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Experimental identification of a prior tensor-valued random field for the elasticity properties of cortical bones using in vivo ultrasonic measurements

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The biomechanical materials are among the most complex mechanical systems. Most often, their micro-structure are complex and random. This is the case for the human cortical bones which are considered in this paper. For such a system, the micro-structure can be altered near its interface with the marrow (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none of the usual theories of porous materials can be applied. For this reason, we present a simplified model with gradient for the elasticity tensor. The elasticity tensor is modelled by a random field. In this paper, the parameters of this probabilistic model are identified with experimental observations in ultrasonic range.

1 Introduction

A cortical bone layer is a biomechanical system that is difficult to model in regard to the complexity level of its microstructure. The experimental identification of its effective mechanical properties at the macroscale is usually carried out using the axial transmission technique which is often modeled with a simplified mean mechanical model. In this paper, the simplified mean mechanical model is a fluid-solid semi-infinite multilayer system (skin and muscles/cortical layer/marrow). It is also assumed that the effective elasticity properties of the solid layer (cortical bone) have spatial variations in the thickness (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none of the usual theories of porous materials can be applied. For these reasons, these systems are often modelled using a simplified mechanical model which corresponds to a rough approximation of the real system. The uncertainties introduced in the construction of this simplified mean model are taken into account with an a priori probabilistic model in which the elasticity tensor is a non-homogeneous and non-Gaussian tensor-valued random field which has been constructed by C. Soize using the information theory and the maximum entropy principle. The parameters of this probabilistic model are (1) the mean value of the effective thickness and the mean value of the elasticity tensor of the cortical bone and (2) the parameters controlling the level of uncertainties which depends on the spatial coordinates. A method and an application are presented for the identification of these parameters using in vivo experimental measurements in ultrasonic range with the axial transmission technique.

2 Simplified model

The properties of the human cortical bone are studied by using *in vivo* measurements obtained with the axial transmission technique: an acoustic pulse is applied on the skin layer in the ultrasonic range and the velocity of the first arriving signal is measured. A simplified model of the human cortical bone with the skin, the coupling gel with a probe that applied the acoustic pulse and the marrow has been developed in [8, 5]. This simplifed model is composed of an elastic solid semi-infinite layer between two acoustic fluid semi-infinite layers. Let $\mathbf{R}(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be the reference Cartesian frame where O is the origin of the space and $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is an orthonormal basis for this space. The coordinates of the generic point $\mathbf{x}$ in $\mathbb{R}^3$ are (x_1, x_2, x_3). The thicknesses of the layers are denoted by h_1, h and h_2.

The first acoustic fluid layer occupies the open unbounded domain Ω_1 , the second acoustic fluid layer occupies the open unbounded domain Ω_2 and the elastic solid layer occupies the open unbounded domain Ω. Let $\partial\Omega_1 = \Gamma_1 \cup \Sigma_1$, $\partial\Omega = \Sigma_1 \cup \Sigma_2$ and $\partial\Omega_2 = \Sigma_2 \cup \Gamma_2$ (see Fig. 1) be respectively the boundaries

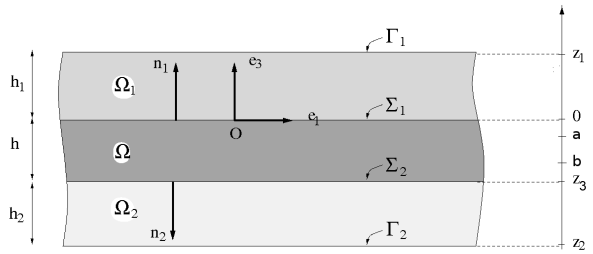

Figure 1: Geometry of the multilayer system

of Ω_1, Ω and Ω_2 in which $\Gamma_1, \Sigma_1, \Sigma_2$ and Γ_2 are the planes defined by

$$\begin{aligned}
\Gamma_1 &= \{x_1 \in \mathbb{R}\,, \quad x_2 \in \mathbb{R}\,, x_3 = z_1\} \\
\Sigma_1 &= \{x_1 \in \mathbb{R}\,, \quad x_2 \in \mathbb{R}\,, x_3 = 0\} \\
\Sigma_2 &= \{x_1 \in \mathbb{R}\,, \quad x_2 \in \mathbb{R}\,, x_3 = z\} \\
\Gamma_2 &= \{x_1 \in \mathbb{R}\,, \quad x_2 \in \mathbb{R}\,, x_3 = z_2\}
\end{aligned}$$

in which $z_1 = h_1$, $z = -h$ and $z_2 = -(h + h_2)$. Therefore, the domains Ω_1, Ω and Ω_2 are unbounded along the transversal directions $\mathbf{e}_1$ and $\mathbf{e}_2$ whereas they are bounded along the vertical direction $\mathbf{e}_3$. A line source modelling an acoustical impulse is applied in domain Ω_1. This line source is defined with a source density Q_1 such that

$$\frac{\partial Q_1}{\partial t}(\mathbf{x}, t) = \rho_1\, F(t)\delta_0(x_1 - x_1^S)\delta_0(x_3 - x_3^S) \quad ,$$

in which $F(t) = F_1 \sin(2\pi f_c t) e^{-4(t\,f_c - 1)^2}$ where $f_c = 1\,\text{MHz}$ is the central frequency and $F_1 = 100\,\text{N}$; ρ_1 is the mass density of domain Ω_1; δ_0 is the Dirac function at the origin and x_1^S and x_3^S are the coordinates of a line source modelling the acoustical impulse. At time $t = 0$, the system is assumed to be at rest. Let $\rho(x_3)$ and $[C(x_3)]$ be the mass density and the effective elasticity matrix of the solid layer at a point x_3 in Ω_1. For a given effective elasticity matrix field $x_3 \mapsto [C(x_3)]$, the displacement field $\mathbf{u}$ in the solid layer Ω and the pressure fields p_1 and p_2 in the two fluids Ω_1 and Ω_2 respectively, are calculated using the fast and efficient hybrid solver presented in [4].

3 Simplified model for a porous medium with gradient

It is well-known that bone medium are made of porous material. However, for the human cortical bones, the pore sizes are not small with respect to the thickness of the cortical layer. In addition, the pore size increases along the transverse direction x_3. In case of osteoporosis, this gradient of porosity is such that, near interface Σ_2, the cortical material is mostly made up of a fluid. No usual theory on porous medium [1, 2, 3] is suitable for modelling such properties.

Hereafter, we then propose an approach that allows the modelling of the elasticity matrix $[C(x_3)]$ to be still constructed within the usual framework of the continuum mechanics. For all x_3 in $[a,0]$, the material in the cortical layer is assumed to be locally an homogeneous transverse isotropic medium and for all x_3 in $[z,b]$ it is assumed to be a fluid. Consequently, (1) for all x_3 in $[0,a]$, we have $[C(x_3)] = [C^S]$ and $\rho(x_3) = \rho^S$; (2) for all x_3 in $[z,b]$ we have $[C^F]$ and $\rho(x_3) = \rho_2$; where $[C^S]$ is the elasticity matrix of a transverse isotropic medium, $[C^F]$ is the elasticity matrix of a fluid medium, ρ^S is the mass density of the cortical layer without taking into account the porosity and ρ_2 is the mass density of the second fluid (the marrow). All components of $[C^S]$ are zeros except the following

$$[C^S]_{11} = \frac{e_L^2(1-\nu_T)}{(e_L - e_L\nu_T - 2e_T\nu_L^2)}$$
$$[C^S]_{12} = \frac{e_T e_L \nu_L}{(e_L - e_L\nu_T - 2e_T\nu_L^2)} ,$$
$$[C^S]_{22} = \frac{e_T(e_L - e_T\nu_L^2)}{(1+\nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} ,$$
$$[C^S]_{23} = \frac{e_T(e_L\nu_T + e_T\nu_L^2)}{(1+\nu_T)(e_L - e_L\nu_T - 2e_T\nu_L^2)} ,$$
$$[C^S]_{44} = g_T \quad , \quad [C^S]_{55} = g_L \quad ,$$

with $[C^S]_{22} = [C^S]_{33}$, $[C^S]_{12} = [C^S]_{13} = [C^S]_{21} = [C^S]_{31}$, $[C^S]_{23} = [C^S]_{32}$ and $[C^S]_{55} = [C^S]_{66}$ and where e_L and e_T are the longitudinal and transversal Young moduli, g_L and g_T are the longitudinal and transversal shear moduli and ν_L and ν_T are the longitudinal and transversal Poison coefficients such that $g_T = e_T/2(1+\nu_T)$. All components of $[C^F]$ are zero except $[C^F]_{11}$, $[C^F]_{12}$, $[C^F]_{13}$, $[C^F]_{21}$, $[C^F]_{22}$, $[C^F]_{23}$, $[C^F]_{31}$, $[C^F]_{32}$,$[C^F]_{33}$ that are all equal to $\rho_2\, c_2^2$. The model of $[C(x_3)]$ and $\rho(x_3)$ is the following

$$[C(x_3)] = (1-f(x_3))\,[C^S] + f(x_3)\,[C^F] \quad ,$$
$$\rho(x_3) = (1-f(x_3))\,\rho^S + f(x_3)\,\rho_2 \quad ,$$

where $f(x_3) = 1$ if $x_3 < b$, $f(x_3) = 1$ if $x_3 > a$ and $f(x_3) = \sum_{k=0}^{4} a_k\, {x_3}^k$ if $b \le x_3 \le a$ in which coefficients a_0, a_1, a_2, a_3, a_4 are such that $f(a) = 0$, $f(b) = 1$ and the derivative of f with respect to x_3 is such that $f'(a) = f'(b) = 0$.

4 Probabilistic model of the thickness and elasticity matrix of the cortical layer

At the mesoscale modeling, the cortical bone constituting the elastic solid layer is a heterogeneous anisotropic material for which the elasticity properties field is modeled by a matrix-valued random field $[\mathbf{C}] = \{[\mathbf{C}(x_3)]\,, x_3 \in [b,0]\}$. The prior probabilistic model of $[\mathbf{C}]$ is chosen in the ensemble of tensor-valued random field adapted to elliptic operator, defined in [10, 11]. This probability model of the uncertain parameters are constructed by using the maximum entropy principle [9, 6, 7]. For all $b \le x_3 \le 0$, $[\mathbf{C}(x_3)]$ is a positive-definite random matrix which is written as

$$[\mathbf{C}(x_3)] = [L(x_3)]^T\,[\mathbf{G}(x_3)]\,[L(x_3)] + [C_0(x_3)] \quad ,$$

in which the deterministic matrix $[C_0(x_3)]$ is positive-definite and the matrix $[\mathbf{G}(x_3)]$ is a positive-definite random matrix; these two matrices are defined below. In the definition of $[\mathbf{C}(x_3)]$, an upperscript T denotes the transpose operator. By construction, one has

$$E\{[\mathbf{C}(x_3)]\} = [C(x_3)] \quad , \quad \forall x_3 \in [b,0] \quad ,$$

in which $[C] = \{[C(x_3)]\,, x_3 \in [b,0]\}$ is the mean value field defined in the previous section and the operator $E\{\cdot\}$ denotes the mathematical expectation. Positive-definite matrix $[C_0(x_3)]$ must be such that, for all x_3 in $[b,0]$, $[C(x_3)] - [C_0(x_3)]$ is positive-definite. The $n \times n$ (with $n = 6$) upper triangular matrix $[L(x_3)]$ corresponds to the Cholesky decomposition of the positive-definite matrix $[C(x_3)] - [C_0(x_3)]$, that is to say $[C(x_3)] - [C_0(x_3)] = [L(x_3)]^T\,[L(x_3)]$. The matrix-valued random field $[\mathbf{G}] = \{[\mathbf{G}(x_3)],\, x_3 \in \mathbb{R}\}$ is defined as a non-linear mapping of 21 independent second-order centered homogeneous Gaussian random fields $U_{jj'} = \{U_{jj'}(x_3),\, x_3 \in \mathbb{R}\}$ with $1 \le j \le j' \le n$ for which the autocorrelation functions $R_{U_{jj'}}(\xi) = E\{U_{jj'}(x_3+\xi)U_{jj'}(x_3)\}$ are all chosen equal to a same unique function $(2\,\ell/\pi\,\xi)^2 \sin^2(\pi\,\xi/2\,\ell)$ depending only on a spatial correlation length denoted by ℓ. The explicit expressions of this non-linear mapping can be found in [10, 11]. Let parameter δ be the dispersion coefficient defined as $\delta^2 = (E\{\|[\mathbf{G}(x_3)]\|_F^2\} - 1)/n$. The probability density function $P_{[\mathbf{G}(x_3)]}$ of random matrix $[\mathbf{G}(x_3)]$ with respect to the measure $d\tilde{A} = 2^{n(n-1)/4} \prod_{1 \le i \le j \le n} [A]_{ij}$ on the set $\mathbb{M}^+$ of the symmetric positive $n \times n$ real matrices is then written as [10, 11]

$$p_{[\mathbf{G}(x_3)]}([A]; x_3) = \mathbb{1}_{\mathbb{M}^+}([A])\, c_n\, (\det[A])^{b_n}\, \exp\{-a_n \mathrm{tr}[A]\} ,$$

in which $a_n = (n+1)/(2\delta^2)$, $b_n = a_n(1-\delta^2)$ where δ is a dispersion coefficient and where c_n is a normalization constant and $\mathbb{I}_{\mathbb{M}^+}([A])$ is equal to 1 if $[A]$ belongs to $\mathbb{M}^+$ and is equal to zero if $[A]$ does not belong to $\mathbb{M}^+$. It can be seen that $p_{[\mathbf{G}(x_3)]}$ does not correspond to the probability density function of a Gaussian random matrix. In addition, the probability density functions $p_{[\mathbf{C}(x_3)]}$ of random matrix ($[\mathbf{C}(x_3)]$ with respect to the measure $d\tilde{A}$ on the set $\mathbb{M}^+$ is written as

$$p_{[\mathbf{C}(x_3)]}([A]; x_3)$$
$$= c_{\mathrm{norm}} \mathbb{1}_{\mathbb{M}^+}([A] - [C_0(x_3)])\, \det([A] - [C_0(x_3)])^{\lambda - 1}$$
$$\times \exp\{-\frac{n-1+2\,\lambda}{2}\, \mathrm{tr}(([C(x_3)] - [C_0(x_3)])^{-1}([A] - [C_0(x_3)])\} ,$$

where $\mathrm{tr}(\cdot)$ is the trace operator; c_{norm} is a normalization constant; λ is a positive real parameters that depends on the statistical fluctuation of random matrices $[\mathbf{C}(x_3)]$. It can be seen that $p_{[\mathbf{C}(x_3)]}$ does not correspond to the probability density function of a Gaussian random matrix.

For the application to the cortical bone, we do not have any information concerning matrix $[C_0(x_3)]$ which is only introduced to preserve the ellipticity property of the stiffness operator. This matrix can be chosen, for x_3 in $[b,0]$, as $[C_0(x_3)] = \eta_0\,[C(x_3)]$ in which $0 < \eta_0 < 1$. In this case, η_0 can be chosen very small if no information concerning $[C_0(x_3)]$ is available.

With such a stochastic modeling, the displacement field of the elastic solid layer and the two acoustic pressure fields of the acoustic fluid layers are random fields denoted by $\mathbf{U}$, P_1 and P_2.

5 Application

In a previous paper [5], the components of matrix $[C^S]$ has been identified with an experimental database using mea-

surement of the first arriving signal with the axial transmission technique. For the experimental configuration, a device has been designed and is made up of $n_R = 14$ receivers and 2 transmitters. A coupling gel is applied at the interface between the device and the skin of the patient. Each transmitter generates an acoustical impulse in the ultrasonic range that propagates in the coupling gel, the skin, the muscle, the cortical bone and the marrow. The axial transmission technique consists in recording these signals at the $n_R = 14$ receivers located in the device. The first arriving contribution of the signal (FAS) is considered. Following the signal processing method used with the experimental device, the velocity of FAS is determined from the time of flight of the first extremum of the contribution. This experimental database allows the components of matrix $[C^S]$ to be identified (see [5]) and we obtained $\rho^S = 1598.8$ kg.m^{-3}, $e_L = 17.717$ GPa, $\nu_L = 0.3816$, $g_L = 4.7950$ GPa, $e_T = 9.8254$ GPa, $\nu_T = 0.4495$ and $\delta = 0.1029$. In this paper, we are interested by the identification of parameter a which represents the thickness of the healthy part of the cortical bone. Let the random variable Q be defined by

$$Q = \int_0^T \sum_{k=1}^{n_R} |P_2(t, x_1^k)|^2 \, dt \quad ,$$

where T is the duration of an experimental signal and x_1^k, with $k = 1, \ldots, n_R$ are the positions of the receivers along direction $\mathbf{e}_1$. For each experimental measurement $m = 1, \ldots, N_{\exp}$, the signal recorded by the receivers x_1^k with $k = 1, \ldots, 14$ is denoted as $t \mapsto p^m_{2,,\exp}(t, x_1^k)$. Those signals are modeled as $N_{\exp}$ realizations $P_{2,,\exp}(\cdot, x_1^k; \theta_1), \ldots, P_{2,,\exp}(\cdot, x_1^k; \theta_{N_{\exp}})$ of a real-valued random field $P_{2,,\exp}(\cdot, x_1^k)$ indexed by $[0, T]$. We then introduce the random variable Q_{ext} defined by

$$Q_{\text{ext}} = \int_0^T \sum_{k=1}^{n_R} |P_{2,,\exp}(t, x_1^k)|^2 \, dt \quad .$$

The m-th realizations $Q_{\text{ext}}(\theta_k)$ is then computed by

$$Q_{\text{ext}}(\theta_m) = \int_0^T \sum_{k=1}^{n_R} |p^m_{2,,\exp}(t, x_1^k)|^2 \, dt \quad .$$

The identification of a is carried out by minimizing the cost function

$$F(a, \ell) = \frac{(\underline{Q}_{\text{ext}} - \underline{Q})^2}{\underline{Q}^2_{\text{ext}}} + \frac{(\delta_{Q,\text{ext}} - \delta_Q)^2}{\delta^2_{Q,\text{ext}}} \quad ,$$

where $\underline{Q}_{\text{ext}}$ and $\delta_{Q,\text{ext}}$ are the mean values and the dispersion coefficient of Q_{ext} which are estimated by with $N_{\exp}$ realizations $Q_{\text{ext}}(\theta_m), \ldots, Q_{\text{ext}}(\theta_{N_{\exp}})$. In addition, $\underline{Q}$ and δ_Q are the mean values and the dispersion coefficients of Q for a given thickness a and a given spatial correlation length ℓ. For the application, the experimental measurements $p^m_{2,,\exp}(t, x_1^k)$ have been computationally constructed for a cortical bone with $a = 2.5 \times 10^{-3}$m, $b = 4 \times 10^{-3}$m and $\ell = 2 \times 10^{-4}$m.

In Fig. 2, the graph of $a \mapsto F(a, \ell)$ is shown for different values of ℓ. It can be seen that the minimal values of functions $a \mapsto F(a, \ell)$ are reached at $a^{\text{opt}} = 2.33 \times 10^{-3}$ for the different values of the spatial correlation length ℓ.

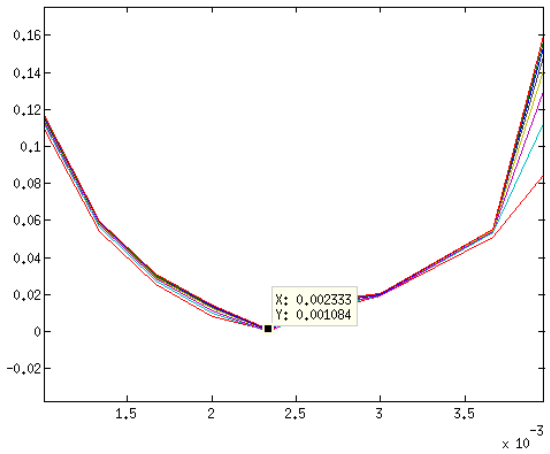

Figure 2: graph of $a \mapsto F(a, \ell)$ is shown for different values of ℓ. Vertical axis: $F(a, \ell)$. Horizontal axis: a

6 Conclusion

In this paper we have considered the transient dynamical response of a multilayer system submitted to an impulse in the ultrasonic range. The application concerns a biomechanical system: the human cortical bone. This system is really tricky to be modelled due to the lack of knowledge on its micro-structure. For such a system, the micro-structure can be altered near its interface with the marrow (osteoporosis). A gradient of porosity is then observed in the thickness direction but, in this case, none of the usual theories of porous materials can be applied. This is the reason why we have proposed a simple model of the elasticity tensor for media with a gradient of the porosity in order to take into account the alterations of the cortical bone micro-structure. Thus, inside the solid layer, the constitutive equation of the solid goes to the constitutive equation of the fluid (the marrow). We have taken into account the uncertainties by substituting the elasticity tensor with a random field for which the probabilistic model has been constructed using the maximum entropy principle. A methodology has been proposed for the identification of the parameters of the probabilistic model. An application is presented for which the thickness of the healthy part of the cortical bone of the probabilistic model has been identified as the optimal parameter of a cost function which corresponds to the total energy of the random pressure pressure field.

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