Influence of the reed flow on the intonation of the clarinet

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The playing frequency of a reed instrument is generally mainly imposed by the resonator. Nevertheless, numerous other factors have an influence on the playing frequency. Among those, the volume velocity pulsed by the reed must be taken into account in the models as it lowers significantly the playing frequency. At first order, this volume velocity can be taken into account by adding an equivalent volume of about 1 cm$^3$ at the entrance of the pipe. However, the concept of equivalent volume is theoretically only valid for linear vibrations i.e., for the clarinet: the non beating reed regime. In the beating regime, this volume is likely to vary quite a lot with the mouth pressure which can lead to intonation problems. In addition, this volume depends much on the reed opening, but experiments performed with an artificial mouth show that for usual openings (0.3-0.7 mm) the intonation remains surprisingly stable. This suggests that the reed and mouthpiece makers know how to control this problem by a well chosen design of the lay. A counter-example is obtained by using a Claripatch™ especially designed so that intonation is particularly difficult to control.

1 Introduction

The playing frequency of a clarinet is known to be mainly controlled by the pipe [1,2]. Recently, a large number of papers have shown that the mouth cavity might play an important role, especially for specific playing technique such as pitch bending or altissimo [3, 4, 5]. The reed motion is also known to play a role but it is usually considered as equivalent to a constant added volume [6]. Its role on the intonation as been neglected and the aim of the present paper is to show that it might have an important role on the stability of the intonation and that reed and mouthpiece makers have to deal with this problem in order to make easily playable instruments.

We first give (section 2) a simplified theory of the influence of the reed motion on the playing frequency, in the case of the non beating reed regime (section 2.1) and in the case of the beating reed regime (section 2.2). Then, experiments with an artificial mouth are performed (section 3) in which the variation of the playing frequency is investigated as a function of the mouthpressure (section 3.1) and as a function of the embouchure (section 3.2). Finally (section 4) the experimental results are confronted to the experience of a professional clarinet player (third author) who demonstrates the possibility of designing a mouthpiece with an unstable intonation.

2 Reed equivalent volume theory

2.1 Non beating reed (linear regime)

Neglecting the reed inertia and damping the reed can be seen as an ideal spring and the reed tip aperture $h(t)$ is proportional to the pressure drop $\Delta p(t)$ between the mouth and the mouthpiece ($\Delta p(t) = P_m - p(t)$ with $P_m$ the mouth pressure and $p(t)$ the pressure in the mouthpiece, see Fig.1). So, it can be written:

$$h(t) = C_r \Delta p(t).$$

(1)

where $C_r$ is the reed compliance.

For the non beating reed regime, the reed can be considered as a linear spring (see [7] for experimental evidence). So, the mouthpressure being considered as constant, the reed motion equation can be written in the frequency domain:

$$H(\omega) = C_r P(\omega)$$

(2)

$P(\omega)$ being the acoustic pressure in the mouthpiece.

![Figure 1: Schematic view of a clarinet mouthpiece and notations](image)

The reed motion induces a flow $u_r = S_r \dot{h}$ where $S_r$ is the vibrating surface of the reed which can reasonably be considered as constant for the non beating reed (see again [7] for experimental evidence). The reed compliance can then be expressed in the frequency domain in terms of an equivalent acoustic admittance $Y_r = U_r/P$ where $U_r$ is the volume velocity in the frequency domain and $Y_r = jC_r \omega S_r$. It can also be expressed in term of an equivalent volume, writing:

$$V_{eq} = \rho c^2 C_r S_r = \gamma P_{atm} C_r S_r.$$

(3)

where $P_{atm}$ is the atmospheric pressure and $\gamma = 1.4$.

It is important to notice that the equivalent volume is different from the displaced volume $V_{dis}$ which is, considering an harmonic oscillation, proportional to the amplitude $|P|$ of the acoustic pressure and is given by:

$$V_{dis} = 2C_r S_r |P|.$$

(4)

So, the displaced volume is usually much smaller than the equivalent volume as:

$$\frac{V_{dis}}{V_{eq}} \approx \frac{2|P|}{\gamma P_{atm}}.$$  

(5)

and $|P| \ll P_{atm}$.

As the reed volume velocity adds up to the flow entering in the pipe the reed motion can be seen as a corrective term to the admittance of the pipe: $Y = Y_{in} + Y_r$. As a first approximation the clarinet can be considered as an open cylinder and $Y_{in} = -j \frac{S}{\rho c} \cot(kL)$ where $L$ is the effective length of the clarinet including eventually side holes effects and radiation.

Eigenfrequencies $f_n$ which are assumed to be close to the playing frequency are then given by:

$$f_n = \frac{(2n+1)c}{4(L + \Delta L)}.$$  

(6)
where $\Delta L = \rho c^2 C_r S_r / S = V_{eq} / S$ and $k\Delta L << 1$. 

The reed motion can then be seen as a length correction added to the effective length of the pipe. This result is well known and its practical importance has been shown by Dalmont et al. [6] although it was not demonstrated that the difference between the playing frequencies and the resonances frequencies are only due do the reed volume velocity. In practice this length correction is typically in the range of 10mm depending on the reed compliance.

Now, all the previous results are valid only in a linear context. This is obviously not the case for the beating reed regime which is certainly the more common way of playing clarinet. In the following we show how the concept of equivalent volume or length correction can be extended to the beating reed regime.

### 2.2 Beating reed (non linear regime)

When a system is non linear, the concept of impedance (or admittance) is no more valid. However, in the case of a slight non-linearity, it can be useful to extend this concept to the non linear situation.

When the reed is beating, Eq.1 can be reasonably replaced by:

$$h(t) = h_0,$$

where $h_0$ is the reed tip opening at rest ($\Delta p=0$). The displaced volume is constant and given by $V_{dis} = S_r h_0$. It is no more proportional to the acoustic pressure amplitude. Assuming that Eq. (5) remains valid the equivalent volume is then given by:

$$V_{eq} = \frac{\gamma P_{am} S_r h_0}{2|\gamma|}.$$  

(8)

So, the equivalent volume diminishes when the pressure increases and consequently up to the beating reed threshold the frequency tends to increase with the amplitude. It might be noticed that the continuity at the beating reed threshold is ensured as in that case $|\gamma| = h_0/(2C_r)$ and consequently $V_{eq} = \gamma P_{am} S_r C_r$. This result ignores the fact that the signal is not harmonic and that the shape of the signal might play a role but this can be considered as a minor effect.

### 3 Experiments

#### 3.1 Influence of mouthpressure for a given embouchure

In order to evaluate the influence of the reed motion on the playing frequency a first experiment has been realized by using an artificial mouth allowing the measurement of the pressure in the mouth $p_m$, the pressure in the mouthpiece $P$ and the reed tip displacement. The reed tip displacement is obtained with a laser vibrometer after an integration of the velocity signal. During the experiment the mouthpressure is varied from the threshold of oscillation to the extinction beyond which the reed channel is closed.

![Figure 2: Playing frequency versus mouthpressure for a given embouchure.](image)

![Figure 3: reed tip displacement amplitude versus mouthpressure (measurement).](image)
It might be noticed that the acoustical length $L$ of the pipe including the mouthpiece volume is not directly measured but deduced from the playing frequency when the reed closes $f_{ext}$ that is here $L = c/(4f_{ext})$ with $f_{ext} = 356Hz$.

### 3.2 Influence of embouchure parameters

In order to evaluate the influence of the reed parameters on the playing frequency, a series of experiments has been realised by using an artificial mouth allowing the measurement of both the pressure in the mouth $p_m$ and the pressure in the mouthpiece $p$. A force sensor, made with strain gauges stuck on the support of the lip, measures the force of the artificial lip on the reed in order to have a control of the reed opening. The mouthpiece is a RV40 by Vandoren and the reed a plasticover by Rico. For a given embouchure (that is a given set of reed parameters) the non linear characteristic is measured using the procedure described in [8] and also used in Almeida et al. [9]. From the non linear characteristic, the embouchure parameters, $p_M$ and $u_d$ can be determined from which the two parameters $C_r$ and $h_0$ can be deduced (see table 1). Then, for the same embouchure with a 50cm tube, the bifurcation diagram is recorded and the frequency is determined as a function of the mouthpressure. This experiment is repeated for different values of the force exerted by the artificial lip on the reed. The set of measurements is the one presented in [10] which was focused on the oscillations thresholds.

<table>
<thead>
<tr>
<th>$h_0$ mm</th>
<th>$p_M$ hPa</th>
<th>$C_r$ mm/kPa</th>
<th>$f_{beat}$ Hz</th>
<th>$\Delta L_{eq}$ cm</th>
<th>$S_r$ cm$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.10</td>
<td>73.8</td>
<td>0.149</td>
<td>135.9</td>
<td>3.10</td>
<td>2.98</td>
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<tr>
<td>0.86</td>
<td>74.4</td>
<td>0.116</td>
<td>137.9</td>
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<td>2.70</td>
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<tr>
<td>0.73</td>
<td>71.0</td>
<td>0.103</td>
<td>139.1</td>
<td>1.65</td>
<td>2.29</td>
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<tr>
<td>0.68</td>
<td>66.7</td>
<td>0.101</td>
<td>140.1</td>
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</tr>
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<td>0.49</td>
<td>61.1</td>
<td>0.080</td>
<td>141.2</td>
<td>0.73</td>
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</tr>
<tr>
<td>0.41</td>
<td>60.3</td>
<td>0.068</td>
<td>141.4</td>
<td>0.64</td>
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</tr>
<tr>
<td>0.28</td>
<td>51.7</td>
<td>0.054</td>
<td>141.8</td>
<td>0.47</td>
<td>1.26</td>
</tr>
</tbody>
</table>

where $p_M = h_0/C_r$; $\Delta L_{eq} = c/(4f_{beat}) - L$ with $L=60cm$ and $S_r = S\Delta L_{eq}/(p^2M C_r)$.

The results, for various values of the opening $h_0$, are plotted on figure 4. The dependence of the playing frequency on the reed opening is obvious. The effect can be rather large, i.e. here about a half tone, but it might be pointed out that some embouchures are completely unrealistic, the reed being left almost free for the larger $h_0$ value. Realistic values correspond to $h_0 < 0.7$ and the fluctuations of the playing frequency are then much more limited, i.e. in that case <10cent. The effect of the embouchure for $h_0=1.1mm$ corresponds to a length correction of 3cm whereas it is less than 1cm for $h_0=0.7$ which corresponds more to the results of the literature [6] (see table 1).

An equivalent length $\Delta L_{eq}$ for the reed can be deduced from the frequency shift between the playing frequency of the tube and the resonance frequency of the tube (143Hz). The equivalent volume below the beating threshold could also be calculated knowing the compliance of the reed $C_r$ and the vibrating surface of the reed $S_r$.

Figure 4: Playing frequency versus mouthpressure for various reed opening (from 1.1mm to .28mm from bottom to top).

Unfortunately the vibrating surface of the reed is not known. An attempt to calculate this surface can be done knowing the compliance of the reed and the equivalent length $\Delta L_{eq}$ (see table 1). However, the values obtained for the vibrating surface have to be considered cautiously: the vibrating surfaces obtained seem rather too large and it seems that the equivalent volume of the vibrating reed is not the only cause of the frequency shift. Indeed, studies by Wilson &Beavers [11], recently revisited by Silva et. al. [7], have shown experimentally and analytically that when the reed is not considered as a single spring, hence possessing its own resonance frequency, the acoustic flow generated by the reed displacement is not the only cause of the lowering of the playing frequency at the oscillation threshold. Simulations (figure 5, for detail on the simulations, see [12]) show a behaviour very similar to the one displayed on figures 2 and 4: the playing frequency decreases up to the beating-reed threshold ($P_m/P_M$ around 1/2) and then increases. The more $h_0$ is large, the more the average playing frequency is low and the more the frequency variations are important.

Figure 4: Playing frequency versus dimensionless blowing pressure $P_m/P_M$ for various reed opening $h_0$ (mm).

Dotted line: model including reed dynamics only.
Thick line: model including reed dynamics and reed volume velocity ($V_{eq}= 1cm^3$).
4 Discussion

Experiments show that intonation might vary quite a lot with both the mouthpressure and the embouchure. However, in practice, pressure and reed opening ranges are much more limited than in the experiments. The frequency shift is then not more than ±-10 cent in both cases. This could be much more important, and it seems to us that the design of the reed and mouthpiece is optimized in order to have a stable intonation. Especially, as suggested by previous work [8], the vibrating length of the reed should not vary much in order ensure a stable intonation.

A demonstration of this possibility of designing a mouthpiece with an unstable intonation is made by using a specially designed Claripatch™. A Claripatch™ is a thin wedge of variable thickness (typically 0.015-0.105mm), which is inserted between the mouthpiece and the reed. This modifies the curvature of the lay, as described in the patent US6921853. A augmentation of the curvature of the lay in a portion comprised between about 15 and 25 mm from the reed tip (therefore located under the lip) followed by an reduction of the curvature from 25 to 35 mm (located between lip and ligature) increases the sensitivity of the lip pressure toward pitch control (“jazzy” sound), whereas the opposite modification reduces this flexibility (“classical” sound). This effect can be partially cancelled by taking a long embouchure (applying the lip nearer from the ligature) or a short embouchure in the second case.

5 Conclusion

Our work demonstrates that the reed motion is a parameter which might have an important influence on the intonation of the clarinet. In practice, it can reasonably be considered as a constant length correction but this result is rather paradoxical: indeed, our work suggests that the reed makers know how to solve the problem of intonation which masks the fact that the intonation can be more or less stable depending on the reed and mouthpiece design. Our interpretation remains to be validated by experiments showing how the playing frequency is related to the vibrating surface of the reed.

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References