

Optimization of electric shunt resonant circuits for electroacoustic absorbers

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Sound absorption can be achieved in a relatively simple manner with passive or active electroacoustic absorbers. Electroacoustic absorbers are basically loudspeakers, the membrane of which is used as an absorber/reflector of sound, and the electric terminals of which are connected to a dedicated electric load. This electric shunt can be a simple electric resistor, or an active device employing feedback control on acoustic and/or electric quantities (aka. direct impedance control). Whilst the latter allows significant broadening of acoustic absorption performances that could not be obtained with simple passive resistors, it is also possible to find passive shunt alternatives that allows interesting acoustic performances without the use of expensive and energy consuming electric disposals. A simple configuration of shunt electric resonator is proposed as an alternative to active shunt for broadening the performances of passive electroacoustic absorbers.

1 Introduction

Turning an electrodynamic loudspeaker into a sound absorber can be obtained in very straightforward manners, the mechanical equipment of the loudspeaker acting naturally as a 1 degree of freedom mechanical resonator, thus capable of absorbing some sound energy at the resonance. These performances can already be slightly tuned with passive treatments, either by providing additional acoustical or electrical damping, or by adding mass to the moving parts or increasing the stiffness with an enclosure, in order to shift the peak resonance apart from the natural frequency. In [1], the effect of increasing the moving mass, that of varying the compliance by tailoring the volume of the cabinet for instance, or simply adding damping by filling the enclosure with absorbing materials are detailed through designed experiments. Then, the only way to improve performances without affecting the design of the transducer or enclosure is then to apply active control strategy that could alter the dynamics of the system [2]. Here, an alternative solution is proposed for preventing feeding back the loudspeaker with positive electrical energy.

The technique of impedance matching using passive analog electrical networks was first introduced on piezoelectric materials. In [3], a resistor is connected to a piezoelectric material so as to create artificial damping to a structure to which it is attached. Such electrical shunt networks are very easy to implement, cheap and do not require any power for operation. However, it was shown that damping performance with a purely resistive shunt connected to piezoelectric materials is very poor, and in some applications even not measurable. More efficient are the resonant shunt circuits which are obtained by combining resistive and reactive components so as to achieve single- or multi-mode damping. As discussed in [3], shunting piezoelectric materials with a resistor and inductor introduces an electrical resonance which can be optimally tuned to structural resonances. Once bonded to a structure, the shunted piezoelectric material works as a mechanical vibration absorber.

The technique has been transferred to electrodynamic loudspeakers afterward. By analogy with piezoelectric shunt damping, a basic analog network connected across the terminals of a loudspeaker can be used to damp resonant sound fields [2, 4]. The objective here is to globally reduce the sound field without the need of collocated pressure or velocity sensors, as for feedback controlled loudspeakers. The key issue is to properly design an electrical matching circuit which, when connected across the transducer terminals, improves the dissipation of acoustic energy at the diaphragm. In practice, electrical coupling can be achieved with the help of basic analog networks made up of a combination of positive or negative, reactive or resistive components, in series or in parallel. The resulting shunt electrical impedance is commonly designed to dissipate the electrical energy which has been converted from mechanical energy by the transducer.

In the following, we will study several electrical network topologies that allow the tuning of an electroacoustic resonator without the need of any external acoustic sensor. The idea is to examine how the parameters of the resonator (damping, mass and stiffness) may be affected by electrical means. The interaction between an electrodynamic loudspeaker and various arrangements of electrical components, resistive and reactive, is first modeled. Advantages provided by combining some electrical matching networks with a loudspeaker are then quantified. As a conclusion, general remarks regarding limitations and potential applications in actual situations are discussed.

2 Principle of matching electrical network

The key problem of the matching network technique is to find a very simple analog circuit that efficiently allows sound energy dissipation at the loudspeaker diaphragm. The basic idea is to produce a kind of regulation of electrical quantities in the coil circuit. In order to make a current flow, the circuit must be closed in some way. With no electrical load (transducer terminals in open circuit), there is a voltage drop across the output and the loudspeaker behaves as a damped harmonic oscillator with no possibility of adjustment. The general idea is to derive the desired relationship between current and voltage using a two-terminals network, and to combine it with the characteristic equations of the loudspeaker.

2.1 Formulation of the problem

Let's consider the closed-box electrodynamic loudspeaker of fig. 1



Figure 1: The shunt loudspeaker.

Expressed in the frequency domain, the equations coupling the electrical and mechanical parts of the loudspeaker are given as [2]:

$$SP(j\omega) = \left(j\omega M_{ms} + R_{ms} + \frac{1}{j\omega C_{ms}} + \frac{\rho c^2 S^2}{j\omega V_b}\right) V(j\omega) - Bl I(j\omega)$$

$$E(j\omega) = \left(j\omega L_e + R_e\right) I(j\omega) + Bl V(j\omega)$$
(1)

where $\omega = 2\pi f$, f being the frequency of the harmonic disturbance in Hz, and

- V is the normal velocity of the diaphragm, in $m s^{-1}$
- I is the electrical current, in A
- *P* is the total exogenous sound pressure acting at the front of the diaphragm, in Pa
- *E* is the electric voltage applied to the electrical terminals, in V
- M_{ms} is the total moving mass of the diaphragm, in kg

 C_{ms} is the mechanical compliance of the suspension, in N m⁻¹

- R_{ms} is the mechanical resistance, in N s m⁻¹
- R_e is the electrical resistance of the coil, in Ω
- L_e is the electrical inductance of the coil in H
- Bl is the force factor, in N A⁻¹
- S is the diaphragm area, in m^2
- V_b is the enclosure volume, in m³
- ρ is the air density, in kg m⁻¹
- c is the sound celerity in the air, in $m s^{-1}$

In the following, we will denote C_{mc} the total equivalent compliance of the enclosed loudspeaker, such as $\frac{1}{C_{mc}} = \frac{1}{C_{ms}} + \frac{\rho c^2 S^2}{V_b}$.

Behaving as a motor, the loudspeaker is fed by an electrical voltage E, and the electrical current I flowing in the voice-coil is responsible of a mechanical Laplace force F =Bll which actuates the acoustic medium with a pressure P. In reaction, the medium exerts a pressure force SP on the diaphragm, leading the diaphragm to vibrate with the velocity V, depending on its mechanical impedance $Z_{mc}(\omega) =$ $R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{mc}}$. The voice-coil moving with velocity V in the air gap induces a voltage drop (electromotive force) -BlV between the electric terminals, thus modulating the electrical current I circulating in the coil, depending on the electric load. From a control perspective, the problem is then to implement a functional relationship between the electrical variables at the loudspeaker's terminals, that is to say between E and I, in order to assign a certain vibrating velocity depending on the external sound pressure. To that purpose, different types of electrical networks can be used to implement the functional relationship that will shape the current after the electromotive force generated by the pressure force.

2.2 Shunted loudspeakers

When the loudspeaker is loaded with an electric network of equivalent impedance Z_L , the second equation of Eq. 1 can be written as:

$$E(\omega) = -Z_L(\omega)I(\omega) = (j\omega L_e + R_e)I(\omega) + BlV(\omega)$$
(2)

Then, the normalized acoustic admittance presented by the loudspeaker diaphragm can be derived in a straightforward manner as:

$$Y(\omega) = \rho c \frac{V(\omega)}{P(\omega)}$$

= $\rho c S \frac{j\omega}{(j\omega)^2 M_{ms} + j\omega R_{ms} + \frac{1}{C_{mc}} + \frac{j\omega (Bl)^2}{j\omega L_e + R_e + Z_L(\omega)}}$ (3)

Obviously, this result can also be derived in order to process the resulting sound absorption coefficient α defined as:

$$\alpha(\omega) = 1 - \left|\frac{1 - Y(\omega)}{1 + Y(\omega)}\right|^2 \tag{4}$$

It can be observed that the electric load Z_L can provide additional damping to the acoustical resonator constituted by the loudspeaker diaphragm, which results in more or less sound absorption. Moreover, if the load Z_L has reactive components, the electric load can also modify the total dynamic mass and compliance. In other words, the electric load is seen by the acoustic disturbance as additional mechanical impedance (combination of resistances, masses and compliances), thus paving the way to a totally sensorless manner to affect the dynamics of the diaphragm. The following will derive this result for different particular cases.

3 Effects of shunt electric networks on the acoustic properties of electroacoustic absorbers

In the following simulations (and in the following experimental measurement), we will consider different examples of shunt electric networks on a Visaton AL170 low-midrange loudspeaker (Thiele-Small parameters given in Table 1), with an enclosure of $V_b = 101$. At 20°C in the air, $\rho = 1.18$ kg.m⁻³ and c=343 m.s⁻¹. The different shunt configurations are listed in Table 2.

Table 1: Thiele-Small parameters of the Visaton AL 170low-midrange loudspeaker.

Parameter	Notation	Value	Unit
dc resistance	R_e	5.6	Ω
Voice coil inductance	L_e	0.9	mH
Force factor	Bl	6.9	$N A^{-1}$
Moving mass	M_{ms}	13	g
Mechanical resistance	R_{ms}	0.8	$N m^{-1} s$
Mechanical compliance	C_{ms}	1.2	$\rm mmN^{-1}$
Effective area	S	133	cm ²
Natural frequency	f_0	38	Hz

Table 2: Parameter settings for the simulations and experiments with electrical matching networks.

Topology	Case	R_s	L_s	C_s
Open circuit	A	∞	-	-
R shunt	F	4.7 Ω	-	-
Parallel RC	G	34.8 Ω	-	550 <i>µ</i> F
Parallel RL	Н	55.0 Ω	6.5 mH	-
	I	1.5 Ω	15 mH	177 μF
Series RLC	J	1.5 Ω	8.3 mH	406 µF
	K	1.0 Ω	5.5 mH	550 µF

3.1 Shunt resistor

The simplest manner to turn the electroacoustic loudspeaker into a sound absorber is to plug a passive electrical resistance R_s at its terminals [4, 2], thus leading to the normalized acoustic impedance

$$Y(\omega) = \rho c S \frac{j\omega}{(j\omega)^2 M_{ms} + j\omega \left(R_{ms} + \frac{(Bl)^2}{R_e + R_s}\right) + \frac{1}{j\omega C_{mc}}}$$
(5)

It is then obvious that the electric resistance loading the loudspeaker terminals, affects the loudspeaker diaphragm dynamics as it is seen as an additional mechanical resistance $\frac{(Bl)^2}{R_e+R_s}$ damping even more the velocity of the diaphragm when a exogenous pressure force is exerted on its front face. If $R_{ms} < \rho cS$ and $R_e < \frac{(Bl)^2}{\rho cS}$ there even exists an optimal electric load R_{opt} , for which the total resistance of the diaphragm equals the value ρcS (characteristic impedance of the medium converted as a mechanical impedance for the diaphragm of surface *S*):

$$R_{opt} = \frac{(BI)^2}{\rho c S - R_{ms}} - R_e \tag{6}$$

If the loudspeaker is loaded with such shunt resistor value, the diaphragm will be seen as an acoustic impedance surface matching the specific impedance of the air, thus leading to total absorption at the resonance of the loudspeaker defined by $f_s = \frac{1}{2\pi} \frac{1}{\sqrt{M_m C_{mr}}}$:

$$\alpha(f_s) = 1 \tag{7}$$

3.2 Shunt RLC resonators design

The considered electric network topology will only be limited to parallel RC, parallel RL, and series RLC resonators, as illustrated in Fig. 2. In electrical engineering, a network is a collection of components interconnected in series or in parallel. In a series circuit, components are connected along a single electric flow path, so the same current flows through each one [5]. In a parallel circuit, each dipole is connected to the same electrical terminals.

3.2.1 Parallel RC network

The shunt electrical impedance of the parallel RC network of fig. 2(a) that is connected to the transducer terminals



Figure 2: Topology of the different matching network: (a) Parallel RC, (b) Parallel RL, (c) Series RLC. It is assumed that all reactive components are ideal (lossless) and all linear.

can be written as

$$Z_L(\omega) = \frac{R_s}{1 + j\omega R_s C_s} \tag{8}$$

Then the normalized acoustic impedance at the diaphragm can be written as:

$$\frac{V(\omega)}{P(\omega)} = \frac{a_3(j\omega)^3 + a_2(j\omega)^2 + a_1j\omega}{b_4(j\omega)^4 + b_3(j\omega)^3 + b_2(j\omega)^2 + b_1j\omega + b_0}$$
(9)

where

$$a_{3} = SL_{e}R_{s}C_{s}$$

$$a_{2} = S(L_{e} + R_{e}R_{s}C_{s})$$

$$a_{1} = S(R_{e} + R_{s})$$

$$b_{4} = M_{ms}L_{e}R_{s}C_{s}$$

$$b_{3} = M_{ms}L_{e} + (M_{ms}R_{e} + R_{ms}L_{e})R_{s}C_{s}$$

$$b_{2} = M_{ms}R_{e} + R_{ms}L_{e} + (R_{ms}R_{e} + \frac{L_{e}}{C_{mc}} + (Bl)^{2})R_{s}C_{s}$$

$$b_{1} = R_{ms}R_{e} + \frac{L_{e}}{C_{mc}} + (Bl)^{2} + \frac{R_{e}}{C_{mc}}R_{s}C_{s}$$

$$b_{0} = \frac{R_{e}}{C_{mc}}$$
(10)

3.2.2 Parallel RL network

The shunt electrical impedance of the parallel RL network that is connected to the transducer terminals can be written as

$$Z_L(\omega) = \frac{j\omega R_s L_s}{R_s + s L_s} \tag{11}$$

Combining Eq. (11) with the characteristic equations of the loudspeaker yields the following expression for the specific acoustic admittance

$$\frac{V(j\omega)}{P(j\omega)} = \frac{a_3(j\omega)^3 + a_2(j\omega)^2 + a_1j\omega}{b_4(j\omega)^4 + b_3(j\omega)^3 + b_2(j\omega)^2 + b_1j\omega + b_0}$$
(12)

where

$$a_{3} = SL_{e}L_{s}$$

$$a_{2} = S(L_{e}R_{s} + L_{s}R_{e} + L_{s}R_{s})$$

$$a_{1} = SR_{e}R_{s}$$

$$b_{4} = M_{ms}\frac{a_{3}}{S}$$

$$b_{3} = M_{ms}\frac{a_{2}}{S} + R_{ms}\frac{a_{3}}{S}$$

$$b_{2} = M_{ms}\frac{a_{1}}{S} + R_{ms}\frac{a_{2}}{S} + \frac{1}{C_{mc}}\frac{a_{3}}{S} + (Bl)^{2}L_{s}$$

$$b_{1} = R_{ms}\frac{a_{1}}{S} + \frac{1}{C_{mc}}\frac{a_{2}}{S} + (Bl)^{2}R_{s}$$

$$b_{0} = \frac{1}{C_{mc}}\frac{a_{1}}{S}$$
(13)

3.2.3 Discussions

Eqs. (9) and (12) indicate that the loudspeaker, when combined to a parallel RC or RL shunt network, is not anymore a 1 degree of freedom resonator and behaves more as a fourth-order system. The coefficients of Eqs. (10) and (13) show that the connection of a parallel RC or RL network across the transducer terminals affects the mass, compliance and damping of the electroacoustic resonator. More specifically, it can be seen on Fig. 3 that the shunt capacitance C_s is seen as an additional mass $M_{me} = (Bl)^2 C_s$, whereas the shunt inductance $L_e + L_s$ is seen as an additional stiffness $\frac{1}{C_{me}} = \frac{(Bl)^2}{L_s + L_c}$.



Figure 3: Normalized admittance simulated on parallel RL/RC configurations (up: magnitude in dB; bottom: phase in rad).

3.2.4 Series RLC network

An alternative design can be achieved by using a resonant series circuit. The corresponding schematics is shown in Fig. 2(c). Compared to a parallel arrangement of resistive and reactive dipoles, a series RLC network splits the voltages in a frequency-dependent way.

The shunt electrical impedance of the series RLC network that is connected to the transducer terminals can be written as

$$Z_L(\omega) = R_s + j\omega L_s + \frac{1}{j\omega C_s}$$
(14)

Combining Eq. (14) with the characteristic equations (1) of the loudspeaker yields the closed-form expression for the specific acoustic admittance

$$\frac{V(\omega)}{P(\omega)} = \frac{a_3(j\omega)^3 + a_2(j\omega)^2 + a_1(j\omega)}{b_4(j\omega)^4 + b_3(j\omega)^3 + b_2(j\omega)^2 + b_1(j\omega) + b_0}$$
(15)

where

$$a_{3} = S(L_{e} + L_{s})$$

$$a_{2} = S(R_{e} + R_{s})$$

$$a_{1} = \frac{S}{C_{s}}$$

$$b_{4} = M_{ms}\frac{a_{3}}{S_{2}}$$

$$b_{3} = M_{ms}\frac{a_{1}}{S} + R_{ms}\frac{a_{3}}{S_{2}}$$

$$b_{2} = M_{ms}\frac{a_{1}}{S} + R_{ms}\frac{a_{2}}{S} + \frac{a_{3}}{SC_{ms}} + (Bl)^{2}$$

$$b_{1} = R_{ms}\frac{a_{1}}{S} + \frac{a_{2}}{SC_{ms}}$$

$$b_{0} = \frac{1}{C_{mc}}\frac{a_{1}}{S}$$
(16)

When combined to a series RLC network, the loudspeaker becomes a fourth-order system. As for the previous cases, it is not obvious to see the effect of the RLC series network on the dynamic behavior of the loudspeaker.



Figure 4: Normalized admittance simulated on series RLC configurations (up: magnitude in dB; bottom: phase in rad).

However, if we draw the equivalent acoustic circuit corresponding to the loudspeaker connected with a series RLC resonator, an equivalent mechanical impedance can be derived almost easily as [6]:

$$Z_{me}(j\omega) = \frac{(Bl)^2 j\omega}{\left(L_e + L_s\right)(j\omega)^2 + \left(R_e + R_s\right)j\omega + \frac{1}{C_s}}$$
(17)

The natural frequency of this resonant circuit of Eq. (17) is then $\omega_s' = 1/\sqrt{(L_e + L_s)C_s}$.

From Eq. 17, we can derive the asymptotic behaviors of the active mechanical impedance achieved with passive shunt series RLC resonator as:

$$Z_{me}(j\omega) \simeq -\frac{\omega^2 (Bl)^2 C_s}{j\omega} \quad \text{and} \quad \omega \ll \omega_0'$$

$$Z_{me}(j\omega) \simeq \frac{(Bl)^2}{R_e + R_s} \quad \text{and} \quad \omega \approx \omega_0'$$

$$Z_{me}(j\omega) \simeq j\omega \left(-\frac{(Bl)^2}{\omega^2 (L_e + L_s)}\right) \quad \text{and} \quad \omega \gg \omega_0'$$
(18)

It is now almost straightforward to identify from Eq. 18 the corresponding equivalent components obtained from this additional mechanical resonator, that is, the equivalent compliance C_{me} , resistance R_{me} and mass M_{me} as functions of the radial frequency ω :

$$C_{me}(j\omega) \simeq -\frac{1}{\omega^2 (Bl)^2 C_s}$$

$$R_{me} \simeq \frac{(Bl)^2}{R_e + R_s}$$

$$M_{me}(j\omega) \simeq -\frac{(Bl)^2}{\omega^2 (L_e + L_s)}$$
(19)

The equivalent components of the electroacoustic resonator resemble then a constant positive resistance R_{me} in series with a negative mechanical compliance C_{me} and a negative mass M_{me} , which magnitudes decrease with frequency. This results illustrates a simple manner to achieve a mass and stiffness reduction of the electroacoustic absorber without requiring complex active impedance control system.

4 Experimental results

In order to assess experimentally the acoustic performance when using electrical matching networks, a closed-box Visaton AL-170 low-midrange loudspeaker is employed as an electroacoustic resonator. The specific acoustic admittance ratio and absorption coefficient are assessed after ISO 10534-2 standard [7], as depicted in Fig. 9.

In this setup, an impedance tube is specifically designed (length L = 3.4 m and internal diameter $\phi = 150$ mm), one termination of which is closed by an electroacoustic resonator, the other end being open with a horn-shape termination so as to exhibit anechoic conditions [8]. A source loudspeaker is wall-mounted close to this termination. Two holes located at positions $x_1 = 0.46$ m and $x_2 = 0.35$ m from the electroacoustic resonator are the receptacles of 1/2" microphones (Norsonic Type 1225 cartridges mounted on Norsonic Type 1201 amplifier), sensing sound pressure $p_1 = p(x_1)$ and $p_2 = p(x_2)$. The transfer function $H_{12} = p_2/p_1$ is processed through a Pulse Bruel and Kjaer multichannel analyzer.

In this paper, the practical realization of parallel and series RLC resonators is not detailed.

These experimental results then confirm the theory. The connection to parallel RC or RL networks can only present positive mass and stiffness, together with positive mechanical damping, thus providing limited degrees of freedom to tune the electroacoustic absorber properties. On the contrary, it appears quite interesting to plug the loudspeaker to a series RLC network, capable of achieving negative mass and stiffness, to the electroacoustic absorber, as observed with active impedance control devices (see [2], § III.D). The main advantage here is that this negative acoustic properties are obtained without the need of external power source, the shunt series RLC network being a passive device.

5 Conclusion

This study has shown that series RLC network used as shunt for electroacoustic absorbers are good candidates for varying the acoustic properties of the resonator without the



Figure 6: Normalized admittance measured on parallel RL/RC configurations (up: magnitude in dB; bottom: phase in rad).



Figure 7: Normalized admittance measured on series RLC configurations (up: magnitude in dB; bottom: phase in rad).



Figure 8: Absorption coefficients measured on parallel RL/RC configurations.



Figure 5: Picture of the experimental setup.



Figure 9: Absorption coefficients measured on series RLC configurations.

need of active devices. This is an interesting result that could pave the way to very cheap and stable semi-active sound absorbers devices, with broadband absorption capabilities. Further developments are undertaken to ensure practical realization of such passive shunt.

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