Modal analysis comparison of two violins made by A. Stradivari

S. Le Conte[a], S. Le Moyne[b] and F. Ollivier[b]

[a]Cité de la musique, 221 avenue jean jaures, 75019 Paris, France
[b]Institut jean le rond d'alambert, 2 pl de la gare de ceinture, 78210 Saint Cyr L'Ecole, France
sleconte@cite-musique.fr

The Music Museum in Paris keeps in its collection five violins made by the maker Antonio Stradivari, named Stradivarius (1644-1737). Two of them, called the *Davidoff* and the *Tua* have been made according the same mould, at the same period and the most important using the same wood. Even though the violins has not the same history there are kept in their integrity. To evaluate the difference in the making process of this two musical instrument an experimental modal analysis of the soundboard was performed by processing its sound field. A non intrusive method, the Impact Planar Nearfield Acoustic Holography, was used. This technique, developed by the authors, implements the well known inverse method NAH on the basis of the acoustic impulse response field and is well adapted to modal analysis. To quantify the shift between the two responses a comparison has been made with a reproduction of another famous violin called the Allard made.

1. Introduction

Antonio Stradivari, Stradivarius says (1644-49-1737), is the most famous violin maker of all time. Surrounded by legends, the perfection of his instruments has fueled a mythology. Creative genius, it is nonetheless a craftsman thoughtful, methodical, heir to a prestigious tradition. Antonio Stradivari would have made about a thousand instruments which about six hundred and fifty have survived. The Musée de la musique in Paris maintains five violins, showing changes in his making process, a guitar and a pocket. In its collection, the museum keeps two violins which have the reputation to be "twins" called the *Davidoff* (E.1111) and the *Tua* (E. 1932) (figure 1). This reputation is based first of all on the year of facture. They were made in 1708 and this date has been confirmed thanks to dendrochronology [1-2]. This study has also shown that the wood used by Stradivari came from the same place and could come from the same supply. The second reason is the making process itself. The two violins would have the same dimensions (but they have not the same history and today the dimensions are nearly the same - ± 1mm for the back length-). The last reason, from the violin maker expert point of view is that the two backs are made of flat sawn maple. Today less than ten instruments from Stradivari are known to be made according this process.

Figure 1: Left : Violin *Tua* (E.1932) and *Davidoff* (E.1111) from Musée de la musique; Right: Davidoff back.

The nearfield acoustical holography (NAH) has been used in the museum to evaluate the mechanical behaviour of an historical harpsichord [3]. We have shown the many advantages of the technique in the dynamical characterization of a cultural heritage musical instrument. We propose to evaluate the similarities and the differences between 2 violins supposed to be duplicate. To be able to compare the differences measured the same measurement will be hold on a third violin – a fac-simile of another famous violin (the *Alard* made by Guarneri del Gesu in 1742 in Italy) kept in the museum - supposed to be very different but with the same process for the back (only one piece of maple).

2 Impulse Nearfield Acoustical holography

2.1 Principle

This technique is here used in the aim of achieving a structural modal analysis. The NAH process of planar harmonic pressure fields is exhaustively described in [5], its adaptation for impulse source excitation (IPNAH) was primary presented in [6].

The impulse response of the vibrating source is measured in term of radiating acoustic field with a microphones array. The impulse response is obtained by a point shock excitation of the structure.

The vibration behaviour of the source is then deducted, in terms of normal vibration velocity, with the help of an inverse calculation method based on spatial 2D Fourier transforms.

Compared to more classical experimental modal analysis methods (laser vibrometry, piezoelectric accelerometers) that measure directly the vibration behaviour of the structure, one must keep in mind the inverse calculation hypothesis approximations used:

- the vibration source is supposed to be planar
- the vibration source is reconstructed on a virtual planar rectangle of the same size as the microphones array.
- The source distribution is supposed to be continuous.
- Moreover measurement is performed in the near acoustic field, evanescent components are partially covered by noise and the information they contain is then lost with the noise filter NAH operation.

From the last two points results a lack of precision in the vibration field reconstruction in the vicinity of edges areas where sharp discontinuities are present. These defaults were shown to be minor.

IPNAH has some interesting advantages, especially in case of fragile structures like ancient musical instruments, compared to classical methods:

- except for the excitation process, it is a non contact method
- since an important number of grid points (120) can be measured at the same time, the number of shocks on the structure is strongly limited.
- The measurement time is comparatively very short

In the present study it took around twenty minutes for recording the set of 1920 point impulse responses to be processed.

Following the measurement stage, a classical frequency analysis provides a set of harmonic hologram pressure fields ($Ph(\omega,x,y,z_h)$) over the desired frequency band.

2.2 Nearfield Holography Process

The successive steps implemented for processing the harmonic acoustic fields obtained previously follow the description presented in [3]. The first step consists, by means of a 2D spatial Fourier transform, in converting the measured harmonic pressure field $Ph(\omega,x,y,z_h)$ from the real space domain into its k-space representation $Ph(\omega,k_x,k_y,z_h)$.

The second step consists in conditioning the obtained spatial spectrum in order to eliminate the high spatial frequency noise brought by the measurement process. This is done applying a low-pass Veronesi filter, with a cutoff wave number of k_c. The filtered k-spectrum is denoted $KPhf(\omega, k_x,k_y,z_h)$.

In our case, the objective is to reconstruct the normal velocity V_s of the structure Therefore the following operation, called the back propagation process, is modeled with an operator $G_{PV} = E_{PV}(\omega,k_x,k_y).H(\omega, k_x,k_y, z_h\text{-}z_s)$.

H stands for the exponential propagator $exp(jk_z(z_h\text{-}z_s))$, where $k_z = (k^2\text{-}(k_x^2+k_y^2))^{1/2}$ is purely imaginary for evanescent components of the field, and real for the propagating components.

The operator $E_{PV} = k_z/\rho ck$ is independent of the source-hologram distance and derives directly from Euler's equation. Its effect is to transform pressure into normal velocity.

After the back propagation process of the spatial spectra onto the source plane, the ultimate step brings back to the real space, consisting of an inverse 2D spatial Fourier transform.

3. Experimental set-up

Figure 2 : experimental set up for the soundboard response

The experimental setup used for the three instruments (2 violins from Stradivari and a third one) was inspired by a precedent study on a Couchet harpsichord [3]. The impulse responses of the soundboards and the back are measured in the semi-anechoic room of the museum. Ambient temperature and hygrometry were controlled during the whole duration of the experiment and are the same as in the museum to avoid wood reaction due to hygrometry. The strings were chosen not to be removed for conservation reasons to avoid an additional stress cycle on the structures. The three violins are tuned at the same tension to avoid measuring a difference due to the pre-stress of the soundboard [4]. The strings were muffled to not disturb the acoustic pressure field measured. For the same reason and to muffle the vibration of the back the violon was simply supported on a foam cradle (figure 2). To record the back response the violins were placed horizontally on three carved low-density open-cell foam blocks at the neck joint and the endblock.(figure 3).

Figure 3 : Positioning of the violin for the back response measurement

A point impulse excitation of the soundboard and of the back is provided by an automated hammer driven by an electromagnet that produces a reproducible shock (figure 4). The excitation point is the same for all instruments and is chosen on the bridge in the treble area as it is usually done for modal analysis of soundboards [7] for the soundboard and for the back responses.

Figure 4 : muffling system, excitation and reference accelerometer

A 12 by 10 electret microphones array, with a 50 mm step, has been used to collect the pressure field. So as to fit the measurement grid, the array is moved according to 16 positions. The 120 impulse pressure responses for each position of the array are collected using a home made 128 channels synchronous digital recorder. Each measurement associated to one shock on the soundboard has to be phase referenced. Therefore an accelerometer has been positioned on the soundboard near the excitation point (figure 4), and its constant impulse response is systematically recorded along with the acoustic signals.

The resulting acoustic impulse response field is measured over a parallel plane at a distance $z_h = 136$ mm, the smaller possible here for technical reasons of accessibility. The field is finally sampled according to a thin grid with a 12.5 mm step.

4. Results

Many studies have been done on the vibrational behaviour of the violin. Indeed from 1830 Savart wondered what could be the sound of the free plates before including in the whole instrument (supposed to be a “good” one). He worked with the violin maker Vuillaume and with the physicist Chladni [8]. Later Hutchins developed a technique known as free-plate tuning for maker based on the Chladni patterns. One of the first modal analysis of a violin was made in the early 80’s [9-10]. The modes of the violin have been identified. The figure 5 presents the average spectra on the measurement grid for the three violins, on the top is the soundboard side and on the bottom, the back side. The frequency responses are typical for violins. We can notice, on the top, a hill around 500 Hz with several peaks superimposed.

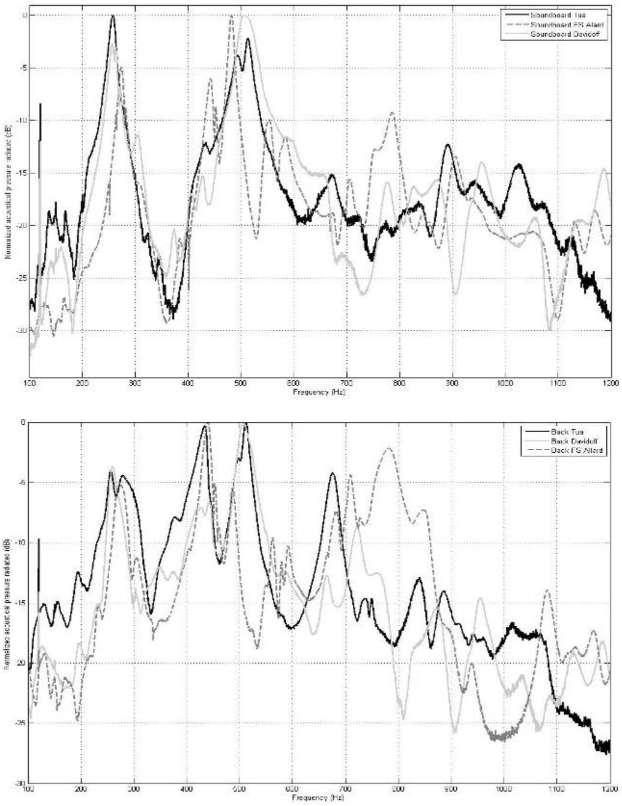

Figure 5: Average normalized preasure radiated from the soundboard (top) and from the back (bottom) for the 3 violins

It can be noticed that the two spectra from the two violins made by Stradivari are much closed. The spectra from the fac simile are shift in frequency.

The first resonance, called A_0 [11] is the cavity resonance. This resonance is a Helmholtz resonance, by definition it depends only geometrical parameters. The black line and the grey one (from the violins made by Stradivari) are superposed around 257 Hz. We can observe also this resonance from the back. This measure means that the two violins have the same ration opening surface over internal volume. The dot line (fac simile) for the soundboard (and for the back) is shift in frequency around this peak (274 Hz). Indeed, the length of the back is 353.5 mm for the fac simile and 359 mm for the other two violins.

According to Schleske who has examined by modal analysis almost 90 violins [11] the B_1 mode could be regarded as a kind of leading mode for the tonal colour of the instrument. This mode is a corpus mode strongly radiated one [11]. This mode is located in the frequency bandwidth [500-600] Hz. For the *Tua* and the *Davidoff* we measured respectively 511 Hz and 506 Hz (less than 1 % shift) on the back responses, for the fac simile this mode is at 481 Hz (almost 6% shift compare to the *Tua*).

Over 600 Hz, we can notice the called table modes. It is impossible to find similarities on the *Tua* and *Davidoff* spectra. Indeed, after inside examination of these violins, it is possible to see many repaired cracks on the *Davidoff* soundboard, and few insect tunnels on the *Tua* back (figure 6).

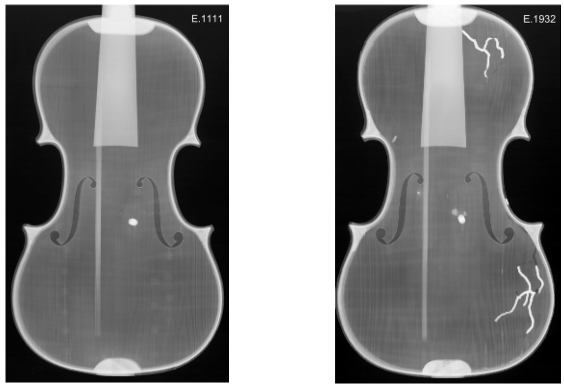

Figure 6 : radiography of the Davidoff (left) and of the Tua (right) © Cité de la musique..

Conclusion

The vibrational behaviour of these two violins made by Stradivari has been compared because they have the reputation to be “twins”. NAH has been chosen to evaluate the similarity between these instruments because the technique offers many advantages for the Cultural Heritage objects. We have shown that for the modes which are deeply geometrical dependant (Helmholtz and corpus mode) the similarity is strong between the *Tua* and the *Davidoff* whereas the fac simile is different. For the plate modes (back and soundboard) nothing can be compared in the spectra.

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