

$$\phi^*=\sum_{n=1}^{10}H(n)f(n)\exp\left(\iota\omega\left(\frac{x_1}{c}+q(n)x_3-t\right)\right). \quad (7)$$

3 Mode conversion at piezoelectric porous layer completely immersed in a fluid

We consider the case where a transversely isotropic porous piezoelectric material layer, with thickness H, is loaded with fluid on both sides. A plane wave, making an angle θ with $-x_3$ axis, is assumed to be incident at the interface. This wave results in one reflected mode in upper fluid, five transmitted modes in the transversely isotropic porous piezoelectric layer and one transmitted mode in lower fluid. The wave propagation in fluid is described by the following equations.

In the upper fluid, the displacement evolutions are given by:

$$u_{1u}^f=R\exp\left(\iota\omega\left(\frac{x_1}{c}-q_{fu}x_3-t\right)\right)+\exp\left(\iota\omega\left(\frac{x_1}{c}+q_{fu}x_3-t\right)\right),$$

$$u_{3u}^f=-q_{fu}R\exp\left(\iota\omega\left(\frac{x_1}{c}-q_{fu}x_3-t\right)\right)+q_f\exp\left(\iota\omega\left(\frac{x_1}{c}+q_{fu}x_3-t\right)\right).$$

And the stress field by:

$$\sigma_{33u}=\iota\omega\rho_{fu}\,c\left(R\exp\left(\iota\omega\left(\frac{x_1}{c}-q_{fu}x_3-t\right)\right)+\exp\left(\iota\omega\left(\frac{x_1}{c}-q_{fu}x_3-t\right)\right)\right)$$

For the lower fluid, the formal solution consists of transmitted components and is given by

$$u_1^f=T\exp\left(\iota\omega\left(\frac{x_1}{c}+q_fx_3-t\right)\right),$$

$$u_3^f=q_fc\,T\exp\left(\iota\omega\left(\frac{x_1}{c}+q_fx_3-t\right)\right),$$

$$\sigma_{33}^f=\iota\omega\rho_f^lT\exp\left(\iota\omega\left(\frac{x_1}{c}+q_fx_3-t\right)\right),$$

where $q_{f_{u/l}}=\frac{1}{c}\sqrt{\frac{c^2}{c_{f_{u/l}}^2}-1}$.

Here $c_{fu/l}$ is the longitudinal incident wave velocity in fluid medium. R and T are reflection and transmission coefficients in upper and lower fluids respectively. The boundary conditions on both sides of the plate are given by

(a) Mechanical Boundary Conditions

$$(1-f)\dot{u}_3+f\dot{U}_3=\dot{u}_{3u/l}^f,$$
$$\sigma_{33}+\sigma=\sigma_{33u/l}^f,$$
$$\sigma=f\sigma_{33u/l}^f,$$
$$\sigma_{13}=0,$$

(b) Electrical Boundary Conditions

$$\phi=0,$$
$$\phi^*=0$$

The subscripts u and l in the above expressions correspond with the entities in upper and lower fluids, respectively. f is the porosity of the layer. Inducing displacement components and electric potentials into the boundary conditions on both sides of the plate, we obtain twelve equations in twelve unknowns $f_1,f_2,............,f_{10},R,T$. We can write the resulting set of equations as

$$\mathbf{AX}=\mathbf{B},$$

where

$$\mathbf{X}=\begin{bmatrix}f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 & f_8 & f_9 & f_{10} & R & T\end{bmatrix}^T,$$
$$\mathbf{B}=\begin{bmatrix}cq_{fu} & c\rho_{fu} & cf\rho_{fu} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

$\mathbf{A}=(a_{ij})$ is a matrix of order twelve. The expressions for the reflection and transmission coefficients are given by

$$R=\frac{\Delta_{11}}{\Delta},\ T=\frac{\Delta_{12}}{\Delta},$$

where $\Delta=|a_{ij}|$ and Δ_{11}, Δ_{12} are the determinants of matrices obtained by Cramer´s rule.

4 Analysis of numerical simulation

Based on the analytical expressions obtained in the previous section, we have numerically computed the reflection and transmission coefficient for a particular material $BaTiO_3$ which is considered to be transversely isotropic (6mm). Values for the elastic, piezoelectric and dielectric constants were taken from Gupta and Venkatesh [13] and are given in the Table 1.

Table I: Elastic constants, Piezoelectric constants and Dielectric constants

Elastic constants (GPa)
$c_{11} = 150.4, c_{12} = 65.63,$
$c_{13} = 65.94, c_{33} = 145.5,$
$c_{44} = 43.86, m_{11} = 8.8,$
$m_{33} = 5.2, R = 20$
Piezoelectric constants (C/m^2)
$e_{15} = 11.4, e_{31} = -4.32,$
$e_{33} = 17.4, \zeta_{15} = 4.56,$
$\zeta_{31} = -1.728, \zeta_{33} = 6.96,$
$e_3^* = -3.6, \tilde{\zeta}_3 = -7.5,$
Dielectric constants (nC/Vm)
$\xi_{11} = 10.8, \xi_{33} = 13.1$
$\xi_{11}^* = 11.8, \xi_{33}^* = 13.9$
$A_{11} = 12.8, A_{33} = 15.1$

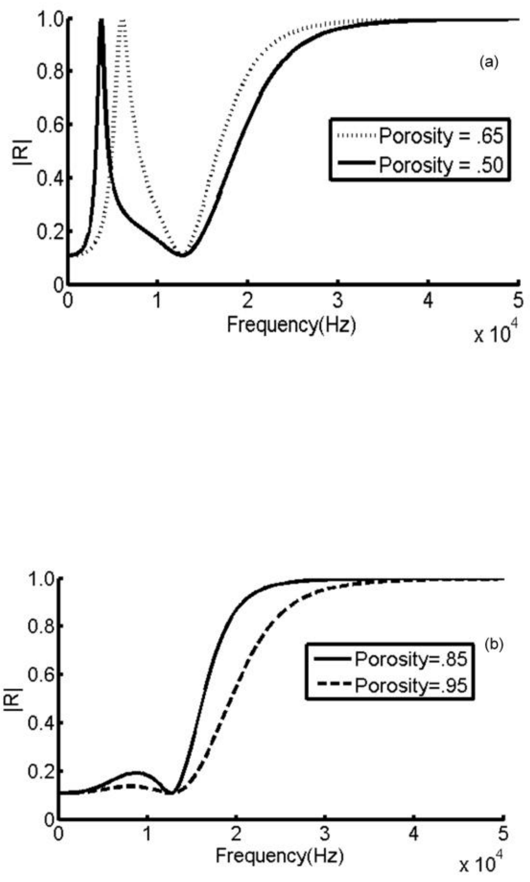

Figure1: Effect of porosity on frequency dependence of a plane sound wave incident at 60° on a 6mm APPM layer of $BaTiO_3$.

Figure 1 depicts the frequency dependence of the reflection coefficient for a plane acoustic wave in air incident on the $BaTiO_3$ layer at an angle 60°. Figure 1a and 1b show the effect of a varying porosity. The reflection coefficient and along with that the acoustic impedance depends linearly on porosity of the layer. That could be quantitatively by considering the gradually introducing the fluid phase, in which sound wave velocity is far lower than in the solid phase. The reduction of the acoustic impedance by increasing the porosity gives a handle for acoustic matching. Figure 2a and 2b demonstrate the effect of thickness of layer along with porosity. Figure 2a shows the effect if the porosity is .65 and figure 2b shows the effect if the porosity is .85. As we notice from the figure, the amplitude attains its maximum and minimum at lower frequency if the layer is thicker. Also the reflected energy is sensitive to small changes in the thickness of the layer. This sensitivity may be due to the fact that small change in the thickness of the layer can result in a relatively large change in the gradient of the velocity profile.

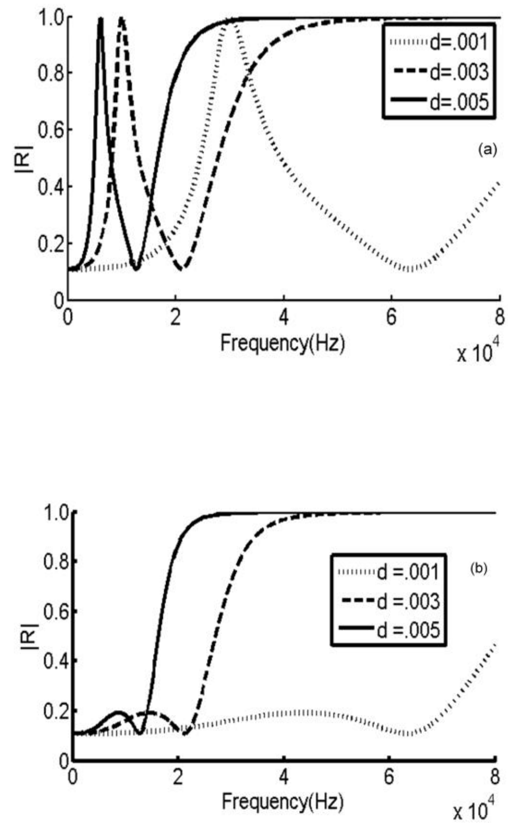

Figure2: Effect of thickness on frequency dependence of a plane sound wave incident at 60° on a 6mm APPM layer of $BaTiO_3$.

Conclusion

Wave propagation in an anisotropic piezoelectric porous layer loaded on both sides with an elastic fluid, was investigated. Analytical expressions for the reflected and transmitted coefficients were obtained. The reflected energy turns out to be quite sensitive to changes in the thickness of the layer. This sensitivity may be due to the fact that small change in the thickness of the layer can result in a relatively large change in the gradient of the velocity profile. The developed calculation tool is expected to have applications for stimulating the performance of smart piezoelectric transducers used for the ultrasonic inspection of composites and structures. Also engineering applications in surface acoustic wave devices, composite materials characterization and smart structures require the analysis of acoustic wave interaction with anisotropic and piezoelectric multilayered structures.

Acknowledgments

The authors would like to thank the KU Leuven Research Council for financial support (project OT/11/064)".

References

[1] W. G. Cady, *Piezoelectricity*, McGraw-Hill, New York (1946)

[2] A.H. Nayfeh, *Wave propagation in layered anisotropic media with applications to composite*, Elsevier, Amsterdam (1995)

[3] R.L. Goldberg, M.J. Jurgens, D. M. Mills, ``Modelling of piezoelectric multilayer ceramics using finite element analysis´´, *IEEE Trans Ultrason. Ferroelect. Freq. Control*, 44 , 1204-1214 (1997)

[4] D.J. Powell, G. Hayward, R.Y. Ting, "Unidimensional modelling of multilayered piezoelectric transducer structures", *IEEE Trans. Ultrason. Ferroelect. Freq. Control*, 45, 667-677 (1998)

[5] B. Honein, A.M.B. Braga, P. Barbone, G. Herrmann, "Wave propagation in piezoelectric layered media with some applications" , *J. Intell Mater. Syst. Struct.*, 2, 542-557 (1991)

[6] V.Y. Zhang, J.E. Lefebvre, C. Bruneel, T. Gryba, "A unified formalism using effective surface permittivity to study acoustic waves in various anisotropic and piezoelectric multilayers", *IEEE Trans. Ultrason. Ferroelect. Freq. Contr.*, 48, 1449-1461 (2001)

[7] A.A.A. Mesquida, R.R. Ramos, F. Comas, G. Monsivais, R. E. Sirvent, "Scattering of shear horizontal piezoelectric waves in piezocomposite media", *Journal of Applied Physics*, 89, 2886-2892 (2001)

[8] C. Cai, G. R. Liu, K.Y. Lam, "A technique for modelling multiple piezoelectric layers", *Smart Mater. Struct*., 10, 689-694 (2001)

[9] T.E. Gomez, F. Montero, "Characterization of porous piezoelectric ceramics: The length expander case", *J. Acoust. Soc. Am.*, 102, 3507-3515 (1997)

[10] T.E. Gomez, A.J. Mulholland, G. Hayward, "J. Gomatam, Wave propagation in 0-3/3-3 connectivity composites with complex microstructure", *Ultrasonics*, 38, 897-907 (2000)

[11] E. Roncari, C. Galassi, F. Craciun, C. Capiani, A. Piancastelli, "A microstructural study of porous piezoelectric ceramics obtained by different methods", *J. European Ceramic Soc.* 21, 409-417 (2001)

[12] C.R. Bowen, A. Perry, A.C.F. Lewis, H. Kara, "Processing and properties of porous piezoelectric materials with high hydrostatic figures of merit", *J. European Ceramic Soc.*, 24, 541-545 (2004)

[13] R.K. Gupta, T.A. Venkatesh, "Electromechanical response of porous piezoelectric materials", *Acta Materialia,* 54, 4063-4078 (2006)

[14] T. Ikeda, *Fundamentals of piezoelectricity*, Oxford Science Publications, Oxford University Press (1996)

[15] Laurel Wucherer, Juan C. Nino, Ghatu Subhash, "Mechanical properties of BaTiO3 open porosity foams", *Journal of the European Ceramic Society* 29, 1987-1993 (2009)

[16] Tobias H. Brockman, *Theory of adaptive fiber composites: From piezoelectric material behaviour to dynamic of rotating structures*, Springer (2009)