

Operational transfer path analysis: theoretical aspects and experimental validation

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^aINSA - LVA, 25 bis avenue Jean Capelle, 69621 Villeurbanne, France ^bDelft University of Technology, Faculty of Aerospace Engineering, M2i, P.O. Box 5, 2600 AA Delft, Netherlands quentin.leclere@insa-lyon.fr Operational transfer path analysis (OTPA) is a diagnosis method aiming to identify and rank noise transmission paths in dynamic systems. The particularity of the method is to require no preliminary acquisition of a transfer matrix between excitation and response dofs, as it is the case for classical TPA approaches. OTPA is based on the identification of a transmissibility matrix between some input and output responses measured for various operating conditions. The first difficulty of the method concerns the definition of this transmissibility matrix, from either a theoretical or experimental point of view. The second difficulty is the use of this matrix for diagnosis purposes, requiring some assumptions leading to potential misunderstandings. Theoretical aspects of the method are firstly discussed in this work. Secondly, an experimental validation is carried out on an academic test setup, and OTPA results are compared to the classical TPA approach.

1 Introduction

Transfer Path Analysis is an engineering method developed in the early 90's to identify and rank contributions of sources exciting a structure [1]. The method is based on the indirect measurement of operating input forces, requiring the prior knowledge of transfer functions between input forces and outputs. More recently, an alternative method, the Operational Transfer Path Analysis, has been introduced [2]. This method is based on the identification of a transmissibility matrix between two sets of responses, one of them being representative of input forces. This approach is attractive because it does not require the measurement of transfer functions, but has been criticized in the literature because of its great uncertainty [3].

The aim of this work is to provide a theoretical and experimental confrontation of these methods. TPA and OTPA are presented in the following theoretical section. An academic experiment is then reported, illustrating the practical advantages and drawback of both approaches.

2 Theory

2.1 Classical Transfer Path Analysis

Classical TPA is based on the indirect quantification of loads exciting the structure. If sources are connected at different positions or directions to the structure, all connection dofs have to be considered as loads. The first step in TPA is to assess frequency response functions (FRF) between excitations and response dofs (accelerations measured on the structure for instance). To do that, the sources are physically disconnected from the structure, and FRF are measured using artificial excitations (roving impact hammer or shakers). The result of these measurements is a transfer system relating excitation to response dofs:

$$X = HF \tag{1}$$

where X and F are column vectors standing for responses and excitations, respectively, and H is the matrix of frequency response functions.

The second step of TPA is to proceed to operational measurements, with sources and structure coupled. The operational responses are measured, and results are stored in a cross spectral matrix S_{XX} made of auto and cross spectra of responses. Then the cross spectral matrix of sources S_{FF} is obtained from the inversion of (1):

$$S_{FF} = H^+ S_{XX} H^{+*}$$
 (2)

where + denotes the regularized pseudo-inverse and * the conjugate transpose. This operation is quite tricky, and the regularization is of prime importance in the success of the method [4].

The individual contribution of excitation dof #j to response #i is then computed following

$$S_{XX}^{\ j}(i,i) = \left| H(i,j) \right|^2 S_{FF}(j,j)$$
(2)

The comparison of S_{XX}^{j} for all values of j allows the ranking of sources (or transmission paths) at response points. It is noteworthy that this source ranking makes sense if individual contributions are not significantly greater than the global contribution. It can be the case if sources are correlated and are cancelling each other at response points, for instance in case of a strong undamped modal behavior. A criterion is proposed here to establish if a ranking of sources is possible:

$$\sum_{j} S_{XX}^{\ \ j}(i,i) \le S_{XX}(i,i) + 3dB$$
(3)

It means that the energetic sum of partial contributions has to be not too greater than the global measured level. If this is not the case, it does not mean that the result is false, it just means that forces are too dependent to be analyzed separately.

2.2 Operational Transfer Path Analysis

In classical TPA, the measurement of FRFs can be very time consuming, requiring the disconnection of active and passive parts. Operational TPA (OTPA) is an alternative to standard TPA, allowing the skipping of this FRF measurement phase. In OTPA, a linear system is considered between two sets of response dofs. The so-called "indicator sensors", noted Y, are placed on the receiving structure side as close as possible to the physical transmission paths. Each indicator sensor is thus related to one excitation force acting on the structure. The "output" set, noted X, is distributed on the receiving structure. The system (1) is formulated for the two types of response dofs :

$$X=HF$$
 $Y=\Phi F$ (4)

The principle of OTPA is then to consider the linear relationship between X and Y :

X=T Y, with T=H
$$\Phi^{-1}$$
 (5)

It can be seen that the matrix T, called transmissibility matrix, is defined only if matrix Φ , relating forces F to

inputs Y, is invertible. It is also worth noting that T is defined for a particular subset of forces, that are supposed to act during operation (see [5] for details).

2.2.1 Estimation of T

The first difficulty of OTPA is the estimation of the transmissibility matrix T, that is not computed using (4) (because the aim of OTPA is to avoid the measurement of FRFs). One way is to formulate (5) using cross spectral matrices of inputs and output obtained from the acquisition of one given operating condition:

$$S_{XY} = T S_{YY} \tag{6}$$

If matrix S_{YY} is invertible (this is unfortunately rarely the case) matrix T is obtained thanks to

$$T = S_{XY} S_{YY}^{-1}$$
(7)

It is important to note that from the mathematical point of view cross spectral matrices are always invertible, because of measurement noise. It is thus important to verify that all eigenvalues are significant.

If matrix S_{YY} obtained from one operating condition is not invertible, it is possible to sum several operating conditions to make the system invertible, or even to process cross spectral matrices obtained from nonstationary operating conditions, like run-up / down [6]. Another possibility is to use a principal component analysis [7] to extract, from each operating condition, eigenvectors corresponding to significant eigenvalues, which can be gathered into one system:

$$[X_1 X_2 \dots X_k] = T [Y_1 Y_2 \dots Y_k]$$
(8)

with k the number of operating conditions, if one principal component only is kept from each operating point. This system can be inverted if eigenvectors are sufficiently linearly independent, using a pseudo-inverse if k is greater than the number of inputs.

An important point is that the rank of X and Y matrices in equation (8) cannot be greater than the number of forces, because each response vector results from the linear combination of the contributions of each force. It can thus be interesting to inspect singular values of X (if the number of output points is greater than the a priori number of forces). Then the number of significant singular values is an indication of the real number of forces exciting the structure. It can be seen as a blind estimation of the number of excitation dofs.

2.2.2 Use of T to characterize transfer paths

The second difficulty is to make a correct use of the transmissibility matrix. The main hypothesis of OTPA is indeed to consider that the participation of one given indicator to outputs through the transmissibility matrix is representative of the contribution of the associated transmission path. (it can be recalled here that each indicator is related to one particular force or transmission path). The participation of one indicator j to one output i is

$$S_{XX}^{J}(i,i) = |T(i,j)|^2 S_{YY}(j,j)$$
 (9)

This participation of indicator i to output j is indeed equal to the contribution of force i if

$$S_{YY}(i,i) = |\Phi(i,i)|^2 S_{FF}(i,i)$$
 (10)

which is correct only if matrix Φ is diagonal. It means that cross contributions of one force to inputs should be zero except for the input corresponding to that force. If it is not the case, an error is introduced here, this is referred as cross coupling between inputs in the literature [6]. The diagonal character of Φ could be verified, but generally matrix Φ is not available. It is thus practically very difficult to quantify this cross coupling, making results of OTPA subject to mistrust.

3 Experimental illustration

3.1 Experimental Setup

TPA and OTPA methods are tested on a simple case: a plate clamped in a window separating two rooms. The reception room is semi-anechoic, the excitation room has no particular acoustic properties. The plate is excited by three physical sources : two shakers and one loudspeaker.



Photo 1: Reception room (antenna of microphones and accelerometers)



Photo 2: Excitation room (shakers, loudspeaker and table mixer)

These three sources can be fed by three independent noise generators. By means of a mixer, it is possible to combine generators and sources to implement different degrees of source correlation. Two accelerometers are placed on the plate, in front of shakers and a microphone is placed in the excitation room close to the loudspeaker. These three sensors represent "indicator sensors" noted Y. The acoustic response in the reception room is measured with an antenna of 25 microphones which represent the "output" set, noted X. A reference set is also acquired, noted F, constituted of mixer outputs sent to physical sources.

Several excitation configurations have been tested, each of them corresponding to a mixer pattern. These patterns are given in table 1.

config	Shaker 3	Shaker 2	Loudspeaker
1	A1	-	-
2	-	A1	-
3	-	-	A1
4	A1	A1	-
5	A1	-	A1
6	-	A1	A1
7	A2	A1	A1
8	A1	A2	A1
9	A1	A1	A2
10	A1	A1	A1
11	A1	B1	C1
A,B,C : Noise generators			
-, 1, 2 : Mixer gain (x0, x1, x2)			

Configurations 1 to 3 are reference tests, for which one source only is active. These configurations are used to measure transfer functions between the reference set and outputs (H), and between the reference set and indicators (Φ). The knowledge of these transfer matrices allows the direct computation of T using equation (5).

Configurations 4 to 9 are different mix patterns, with all physical sources correlated (fed by the same noise generator). These configurations allows the computation of T using equation (8), with k = 6.

In the last configuration, each source is driven by its own generator, constituting 3 uncorrelated sources. This configuration allows T to be calculated theoretically using equation (7).

Configuration 10 is not used for the estimation of T, it is the configuration used for the application of TPA and OTPA (part 2.2.2).

3.2 Validation criteria

We propose two validation criteria:

• For OTPA: the number of significant singular of the responses X obtained for configurations 4 to 10, as described in 2.2.1. Results presented in Figure 1 show that there are three significant singular values of X below 5 kHz, confirming the presence of three excitation devices. Above 5 kHz a fourth singular value becomes significant, caused by a nonlinearity of one shaker.



Figure 1: Singular values of $[X_3X_4 \dots X_{10}]$

• For OTPA and TPA: the criterion that establishes if a ranking of sources is possible, as described in Eq. (3). Figure 2 shows the difference (dB) between the energetic sum of partial contributions and the global measured level. These differences are compared to the proposed limit (3dB). It clearly appears that TPA respects the criterion from 1kHz to 10kHz. For OTPA, the difference is very high until 1 kHz and are is also higher than 3dB between 6 and 9 kHz. The domain of validity of OTPA is therefore restrained in comparison of TPA result.



Figure 2: Criterion on ranking of sources

3.3 Results

3.3.1 Estimation of T

Figures 3 and 4 show transmissibilities computed with three different methods:

- T: transmissibility computed directly from transfer functions using excitation configurations 1 to 3 and equation (5)
- T7: transmissibility computed with Eq. (7), using excitation configuration 11
- T8: transmissibility computed with equation (8), from excitation configurations 4 to 9

Except in low frequencies, the three methods lead to comparable results.



Figure 3: Estimation of transmissibilities with the 3 methods [50-10kHz]



Figure 4: Estimation of transmissibilities with the 3 methods [50-800Hz]

2.2.2 Use of T to characterize transfer paths

The excitation configuration # 10 is used to apply the whole analysis process (one noise generator driving the 3 physical sources). Results are given for contribution of shaker #1 and #2 (Figures 5&6: OTPA and TPA shaker #1 and Figures 7&8: OTPA and TPA shaker #2) and the loudspeaker (Figure 9: OTPA and Figure 10: TPA). Contributions are given in terms of averaged autospectra of the 25 responses X. The measured contributions of the 3 sources, from reference configurations 1 to 3, are also drawn on all figures.



Figure 5: Contribution of shaker n°1 – OTPA method



Figure 6: Contribution of shaker n°1 – TPA method

Results for the shaker 1 are satisfying above 1kHz, for both TPA and OTPA. The identified contribution fits well the measured one, dominating the global level between 5 and 9 kHz. The contribution below 1kHz is well estimated with TPA, but significantly overestimated with OTPA, confirming the criterion given in figure 2.



Figure 7: Contribution of shaker n°2 – OTPA method



Figure 8: Contribution of shaker n°2 – TPA method

Concerning shaker #2, TPA shows that this source is dominant in high frequency, above 9kHz. The identification is not so clear with OTPA because of cross-coupling effects between 3 kHz and 9 kHz. In fact OTPA does not bring a correct separation of contributions of shakers 1 and 2 in the frequency range 6-9kHz.



Figure 9: Contribution of loudspeaker - OTPA method



Figure 10: Contribution of loudspeaker - TPA method

Concerning the loudspeaker, TPA and OTPA methods lead to identical results: this source is the dominant one between 1 kHz and 4 kHz. This good results obtained using OTPA for the loudspeaker is explained by the fact that the indicator sensor chosen for this source (a microphone in the excitation room) is little affected by other sources. The three sources where indeed adjusted to exhibit equivalent overall levels in the reception room. The contribution of shakers through acoustic radiation is approximately the same on both sides of the plate, but the contribution of the loudspeaker is much higher in the excitation room than in the reception room. The contribution of the speaker to the microphone in the excitation room is thus strongly dominant.

Conclusion

TPA and OTPA have been presented theoretically and in the frame of an academic experimental illustration. Different validation criteria have been proposed, allowing to define the frequency range where the separation of contributions is pertinent for both TPA and OTPA. The experiment has shown that these ranges can differ for TPA and OTPA, in the present paper the range is wider with TPA. A very interesting step in OTPA is the inspection of the singular values of the matrix gathering response vectors of different operating conditions. This analysis allows the blind identification of the number of physical connections between the source(s) and the receiving system, even if sources are correlated. The results of the academic illustration are quite satisfying for both methods, even if it can be said that TPA results are better than OTPA results. However, this illustration did not encounter all difficulties and potential pitfalls of more industrial cases, some of them

are addressed in a companion paper [8]. Finally, it should be stated that the ability of TPA and OTPA to establish a meaningful diagnosis strongly depends on the application case, and that a high level of expertise is still required to properly interpret the experimental results.

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