

The effect of wall vibrations on the air column inside trumpet bells

V. Chatziioannou^a, W. Kausel^a and T. Moore^b

^aInstitute of Musical Acoustics, University of Music and Performing Arts Vienna, Anton von Webern Platz 1, 1030 Vienna, Austria

^bDepartment of Physics, Rollins College, Winter Park, Orlando, Florida, Orlando, 32789, USA chatziioannou@mdw.ac.at

The effect of wall vibrations on the sound of brass wind instruments is reported. There is experimental evidence showing that damping the vibrations of a trumpet bell alters the input impedance and transfer function of the instrument. Numerical simulations suggest that it is the axi-symmetric oscillations of the walls of the instrument that are responsible for the observed effects. Indeed, a finite element analysis of the effects of such oscillations on the air column yields results qualitatively similar to those of experiments. However, due to the complexity of the geometry and boundary conditions of a real instrument, the results of simulations do not agree quantitatively with the measured influence of the wall vibrations on the radiated sound from an actual trumpet. For this reason, straight trumpet bells have been manufactured that are expected to have boundary conditions more consistent with the assumptions of the numerical model. The results of both experimental measurements and numerical simulations of these custom-made bells are presented in this study.

1 Introduction

Wave propagation inside a wind instrument has been extensively studied and successfully modelled in the past decades (see for example [1, 2, 3]). The walls of the instrument bore are usually considered as perfectly rigid in such models. However, this is not the case under playing conditions of brass wind instruments, with players claiming that wall vibrations can affect the behaviour of an instrument even though the amplitude is extremely small compared to the dimensions of the bell. This leads to a debate concerning whether such small oscillations can significantly influence the sound radiated from wind instruments. Several researchers have recently dealt with this topic [4, 5, 6, 7] and experiments have shown that the effect of wall vibrations should not be neglected [8, 9].

Apart from the mechanical feedback to the player's lips, there is a coupling between the vibrating walls and the air column inside the instrument, that can affect its input impedance [5]. In fact, it has been shown that even in the absence of mechanical feedback, when the vibrations in the air column are excited by electronic means, differences in the transfer function of the instrument persist [9].

An extensive structural analysis of trumpet bells has been reported in [10], where a simplified mass-spring model was presented that was able to predict the vibrational behaviour of a trumpet taking only axi-symmetric oscillations into account. The results of this model were tested against a finite element study executed using the structural mechanics module of the COMSOL software¹. The performance of the simplified model compared well to the finite element model, rendering it useful for predicting the oscillations of the walls of wind instruments. Furthermore, it has been shown how altering the material properties and the wall thickness of an instrument, as well as the positions of bends, braces and additional masses due to valves or keys can alter the vibrational behaviour of the system.

The long-term objective of this study is to incorporate this model into an algorithm that can calculate the input impedance and transfer function of wind instruments [11], thus taking into account the wall vibrations, when necessary. To achieve this it remains to establish (1) whether it is sufficient to consider only axisymmetric oscillations and (2) what is the coupling mechanism between the vibrating walls and the air column inside the instrument.

Before the formulation of a simplified algorithm that captures the interaction between the walls and the air column it is necessary to obtain a benchmark against which the model will be tested. Therefore, besides performing measurements on trumpet bells that are free to vibrate or damped using sandbags [9], a finite element model was also developed in order to deepen our understanding in the underlying physical phenomena. The implementation of this model was carried out in COMSOL and preliminary results have shown much smaller differences than those predicted by the measurements when damping the wall vibrations [12].

Since it is not easy, due to the complex geometry of a trumpet, to estimate why the model fails to predict the experimental data, a pair of straight trumpet bells have been constructed. The thickness and borelist of these bells were given directly by the manufacturer. It is expected that these bells will be more consistent with the assumptions needed to be taken during the formulation of a numerical model. In fact many uncertainties concerning the locations of bends, braces and additional masses can be eliminated. The experiments and modelling attempts presented in this paper are carried out on these straight trumpet bells.

2 Mechanical modes of vibration

In general, elliptic modes of a trumpet bell oscillate with larger amplitudes than the axial modes. However, these high-Q elliptic modes can only be excited at a certain frequency and therefore can not explain wideband effects due to wall vibrations (as the ones reported in [9]). On the other hand, vibrations along the axis of the bell appear to have an effect over a wider frequency range and, even though their amplitude is significantly smaller, the effective modulation of the cross-sectional area of the air column due to axial vibrations is amplified at the rapidly-flaring region of brass instrument bells.

A three-dimensional finite element simulation of a trumpet bell, that can capture both axial and elliptic modes can be used to demonstrate the limited effect of the latter. The bell was stimulated at the mouthpiece end by a sinusoidal force normal to the mouthpiece plane, plus a small perturbation vertical to the applied force in order to brake the symmetry of the model and ensure that elliptic modes are excited. The displacement at the rim of the bell is plotted in Figure 1 and compared to that calculated by a two-dimensional axisymmetric simulation; it can be observed that the assumption of symmetry along the axis only fails to predict the bell displacement at a limited number of frequencies, corresponding to the elliptic modes of the bell. Furthermore, these modes can be excited under playing conditions only if their resonance frequency is close to the location of an impedance peak of the instrument. In that case they can indeed affect the behaviour of the instrument, but such an interaction can still not explain the wideband differences observed when damping the wall vibrations. Therefore the attempts to model the

¹COMSOL Multiphysics ®



Figure 1: Maximum bell displacement over frequency stimulated by a force applied at the mouthpiece, as calculated using a 2d axisymmetric and a 3d model.

effects of the interaction between the vibrating walls and the air column are confined to studying the axial modes of the instrument oscillations.

Regardless of their significance in this study, elliptic modes can still be used to validate the material properties of the bells. Using a constant bell thickness and standard values for brass density (8400 kg/m³) Young's modulus (110 GPa) and Poisson ratio (0.35), the location of certain elliptical modes was estimated by the model quite close to the frequencies obtained using electronic speckle pattern interferometry (see Figure 2) as discussed in [13]. Discrepancies are still expected due to a non-perfectly circular cross-section of the instrument bell, as well as variations in the thickness of the walls, that is assumed constant in the model. In fact, the thickness of the straight bells, which was given as a constant by the manufacturer, has been measured using a Magna Mika ®8500 thickness gauge; variations of up to 17% were observed along the circumference of the bell and up to 12%along its axis. Such variations are actually expected due to the way trumpet bells are manufactured.



Figure 2: Elliptic modes of the vibration of the trumpet bell, as calculated from a 3D simulation (left) and observed using electronic speckle pattern interferometry (right).



Figure 3: Experimental setup to measure the vibrations of the straight bell.

3 Experiments

The first experiments carried out on the straight trumpet bells were of a structural nature. The bells were suspended at two points (corresponding to the positions of the trumpet braces) 20 and 40 cm away from the rim plane and stimulated at the mouthpiece end by a shaker using a sinusoidal acceleration (see Figure 3). Twelve accelerometers were positioned along the circumference of the rim of the bell and their average is considered, in order to eliminate the effect of elliptical modes. The experiment was carried out twice, inside an anechoic chamber and the results are depicted in Figure 4. The plotted frequency response function is the ratio between the average of the readings for the rim displacement over the mouthpiece displacement. The dashed curve shows the same curve as calculated using the simplified mass-spring model. An extra mass was added at the mouthpiece in the model, corresponding to the mass of the shaker, resulting in a different resonance behaviour compared to that of Figure 1.



Figure 4: Frequency response function (average rim displacement relative to mouthpiece displacement) measured on a straight bell and calculated using the mass-spring model presented in [10].

To measure the effect of the vibrating walls on the behaviour of the instrument, the straight bells were fit with a trumpet mouthpiece and the air column was stimulated using a horn driver. To record the input signal, the driver was connected to the mouthpiece by an adapter that held a microphone between the speaker and the mouthpiece. Another microphone was placed approximately 1 m away from the bell on the center axis to record the output signal, so that the transfer function of the instrument could be calculated. A baffle was placed at the plane of the bell to ensure that no direct sound reached the output microphone from the driving speaker, and the walls and the floor of the room were covered with anechoic foam.

To ensure a precise measurement of the transfer function, a sinusoidal signal lasting 1 s was used to drive the horn driver at each frequency of interest. The frequency of the signal was varied from 100 Hz - 2 kHz in increments of 1 Hz. The signal from the input and output microphones were recorded and the mean amplitude at the driving frequency was calculated in real time for both. The amplitude at the driving frequency was then used to determine the transfer function before changing the frequency of the driving signal. The long sample time allowed for precise measurements as well as ensuring that the bell vibrations had adequate time to reach steady-state at each frequency.

The transfer function was measured for both the case of a free bell and when the vibrations were heavily damped. Damping of the vibrations of the instrument was accomplished by placing sandbags around the bell. Although it is believed that complete damping of the oscillations of the wall was not achieved, the vibrations were significantly reduced and the damped case is henceforth compared to a model with perfectly rigid walls. The resulting transfer function in both cases, as well as the difference between the transfer functions are depicted in Figure 5. Clearly, differences exist between the two curves and they are qualitatively comparable to those measured previously on a complete trumpet [9]. Before attempting to explain the cause of those differences, numerical simulations are used to investigate the wall vibration mechanism.



Figure 5: (top) Transfer function of a straight bell measured 1 m away from the bell as a function of frequency, for the case of a free vibrating bell (blue) and a heavily damped bell (red dashed). (bottom) Difference in the transfer functions caused by damping the bell.

4 Modelling wall vibrations

The finite element model of the bell was implemented in COMSOL using the thermocaoustic-solid interaction module. Such a model can predict the formation of a viscous boundary layer at the walls of the tube, as well as the interaction between the vibrating walls and the air column inside the instrument. The problem is solved in a two-dimensional axisymmetric geometry, which approximately reproduces the conditions of the experiments (see Figure 6). An absorbing perfectly matched layer is used to avoid wall reflections back to the instrument and an extra mass is added at the mouthpiece end to represent the mounting device.



Figure 6: Acoustic pressure field of a straight trumpet bell at 440 Hz, modelled using a two-dimensional axisymmetric geometry.

The vibrations of the system are excited by a vibrating piston at the mouthpiece plane, producing a volume flow with amplitude $U=23 \text{ cm}^3/\text{s}$ at all frequencies (corresponding to an amplitude of 2700 Pa for the mouthpiece pressure at the first impedance peak of the instrument at 191 Hz). The transfer function of the instrument is calculated by dividing the pressure magnitude measured 1 m away from the bell with the pressure magnitude at the mouthpiece. Running the simulation twice, once with the instrument free to vibrate and once with its walls being completely fixed, resulted in very small differences in the transfer function, compared to those expected from the experiments.

Analysing the model reveals that the interaction between vibrating walls and air column is confined to the following: the velocity of the air at the walls of the tube is equated to that of the wall velocity (fulfilling a no-slip boundary condition) and the internal tube pressure is applied as a boundary load to the wall. However, as explained in [9], there is a further effect due to the volume oscillations caused by the vibrating walls.

4.1 Thermodynamic pressure modulation

The ideal gas equation, during an adiabatic process, is given by

$$p(t)V^{\gamma}(t) = n(t)RT, \qquad (1)$$

where the pressure p(t), volume V(t) and number of moles n(t) vary with time, T being the temperature, γ the heat ca-

pacity ratio and *R* the universal gas constant. Thus volume oscillations due to the wall vibrations with an amplitude \hat{V} have to be included in the pressure variation. The formulation of this effect has been carried out in [9], assuming isothermal conditions. Here adiabatic conditions are considered and the effective time varying pressure deviation (p_+) from the equilibrium pressure p_0 can be calculated using a Taylor expansion and neglecting second order terms:

$$p_{+}(t) = \hat{p}e^{i\omega t} - \gamma \frac{p_0}{V_0} \hat{V}e^{i\theta}e^{i\omega t}, \qquad (2)$$

where V_0 is the equilibrium volume of the air column, \hat{p} the oscillating pressure magnitude and θ the phase difference between the internal pressure and the wall oscillations. Hence the extra pressure amplitude due to the wall oscillations is given by

$$\widehat{p_V} = \gamma \frac{p_0}{V_0} \hat{V} e^{i\theta} = \gamma \frac{p_0}{\pi r^2 z} (2\pi r z \hat{s}) e^{i\theta} = \frac{2\gamma p_0}{r} \hat{s} e^{i\theta}, \quad (3)$$

where r is the bore radius, z the length along the bell axis and \hat{s} the amplitude of the radial vibration of the air column.

4.2 Results

Superimposing this extra pressure on the one caused by the piston excitation enhances the effect of the wall vibrations on the transfer function of the bell, bringing it to comparable levels to that of the experimental measurements. Figure 7 shows the transfer function of the bell, calculated 1 m away from the bell, as well as the difference of the two curves.



Figure 7: (top) Simulated transfer function of a straight bell calculated 1 m away from the bell as a function of frequency, for the case of a free vibrating bell (blue) and a fixed bell (red dashed). (bottom) Difference in the transfer functions caused by fixing the bell.

It can be observed that fixing the bell in the numerical simulations and damping it in the experiments produce similar differences to the transfer function of the instrument. In particular it appears that around the first (axial) structural resonance of the bell the effect is maximised. The bell displacement at these frequencies has a significant amplitude in order to modulate the pressure inside the tube. As depicted in Figure 8, the displacement caused by a vibrating piston excitation of the air column is a superposition of the input impedance curve and the displacement curve caused by a mechanical excitation at the mouthpiece.



Figure 8: Input impedance of the free vibrating bell (top) and rim displacement caused by a vibrating piston excitation of the air column (bottom). The dashed curve shows the mechanical resonance of the bell (rim displacement caused by a structural acceleration at the mouthpiece).

There is still a significant difference between the measured differences shown in Figure 5 and the calculated ones, plotted in Figure 7. In particular, there is a large deviation at the low frequency range in the case of the measured data. Such a difference can only be explained by a full-body motion of the bell at low frequencies, possibly including vibrations of the mounting device, excited directly by the driver. Therefore impedance measurements have been carried out, replacing the horn driver with a BIAS impedance head [14]. The rest of the experimental conditions were the same as in the case of the transfer function measurements. The difference in the input impedance for the damped and the free vibrating walls are plotted in Figure 9 and compared to the differences calculated from the finite element model.

The calculated differences have the same order of magnitude with the measured ones, as well as a similar distribution along the frequency axis. It is clear from this plot that damping the wall vibrations is more significant at the third and fourth impedance peaks of the instrument. Such a behaviour is expected if the differences are attributed to the axial vibrations of the bell. As shown in Figure 8, the magnitude of the bell displacement retains a larger value in a frequency range around the axial resonance of the instrument, wherein these two impedance peaks are located.



Figure 9: Difference in the input impedance of the bell caused by the wall vibrations measured experimentally (top) and calculated numerically (bottom). The dashed lines indicate the locations of the impedance peaks.

5 Conclusions

It has been demonstrated, both by experimental measurements as well as using numerical simulations, that the effect of the wall vibrations can significantly alter the input impedance and transfer function of a brass wind instrument. Using straight trumpet bells in order to simplify the boundary conditions of the problem, the measured effect of the wall vibrations could be predicted by physical modelling. The fact that the problem was represented using a two-dimensional axisymmetric geometry, indicates that the axial vibrations of a trumpet bell are sufficient to explain the observed effects.

On the basis of this theoretical explanation of the effect of wall vibrations it is now possible to simulate the wave propagation inside tubes with vibrating walls. In order to model complete wind instruments it is necessary to carefully examine their structural behaviour along the bell axis, and how this is affected by the construction details of each instrument.

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