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MIMO feedback and application to detection

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The feedback effect is well known but unwanted, by sound engineers. It results from a feedback loop between a microphone and a loudspeaker. Recently, it has been shown that we can take benefit of this effect to estimate with a very good accuracy some parameters such as sound speed. More recently, some experimental results have shown the effect of a local perturbation on the top of an ultrasonic waveguide. Here we generalize the concept to MIMO (Multiple Input Multiple Output) system where the feedback effect occurs between an array of emitters and an array of receivers. We propose to model the MIMO feedback effect by introducing a feedback matrix. Thanks to the singular decomposition of this matrix times the transfert matrix, we are able to predict the spatial dependence of the feedback effect either on the emitting array and on the receiving array. In a second part, we present experimental results that are obtained with an array of about 10 microphones and an array of about 10 loudspeakers. Several feedback matrices have been tested. One of them is inspired from time reversal. We have applied this technique to detect a person that goes across this acoustic barrier.

1 Introduction

Usually sound engineers fight acoustic feedback instability between microphones and loudspeakers because it can lead to coloration and instability of sound. An acoustic feedback instability or a Larsen effect appears when a receiver and an emitter are acoustically and electrically connected: the receiver collects a part of the acoustic signal generated by the source, and this analog signal is then transmitted back to the source input by an electrical connection. It is shown in control theory that this closed-loop system is unstable when the electrical gain compensates the acoustic losses and the loop does not induce a phase shift. Because wave propagation generates time delay, the instability can only occur when the time-delay is a multiple of the wave period. Thus the instability consists of a finite set of resonant frequencies with amplitude that exponentially increases with time. But due to non-linear effect (mainly saturation effect), the amplitude reach a high but finite value.

The Larsen effect produces a set of almost pure tones. Because the quality factor of the resonance is very narrow the Larsen effect is very sensitive to any modifications of the system and especially of the propagation properties of the medium. In 2002, Furduev applied this technique to successfully monitor mean water temperature of a sea shelf[Fur02]. In 2006, Weaver and Lobkis showed that the linewidth of the Larsen's effect in a highly reverberating medium depends on the ratio between the spectral power density of the background noise and the intensity of the howl[WLO6]. With their approach, they monitored the temperature and the effect of the water drop in an aluminum block thanks to an ultrasonic feedback. The detection of target is a more recent concern. Lobkis and Weaver studied the relaxation time of the feedback effect when a brief transient load is applied on a cement bar[LW09]. In 2011, Roux *et al.* applied this principle to detect a target in a shallow water environment at ultrasonic scale[RMLRK11].

However, when the feedback is established between only one emitter and one receiver, the spacial diversity is poor. In other words, the frequency shift of the Larsen effect mainly occurs when a temperature fluctuation or a target is localized on the main acoustic path that relies the emitter to the receiver. The sensitivity to modification outside this main path is weaker. If this property is not embarrassing for global changes such as global temperature modification, it can be a major limitation for detection of local modifications. To mitigate this effect, we propose to establish a Larsen effect between a set of N_{Tx} transmitters and N_{Rx} receivers. Compared to a SISO (Single Input Single Output) configuration, the MIMO (Multiple Input

Multiple Output) would increase the sensitivity because it explores several paths. It exists is a large literature on MIMO control systems. A good overview can be found in [Pos96]. This first works on multichannel feedback comes from studies on active wavefront control with several microphones and several loudspeakers. In these works, the aim is to estimate the maximum gain to avoid instability. Due to the field complexity in room, several stability criteria has been proposed. One interesting approach proposed by Ohsmann[Ohs90] is based on the singular value decomposition of the transfer matrix between the loudspeakers and the microphones. [Pol00]. This approach is closely related to the DORT method, a method resulting from works on time-reversal.

In this proceeding, we propose to introduce the theory of the controlled MIMO feedback effect. This theory is based on the singular value decomposition of the product of the transfer matrix \mathbf{H} times the feedback matrix \mathbf{K} . We show some basic experimental results obtained with a versatile experimental set-up. Then we introduced the concept of time reversal feedback. The article finishes with an application of such a feedback to the detection and the localization of a human being that goes across a MIMO Larsen barrier.

2 SISO and MIMO feedback

The schematic view of a SISO feedback between one microphone and one receiver is shown on Fig. 1a). Now a MIMO feedback implies several microphones and receivers (Fig. 1c). When all the parts of the system are linear and time invariant, the SISO and MIMO feedback can be decomposed into a set of linear operations shown on Fig. 1b and Fig. 1d, respectively. In SISO configuration (Fig. 1b), the gain introduced by the feedback loop between Y (at the loudspeaker) and X (external signal at the microphone) is given by

$$G(s) \equiv \frac{Y(s)}{X(s)} = \frac{K(s)}{1 - H(s)K(s)}, \quad (1)$$

where H is the Laplace transform of the impulse transfer function between the loudspeaker and the microphone and K is the feedback impulse response. It includes the effect of the propagation but also the electro-acoustic responses of the microphone and the loudspeaker. The system is unstable when $H(s)K(s) = 1$ for $\Re(s) < 0$. This results can be easily generalized to MIMO configuration. Lets consider N_{Tx} emitters and N_{Rx} receivers, the gain is now a N_{Tx} by N_{Rx} element matrix whose expression is given by

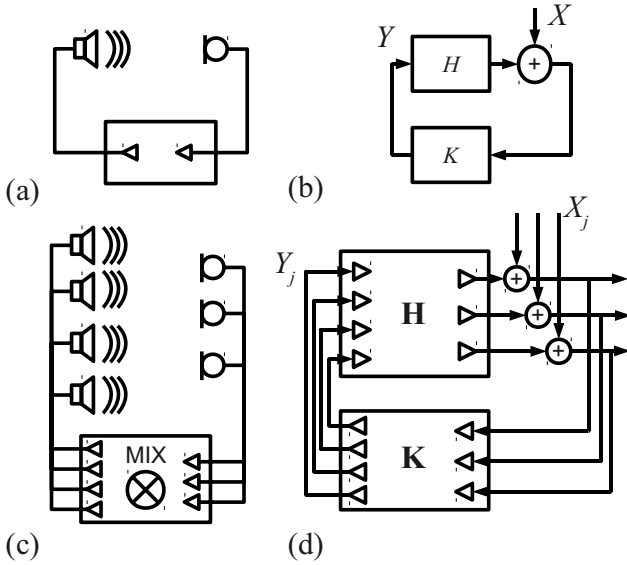


Figure 1: General SISO (Fig. a) and MIMO (Fig. c) feedback schematic representation. When the system is linear, the frequency representation of the SISO and MIMO feedback are represented in Figs. (b) and (d), respectively.

$$\mathbf{G} = (\mathbf{I} - \mathbf{KH})^{-1} \mathbf{K}. \quad (2)$$

The singular value decomposition of $\mathbf{I} - \mathbf{KH}$ yields a simplified expression of the gain

$$\mathbf{G} = \left(\sum_{n=1}^{\min(N_{Tx}, N_{Rx})} \mathbf{U}_n \frac{1}{\lambda_n} \mathbf{V}_n^H \right) \mathbf{K}. \quad (3)$$

Now, the system is unstable if at least one eigen-value of \mathbf{KH} $\lambda_n(s)$ is equal to 0 for $\text{Re}(s) < 0$.

Before saturation, the excitation signal at the loudspeakers is a sum of vectors that grows exponentially with time $\mathbf{U}_n e^{i\omega_L t} e^{-\gamma_L t}$, where $\gamma_L = \Re(s_L)$, $\omega_L = \text{Im}(s_L)$. The complex values of s_L are given by $\lambda_n(s_L) = 0$. Now, when the system is just at the instability threshold it exists only one value such as $\lambda_n(s_L) = 0$ and $\gamma_L = 0$. In such a case, the unstable vector is simply an harmonic function $\mathbf{U}_n e^{i\omega_L t}$. Because $\lambda_n(s_L) = 0$, the resonant angular frequency is given by $\lambda(i\omega)^1$.

While the transfer function \mathbf{H} is imposed by the physical system, the choice of feedback matrix is only limited by the electronic. The choice of the feedback matrix has a huge impact on the Larsen effect. To begin with, we start with a very simple system : a 2 by 2 MIMO feedback in free space. We write the transfer matrix \mathbf{H} as

$$\mathbf{H}(s) = \begin{pmatrix} e^{-\frac{sR_{11}}{c}} & 0 \\ 0 & e^{-\frac{sR_{22}}{c}} \end{pmatrix}.$$

In that case, we assume that the loudspeaker 1 and microphone 1 are completely independent of the loudspeaker 2 and microphone 2. We present 2 characteristic choices for \mathbf{K} . First, the feedback matrix is equal to the eye matrix time a gain scalar g . In such a case, we have 2 completely independent Larsen effects with resonant frequencies are a

¹The frequency expression of the transfer function at angular frequency ω is $F(i\omega)$ where F is its Laplace transform.

multiple of c/R_{11} and c/R_{22} . Like for SISO configuration, the instability appears when $g = 1$. Second, \mathbf{K} is given by an anti-diagonal matrix times g . From a physical point of view, it is like we connect 1 to 2, then 2 to 1. The poles of (2) are deduced from $\det(\mathbf{I} - \mathbf{KH}) = 0$. The unstable frequencies appears when $1 - e^{\frac{i\omega[R_{22}+R_{11}]}{c}} = 0$. As expected, the same equation would be obtained in SISO configuration with a delay time of $R_{22}/c + R_{11}/c$.

In case of a more realistic matrix, the transfer matrix is given by

$$\mathbf{H}(s) = \begin{pmatrix} e^{-\frac{sR_{11}}{c}} & e^{-\frac{sR_{12}}{c}} \\ e^{-\frac{sR_{21}}{c}} & e^{-\frac{sR_{22}}{c}} \end{pmatrix}. \quad (4)$$

Again, for simplicity the amplitude attenuation is not considered. Moreover we assume that the 2 loudspeakers and the 2 microphones are on the 4 corners of a rectangle. ($R_{11} = R_{22}$ and $R_{21} = R_{12}$). Two characteristic equations are worked out from $\det(\mathbf{I} - \mathbf{KH}) = 0$:

$$\begin{aligned} 1 &= 2ge^{\frac{i\omega(R_{11}+R_{12})}{2}} \cos\left(\frac{\omega[R_{21}-R_{11}]}{2}\right) \\ 1 &= 2gie^{\frac{i\omega(R_{11}+R_{12})}{2}} \sin\left(\frac{\omega[R_{21}-R_{11}]}{2}\right) \end{aligned}$$

First, we note that the gain g should be at least larger than 1/2 to observe a feedback effect. The condition to observe the feedback is now much more complex. The resonant frequencies are solution of a transcendent equation. Contrary to the two previous case, there are no periodicity. To illustrate the result of the MIMO feedback, we have computed the spectrum of the signal when the system is excited by a wideband noise source. To avoid saturation, we have adjusted the gain g just below the instability. We compare the spectrum to the resonances deduced from the poles of (2).

3 Experiments

The Larsen effect is established between one linear array made of 8 loudspeakers and one linear array made of 9 microphones. The two arrays are 1.5m away from each other. The feedback is realized thanks a digital electronic with 24 analog inputs/outputs with 24-bit, 96kHz converters. The converted inputs are continuously transferred to a computer memory. The inputs are processed on the fly and sent back to the emitter. Different processing can be performed

- apply a matrix of gain or an attenuation between the microphones and the loudspeaker
- apply a matrix of delay between the microphones and the loudspeaker
- apply a numerical pass-band filter

The first experiment is very simple, we apply a feedback between one input and one output of the electronic (see Fig 3). The feedback gain is above the instability criterion. A saturation effect appears at about 2.5s.

In Fig. 3, we have also plotted the spectrogram. We clearly observe resonances frequencies, but that are not

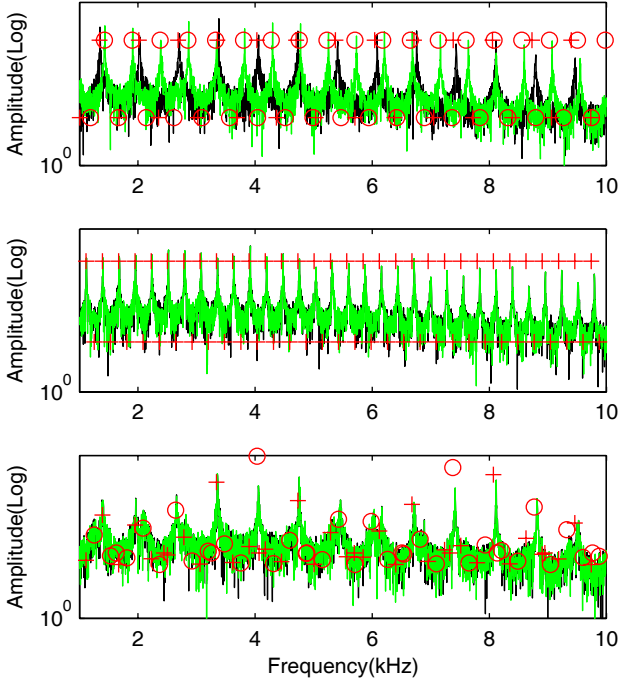


Figure 2: Feedback spectra of channel 1 (black) and channel 2 (green). The cross and circle shows the resonance frequencies deduced from the characteristic equations. The top subplot is obtained in a 2xSISO system. The middle subplot results from the anti-diagonal matrix feedback. The bottom subplot is due to a eye feedback matrix applied on transfer matrix given by (4).

equally spaced as shown in the previous part. Indeed, in a furnished room, the impulse response is complex with many echoes. Consequently a time shift model is not sufficient to describe the Larsen spectrum in a complex environment. On Fig. 4 is plotted the 2 feedback signals and their spectrogram for a 2 by 2 MIMO system.

4 Time-Reversal feedback

Now for detection purpose we are going to mix time-reversal with feedback. To that end, the feedback matrix is built from the transfer function,

$$\mathbf{K} = \beta \mathbf{H}^H \quad (5)$$

A time reversal feedback means that \mathbf{KH} is now a hermitian operator. The SVD is equivalent to an eigenvalue decomposition.

$$\mathbf{G} = \left(\sum_{n=1}^{\min(N_{Tx}, N_{Rx})} \mathbf{U}_n \frac{1}{1 - \lambda'_n} \mathbf{U}_n^H \right) \mathbf{K}, \quad (6)$$

Consequently, feedback instability occurs when $\lambda_n = 1$. There is not phase condition. The feedback compensates the phase shifting due to propagation. However, because the transfer matrix is causal, the feedback is anticausal. To restore causality, ones has to shift in time the time reversed solution, i.e., to $\mathbf{K} \alpha \mathbf{H}^H e^{j\omega\tau}$ where τ is a sufficiently long time to include all the echoes of the impulse responses. In such a case, the stability equation is $1 - \lambda_n \exp(i\omega_n\tau) = 0$, which is identical to the free space one channel criterion. However, this feedback is very difficult to implement experimentally

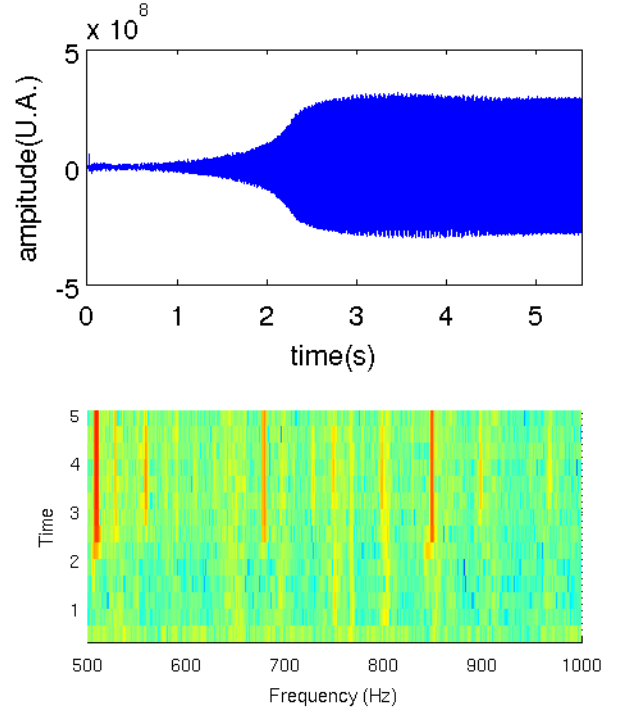


Figure 3: SISO feedback in a furnished room. Top figure: Feedback signal. Bottom : Spectrogram of the feedback signal.

because it requires a huge numerical effort. Nevertheless, we have simulated the feedback from the measurement of the impulse responses between two microphones and two loudspeakers. The result is shown on Fig. 5. We have time reversed only the first 5.3ms of the signal. We observe 3 equally spaced resonances. To restore causality, we have introduced a 5.3ms time shift. It results that the resonance frequencies a 170Hz distant.

5 Detection and localization

By using the time reversed transfer matrix, we also take benefit of the focusing properties of time reversal. This property can be applied for detection a target that goes across the acoustic barrier that results from the Larsen effect. Target detection is a tedious problem because the receivers are completely dazzled by the emitter. The detection of a weak target is therefore difficult. As proposed by Roux et al. [RMLRK11], one solution is to take advantage of the high sensitivity of the Larsen effect. The target signature on the transfer matrix is sufficiently large to modify the Larsen effect. To quantify this effect [RMLRK11] proposed to estimate continuously the feedback gain where the instability threshold occurs. Most of the time, a target increases the transmission losses and therefore the feedback gain has to be increased to reach the instability threshold. To estimate this gain, Roux et al. test the feedback effect for different gain and see where the instability occurs. Here, we propose a new way to estimate almost instantaneously this gain. We continuously adjust the global amplitude of the feedback matrix \mathbf{K} by the way of a factor β to always emits the feedback signal with the same maximum amplitude. First we continuously set the total gain close to 1 at the instability threshold. Second, we can choose the acoustic

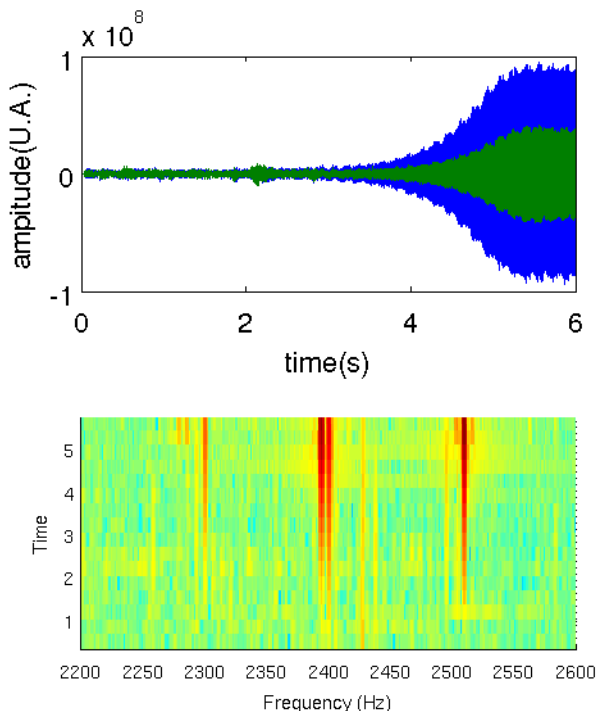


Figure 4: 2 by 2 MIMO feedback in a furnished room. Top figure: The two feedback signals. Bottom : Spectrogram of the first feedback signal.

level of the Larsen effect. By this way, we can stay in a regime where the electronic is not saturated.

As said in the previous section, we cannot implement a full time reversal feedback because it requires a huge numerical effort. However we can overcome this difficulty because we work at the threshold regime. In such a regime, a feedback signal is only composed one frequency. So we continuously compute the frequency resonance and its complex amplitude. From the Fourier transform of the transfer matrix (acquired before), we immediately deduce the amplitude and the phase to apply in order to perform phase conjugation.

We apply this result to detect a human being that goes through this time reversal barrier.

We have done this experiment in a MIMO configuration. The feedback is established between 6 loudspeakers and 2 microphones that are sufficiently away from each others to limit cross-talking. We clearly observe on Fig. 6, the focusing effect on the two microphones. Now, at the 9th displacement, the target is in front of microphone #8. Then at position 17, the target is in front of microphone #2. Finally, around position 30, the target pass in front of the 6 loudspeakers. First we observe that even if one microphone is hidden the Larsen feedback seem still focuses on the other one. When, the human being is in front of the loudspeaker, the wave still focus on the 8th microphone which is the microphone where the amplitude is the strongest. By this way, we can localize the target.

6 Conclusion

Here, we have presented some original works on the MIMO Larsen. In a first part, we have shown some basic properties of MIMO Larsen for different feedback

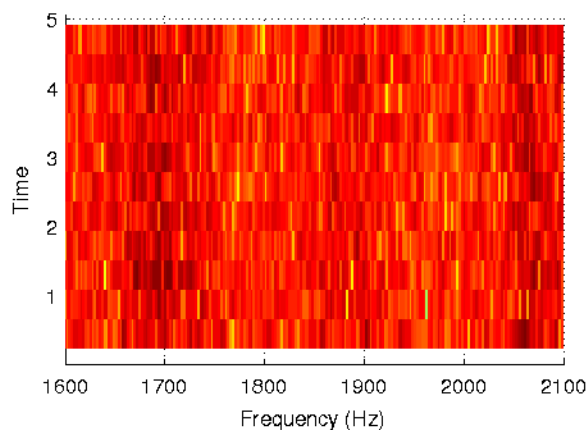


Figure 5: Spectrogram of the feedback signal of a 2x2 MIMO system above the threshold instability level. The feedback matrix is the time reversal of the transfer matrix. The feedback signal has been computed numerically from the experimental acquisition of the transfer function.

matrices. Especially, we have also shown the huge impact of the feedback on the Larsen effect. In second part, we have presented some experimental results obtained with a versatile and powerfull set-up that allows to perform complex real time processing. After two basic feedback experiments, we have introduced the concept of time reversal

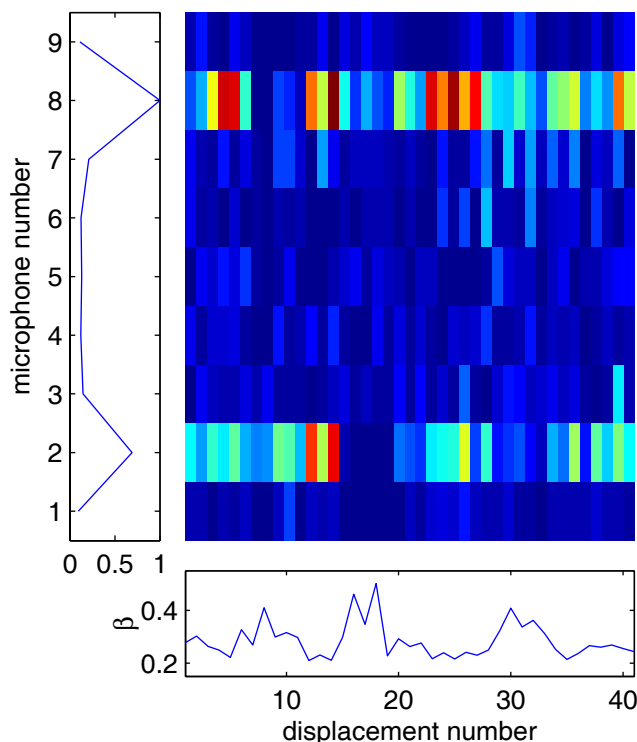


Figure 6: Detection of a human being target with a time reversal feedback between 6 loudspeakers and two microphones (positions #2 and #8). The color map represent the acoustic level on one of 9 microphones (vertical axis) for different position of the human being (horizontal axis). The vertical plot on the left is the average level value at each microphone for all the target positions. The horizontal plot is the β factor for the different positions to reach the instability threshold.

feedback. We have shown that this feedback implies that the Larsen frequencies are equally spaced even in a complex environment. Then we have presented a first demonstration of a MIMO acoustic barrier. The real-time detection of the target is obtained by measuring the feedback gain that have to be applied in order to just compensate the losses. The localization of the target is obtained thanks to the time reversal feedback matrix.

The next steps will consist to apply these results with different feedback matrix, e.g., cyclic matrix in order to explore all the area of the acoustic barrier. Also we can imagine to generate simultaneously several Larsen effects at several frequencies in order to increase the sensitivity and the precision on localization.

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