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Identification of vibration excitation using a regularized Finite Element operator and a deconvolution post-process

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The Force Analysis Technique is an experimental inverse approach that aims to identify vibration sources loading a structure from its measured displacements field. Based on the verification of the structural local dynamic behaviour, the Force Analysis Technique constitutes a challenging inverse problem since boundary conditions and sources outside the studied domain may not be known. In its first developments, the Force Analysis Technique was written for analytically known structures like beams, plates and shells. Recently, its adaptation to more complex structures was made possible thanks to the use of Finite Element Method and model reductions. In this mathematical adaptation, the regularization of the problem is done by a double inversion of the numerical operator, the well known Tikhonov regularization being applied in the second inversion. To correct the smoothing effect on regularized results, it is then proposed to apply a deconvolution post-process using the Richardson-Lucy algorithm. After the description of all methodological steps, experimental results on a L-shaped plate excited by a point force is shown. In addition to the location of the force, it is also shown how the quantification of the force is improved by the deconvolution post-process in comparison with a direct measurement by a piezoelectric sensor.

1 Introduction

The Force Analysis Technique (also known by the acronym RIFF) is an experimental method allowing one to identify vibratory excitations from measured displacement (or velocity) fields. This method is appreciated thanks to its local aspect, and its efficient regularisation based on a low-pass wavenumber filtering [1]. Coupled to the Nearfield Acoustic Holography [2-3], it presents an interesting tool allowing one to identify the vibration causes (forces, moments, pressure fluctuations, etc.) from the radiated acoustic field of a structure [4]. However, its major inconvenient is that it is only applicable to simple structures on which the motion equation must be analytically known [5].

The extension of the method to more complex structures was performed by the use of a Finite Element operator constructed over a subdomain and using free boundary conditions [6]. In order to reduce the large number of Degrees Of Freedom (DOF), some model reductions were also investigated like dynamic condensation [7] and Craig-Bampton reduction [8-9].

The inverse approach needs also a regularisation which is held thanks to a Tikhonov procedure optimized by the L-curve principle. Nevertheless, the use of a mathematical regularisation induces a spatial smoothing effect on the result. To correct this setback, a deconvolution post-process is proposed at the end of the regularisation procedure [9]. The goal of this paper is to present the results obtained on a L-shaped plate using the whole method, from the construction of the FE model, with reductions and condensation, to the deconvolution post-process.

2 Theoretical development

2.1 External forces identification using a FEM-based operator

A structure is supposed to be excited by some harmonic forces at the angular frequency ω. A Finite Element model is used to describe a part of it, which is called subdomain. If necessary and if there are enough measured data, it can obviously be extended to the whole domain. The model contains N_e elements corresponding to N nodes of a mesh called computation mesh. It is on this subdomain that forces will be identified.

Using Free-Free boundary conditions (of the subdomain) the Finite Element model is written in a matrix form at the angular frequency ω:

$$\left(-\omega^2\left[M\right]+\left(1+j\eta\right)\left[K\right]\right)\left\{V\right\}=\left[L\right]\left\{V\right\}=\left\{A\right\}, \quad (1)$$

where $[L]$ is the finite element matrix operator corresponding to the substructure, $\{V\}$ the vector of N translations and $2N$ rotations, $\{A\}$ the vector of exterior mechanical actions corresponding to N forces and $2N$ moments for the whole set of nodes and η is the structural damping factor.

If quantities contained in $\{V\}$ are known (by means of measurements), it becomes possible by respect of equation (1) to identify data in $\{A\}$.

2.2 Craig-Bampton reduction option

In the aim to decrease the total number of DOFs to be measured and modelled, a Craig-Bampton reduction technique is proposed, where some non excited areas (supposed to be known) are merged in superelements. Another objective can also be to delete DOFs corresponding to non accessible (and non excited) parts of the subdomain.

Therefore, the overall DOFs set is splitted in three sets:

1. Set $\underline{b}$, corresponding to the DOFs located at the boundaries (or interface) of all superelements,
2. Set $\underline{i}$, corresponding to interior DOFs of all superelements,
3. Set $\underline{o}$, corresponding to other DOFs, outside all superelements.

The Craig-Bampton reduction consists in calculating a basis representing the DOFs $\underline{b}\cup\underline{i}$:

1. the first part of this basis is composed of the DOFs of $\underline{b}$. It relates them to the ones of $\underline{i}$ by the definition of constraints modes,
2. the second part contains modal DOFs representing the internal modes of the superelement with clamped interfaces nodes. They are called fixed interface modes and they form a new set of DOFs $\underline{\Phi}$.

The reduction can then be applied on the whole model by projecting Equation (1) on the reduced basis. The reduced system is written:

$$\left[L^r\right]\left\{V^r\right\} = \left\{A^r\right\}, \quad (2)$$

where $\left[L^r\right] = \left[T^T\right]\left(-\omega^2\left[M\right] + \left[K\right]\right)\left[T\right]$ is the reduced operator, $\left\{V^r\right\} = \begin{Bmatrix} V_o \\ V_b \\ V_\Phi \end{Bmatrix}$ is the reduced vector of DOFs, $\left\{A^r\right\} = \left[T^T\right]\begin{Bmatrix} A_o \\ A_b \\ A_\Phi \end{Bmatrix}$ is the reduced vector of exterior mechanical actions and $\left[T\right]$ is the projection matrix constructed thanks to the Craig-Bampton method. ^{T} denotes the matrix transpose.

2.3 Substitution of the non measurable DOFs

In practice, some DOFs cannot be directly measured. This is the case of rotations but also, when Craig-Bampton reduction is processed, of the modal DOFs. A dynamic condensation is here proposed in order to substitute all non-measurable DOFs.

In the following, the set of measurable/measured DOFs is noted $\underline{m}$ and the set of non measurable/non measured DOFs is noted $\underline{s}$. Considering that non measured DOFs are not excited by external forces, the dynamic condensation can be performed at each frequency by:

$$\left\{F^{rc}\right\} = \left(\left[L_{mm}\right] - \left[L_{ms}\right]\left[L_{ss}\right]^{-1}\left[L_{sm}\right]\right)\left\{V^{rc}\right\}, \quad (3)$$

which leads to the construction of a reduced and condensed system of equations as follows:

$$\left\{F^{rc}\right\} = \left[L^{rc}\right]\left\{V^{rc}\right\}, \quad (4)$$

where $\left[L^{rc}\right]$, $\left\{F^{rc}\right\}$ and $\left\{V^{rc}\right\}$ are respectively the reduced operator, the vector of unknown forces and the vector of measured DOFs.

2.4 Regularisation and deconvolution post-process

The inverse technique is particularly sensitive to the noise in displacement measurements. A regularisation method must be applied to ensure a robustness.

The Tikhonov method [10] is proposed. It consists in minimising the quantity $\left\|\left[L^{rc}\right]^{-1}\left\{F^{rc^{tikh}}\right\} - \left\{V^{rc}\right\}\right\| + \beta^2\left\|\left\{F^{rc^{tikh}}\right\}\right\|$ by defining the parameter β to obtain the regularised solution $\left\{F^{rc^{tikh}}\right\}$ from $\left\{V^{rc^{tikh}}\right\}$ and the inversed reduced and condensed operator $\left[L^{rc}\right]^{-1}$.

This approach requires two inversions :

- the first is the inversion of the operator $[L^{rc}]$, which is made by a Singular Value Decomposition with no truncation,
- the second is performed on $\left[L^{rc}\right]^{-1}$ and it includes the Tikhonov regularisation [10].

The determination of parameter β relies on the principle of the L-curve which is a plot of the norm of the solution $\left\|\left\{F^{rc}\right\}\right\|$ with respect to the residue $\left\|\left[L^{rc}\right]^{-1}\left\{F^{rc}\right\} - \left\{V^{rc}\right\}\right\|$ for different values of β and where $\|.\|$ is the vectorial 2-norm. The best value for β corresponds to the corner of this L-curve.

A major setback of this mathematical regularisation is that it induces a smoothing effect which corrupts spatial results.

In order to improve the spatial resolution, a deconvolution post-process is performed [9]. It relies on the Richardson-Lucy algorithm which is a well known procedure in astronomical image processing [11-12]. It gave also appropriate results in source maps obtained from beamforming techniques [13-14].

Regularisation yielded to the following equation:

$$\left\{F^{rc^{tikh}}\right\} = \left[L^{rc^{tikh}}\right]\left\{V^{rc}\right\}. \quad (5)$$

The relationship between smoothed results and deconvolved results needs to be constructed. Using the concept of Point Spread Function which derives from impulse response function (or transfer function in the case of spatial results), it is possible to compute the terms of a matrix $\left[H\right]$ which allows to construct the following relationship [9] :

$$\left|F_m^{rc^{tikh}}\right|^2 = \sum_{n=1}^{N_d} H_{mn}\left|F_m^{rc^d}\right|^2, \quad (6)$$

where $\left|F^{rc^d}\right|^2$ is the square of the magnitude of the deconvolved forces.

Equation (5) relates positive pieces of data because it is inversed using the Richardson-Lucy algorithm [11-12]: the deconvolution process is again an inverse problem and the add of a positive constraints on data is the classical approach to regularise it.

3 Experimental validation

3.1 Experimental setup

The experimental validation was performed on a 1.5mm thick L-shaped aluminium plate, as shown in Figure 1. The material and geometrical characteristics are summed up in Table 1.

Measurements were performed using a scanning laser vibrometer while the reference signal is given by an accelerometer. A shaker fed by a periodic chirp signal excites the plate at the location shown in Figure 2. The force applied to the structure is directly measured thanks to a force sensor, in order to allow comparisons with the identified forces between 100 Hz and 3200 Hz.

Table 1: Material and geometry characteristics of the experimental plate.

Thickness (in mm)	Mass density (kg/m^3)	Young modulus (N/m²)	Structural damping
5.1	2700	72E9	1E-4

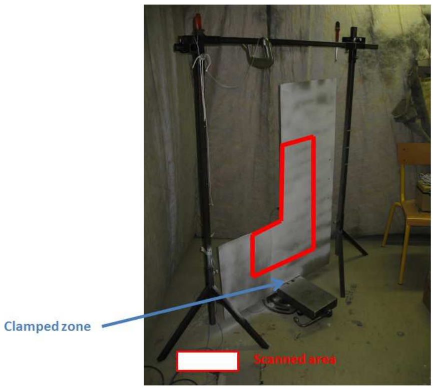

Figure 1: Photography of the experimental setup. The clamped area can be seen and the scanned subdomain is highlighted by the red contour.

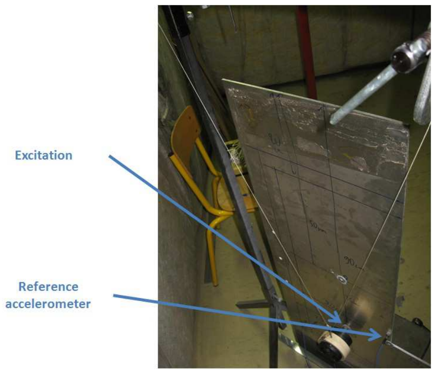

Figure 2: Photography of the back side of the experimental setup. The locations of the shaker and of the reference accelerometer are pointed on these pictures.

The FE model was constructed using a mesh made of triangles with a mean length of 23mm.

A first analysis using only the dynamic condensation and the regularization on a few frequencies allowed one to locate some non excited areas. This analysis cannot be performed at all frequencies because of the measurement and high computational times. But, from the hypothesis of non excited areas it provided, it was possible to perform a Craig-Bampton reduction.

As shown on Figure 3, the studied area is split into 3 parts :

– the exterior area (corresponding to the limits of the studied area, including the free edges),

– the interior area (where the exciting force ”might be” applied),

– the superelement (where no forces are supposed to be applied). In the following, 16 modes are kept, the eigen frequency of the last mode being 3350Hz.

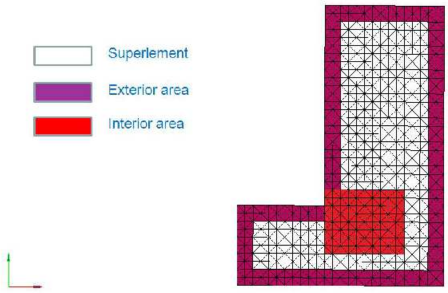

Figure 3: Definition of the mesh with indication of the areas reduced using the Craig-Bampton method.

3.2 Results

The result obtained at 750 Hz from the Craig-Bampton reduction, the dynamic condensation and the Tikhonov regularisation is shown in figure 4. The excitation is visible but its location is fuzzy, due to the Tikhonov regularization. When one wants to estimates the force amplitude, all nodal forces inside the observed pic must be summed. The summation zone is indicated by the blue rectangle.

The result obtained obtained at 750 Hz after the deconvolution post-process is shown in Figure 5.

The accuracy of the force location is obvious. The deconvolution allows one to define a sharper summation area in order to assess the force magnitude at all frequencies. Because of the spatial décorrélations hypothesis beneath deconvolution, the summation is performed using a quadratic sum at all frequencies and results are compared to the direct measurements as plotted in Figure 6. Results are accurate, except on structural resonances.

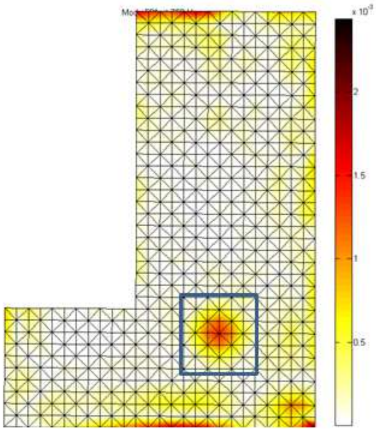

Figure 4: Colormap of the amplitudes of the forces identified on the plate from experimental measurements at 750Hz without deconvolution post-process.

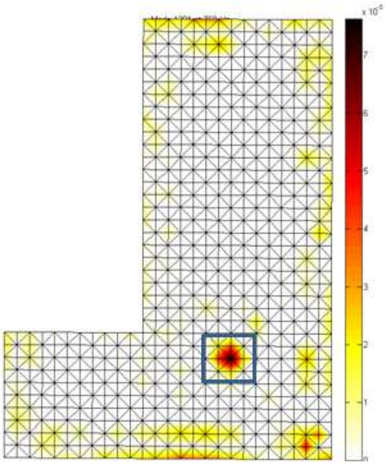

Figure 5: Colormap of the amplitudes of the forces identified on the plate from experimental measurements at 750Hz with deconvolution post-process.

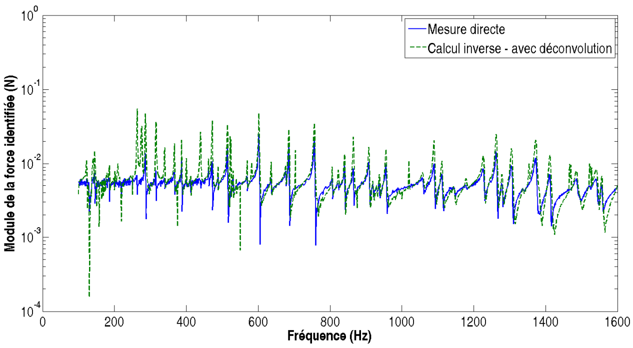

Figure 6: Identified exciting force magnitude compared to direct force measurement modulus, for all frequencies. Inverse results are obtained with the deconvolution post-process.

4 Conclusion

The Force Analysis Technique was recently adapted to complex structures by the use of a Finite Element Model instead of an analytic motion equation. This adaptation consisted in integrating model reductions in order to be applicable in real situations. The Craig-Bampton reduction and the dynamic condensation have shown the possibility to avoid rotation measurements and to reduce drastically the number translation DOFs. As in the classic Force Analysis Technique, the reconstructed forces are very sensitive to the uncertainties in measured data. A regularisation must also be applied. Here, the Tikhonov approach is proposed, but whatever the used method, the regularisation smoothes spatially the result. The Richardson-Lucy algorithm was used to deconvolve results as a post-process to counteract effects of smoothing. The improvement of the results is clearly noticeable. This kind of approach relying on a robust algorithm seems to be particularly efficient and its use in vibroacoustic inverse problems (acoustic holography, beamforming, etc.) should and will be more explored.

Acknowledgments

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