Numerical and experimental characterization of pre-fractal anisotropic stacks

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In this work, acoustic waves transmission through multi-layered anisotropic media is numerically and experimentally investigated. Acoustic properties of pre-fractal samples constituted by a succession of orthotropic layers are studied. Complex media (periodic, disordered or fractal) are known for their remarkable properties as regards to acoustic wave propagation. Fractals appear to be between periodicity ans disorder and their intrinsic proportional transformation impacts directly on waves propagation. Indeed, they represent very well natural irregularities, but they are also built from a repetitive pattern at different scales: they affects the introduced disorder. Bulk waves propagation in multi-layered anisotropic media is theoretically described by the stiffness matrix method (published by Rokhlin and Wang in 2002). From the resolution of Christoffel equation, stresses and displacements in each layer are connected by a matrix formalism. Pre-fractal stacks are numerically and experimentally characterized. They are also compared with classical fractal, periodic and disordered structures.

1 Introduction

Reinforced fiber composites are more and more integrated on industrial structural pieces because of their possibility to reduce drastically structures weight while increasing their rigidity. Composites classical drapings are constituted by a succession of 0°, 90°, ±45° oriented layers. Numerous researches deal with the optimization of these materials from a mechanical point of view [1]. For acoustic waves, anisotropic multi-layered media were first studied in geology [2] because earth's crust present the same transverse isotropy symmetry. Composites materials are now studied mostly in order to improve ultrasonic non-destructive techniques [3, 5, 6, 4, 7]. Here, the draping property is combined with a complex media approach: how can we make fiber-reinforced composites more complex in order to make them more interesting regarding an acoustical point of view and particularly concerning their transmission properties?

Wave damping process combines both classical absorption due to dissipative energy conversion or relaxation process and geometrical scattering phenomena that causes the waves to interfere. Distinction between attenuation processes is not easy and is essentially based on the media characteristics, the precise knowledge of their geometry and the ratio between wavelength and media heterogeneities. Complex media present attractive properties due to their specific topology. Periodic media involve Bragg destructive interferences and possess frequency ranges where waves cannot propagate [8]. Disordered materials are characterized by exponentially localized modes. Propagating in such a medium, waves follow multiple-scattering paths. They interfere strongly and can be totally trapped inside the structure [9].

Quasi-periodic, aperiodic, self-similar (structures similar to themselves at different scales) and fractal media appear to be halfway between periodicity and disorder. They combine division of space and iteration process. They are known to represent well natural irregularities and to attenuate wave propagation. Their vibrational properties have been widely studied since the beginning of the 80's [11, 12, 13, 14, 16]. On the experimental aspect, Bernard Sapoval created a fractal noise-reducing wall [10] and achieved to increase significantly road noise absorption [18]. Craciun *et al.* studied the propagation of sound in a one-dimensional Cantor composites [17] and Gibiat *et al.*, a pre-fractal waveguide [15].

Based on these previous works, multi-layered self-similar anisotropic composites are studied. They are described in the first section of this paper. Their attenuation capacity is experimentally (section 3) and numerically (section 4) investigated trough their high frequency transmission coefficient and the transmission ratio (ratio between the area under the transmission stack curve and the area for a total transmission). They

are also compared with periodic and disordered stacks in order to identify their pre-fractal behavior.

2 Samples and experience description

2.1 Pre-fractal stacks

In this section, samples are presented. They are constituted by a succession of unidirectional plies made from carbon fibers impregnated with a resin matrix (figure 1). The micromechanic theory of homogeneization allows to estimate unidirectional ply elastic properties from the ratio and the elastic properties of elementary components. The rigidity tensor can thus be computed [19]. Each sub-layer belongs to the crystallographic hexagonal system and is characterized by five independent elastic modulus, two orthogonal symmetry planes and a rotation around the orthotropic axis A_6 leaves them unchanged. When orthotropic axis is oriented along x'_1, the rigidity matrix obtained by homogeneization is given by equation 1:
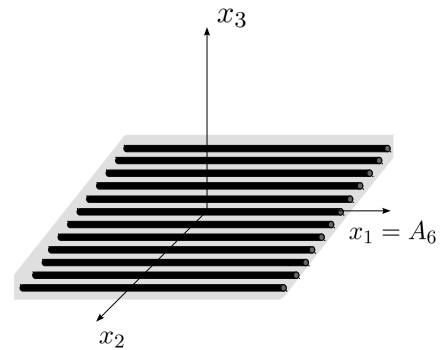

Figure 1: An unidirectional ply and the associated local coordinate system. Orthotropic axis A_6 is oriented along fiber direction x'_1

$$C = \begin{pmatrix} 157.75 & 5.68 & 5.68 & & & \\ 5.68 & 13.88 & 7.41 & & \mathbf{0} & \\ 5.68 & 7.41 & 13.88 & & & \\ & & & 3.24 & & \\ & \mathbf{0} & & & 6.16 & \\ & & & & & 6.16 \end{pmatrix} \text{GPa.} \quad (1)$$

The multilayers are built following auto-similar patterns: Fibonacci or geometric sequence and the Cantor set. Fibonacci samples are based on Fibonacci sequence. Ply orientation follows the sequence : $\phi_{n+2} = \phi_{n+1} + \phi_n$. In order to

maximize the layer to layer difference, the sequence is initialized from 8°. Cantor samples are funded on the Cantor set, created by deleting at each Nth iteration the middle of a set of line segments. Figure 2 clearly shows the self-similarity of the triadic ($\gamma = 3$) Cantor set: each sub-figure framed in gray dotted line is a Cantor set.
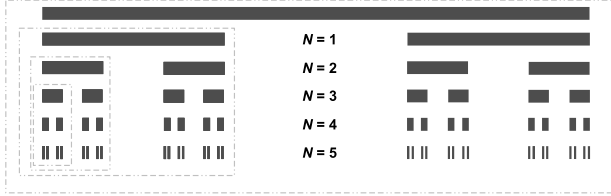

Figure 2: The triadic Cantor set $\gamma = 3$

Two differently oriented orthotropic media A and B are structured according to the Cantor set 2.1.

<div align="center">
A
ABA
ABA BBB ABA
ABA BBB ABA BBBBBBBBB ABA BBB ABA
· · ·
</div>

The last sort of sample is also inspired by the Cantor set, but the successive divisions are made on the central orientation. When the pre-fractal sequence is up to the Nth generation, and the central layer has an orientation φ_0, the structure is a succession of $4N - 1$ layers 2:

$$\left[0 \ \frac{\phi_0}{\gamma^N} \ 0 \ \ldots \ 0 \ \frac{\phi_0}{\gamma^2} \ 0 \ \frac{\phi_0}{\gamma} \ 0 \ \frac{\phi_0}{\gamma^2} \ 0 \ \ldots \ 0 \ \frac{\phi_0}{\gamma^N} \ 0 \right]. \quad (2)$$

To minimize residual stresses due to building process, stacks have to possess a mirror symmetry as regard to the central layer. Table 1 summarizes manufactured samples, their name and corresponding number.

Table 1: Correspondence between sample number, name and stacks.

$n°$	Name	Stack
1	Unidirectional	$[0]_{12}$
2	Fibonacci	$[89\ 55\ 34\ 21\ 13\ 8]_S$
3	45° angular triadic cantor	$[0\ 5\ 0\ 15\ 0\ 45]_S$
4	45° thickness triadic cantor	$[0\ 45\ 0\ 45\ 45\ 45\ 0\ 45\ 0]$

2.2 Experimental system

In order to characterize high frequencies behavior of the samples, ultrasonic measurements in transmission have been made in water. They allow to check if the manufacturing of samples is reproductible and to compare the stacks in terms of transmission coefficient and transmission ratio (ratio between the area under the transmission stack curve and the area for a total transmission).

In an ultrasound tank filled with water, a piezo-electric transducer, with a 5MHz central frequency, generates a sinusoidal wave that travels through the sample and that is received by an array of 32 5MHz central frequency transducers. A goniometer head allows to rotate the sample to change the incident wave angle 3.
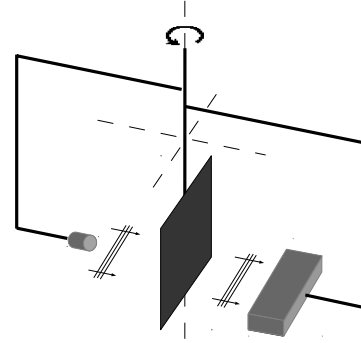

Figure 3: Transducers and sample disposition for ultrasonic transmission experience. Sample can be rotated to vary the incident ultrasonic beam angle

Composite plate anisotropy required a very accurate adjustment of normal incidence and transducer and stack parallelism. This setting is made adjusting transducers positions by dichotomy so that the received signal on the array is maximum, centered, and that 1 and 32 transducers received signals are synchronized. The receiver array allows an average on 32 signals, experimental results are thus more precise.

3 Experimental results: auto-similar stacks characterization

3.1 Experimental transmission

Transmission coefficient is obtained averaging on the N_R receiving transducers, the ratio between the amplitude of the signal propagating through water only S_0 and the amplitude of the signal propagating through the sample and the water S_1 (equation 3).

$$T = \frac{1}{N_R} \sum_{i=1}^{N_e} T = \frac{1}{N_R} \sum_{i=1}^{N_e} \frac{S_1^{\ i}(t)}{S_0^{\ i}(t)} \quad (3)$$

The manufacturing process requires a high precision, because pre-fractal stacks contain many orientations and have thus many chances to deviate from the theoretical stack. In order to check the reproducibility, two copies of each stack have been manufactured and tested. Figure 4 presents experimental results as a function of the incidence angle. Gray continuous and black dotted lines correspond respectively to the first and the second stack copy. Measurement errors correspond to the standard deviation of transmission.

The first remark about these curves is the similarity between the two stack copies. Excluding Fibonacci samples between 25° et 50°, the graphs, if not superposed, follow the
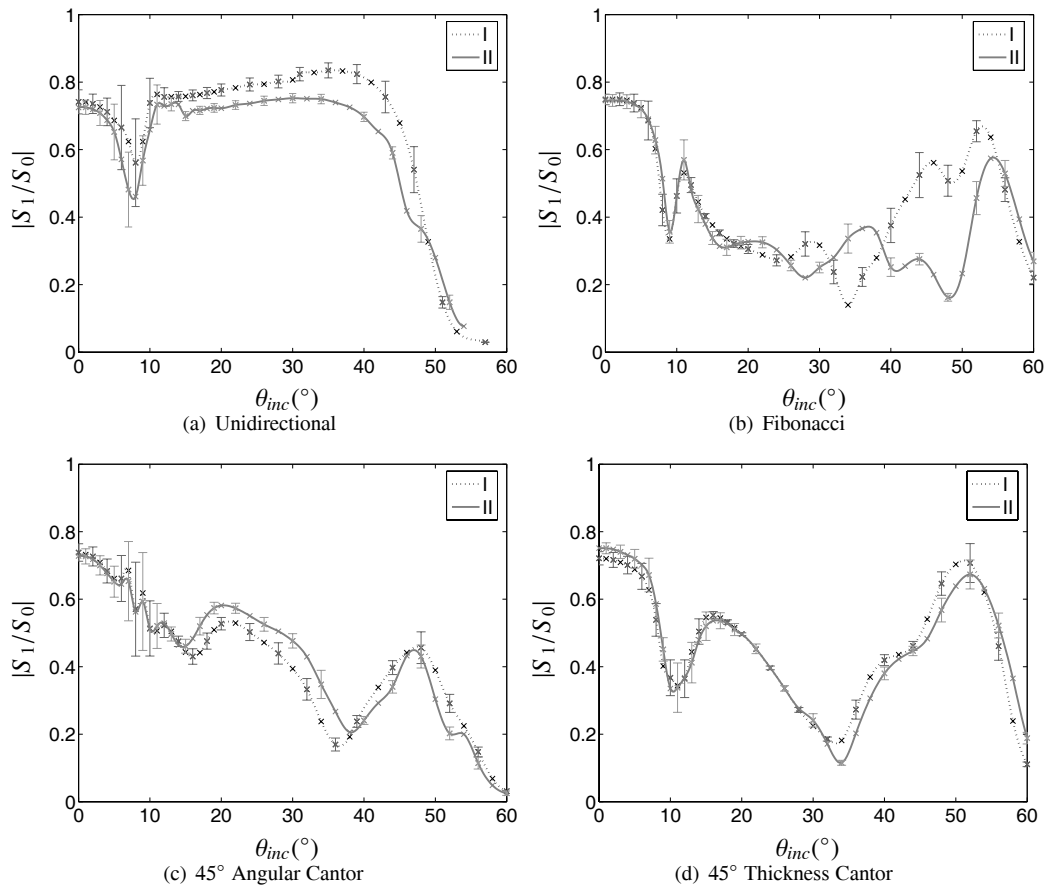
Figure 4: Experimental 5MHz transmission coefficient for four stacks: unidirectional (a), Fibonacci (b), 45° angular (c) and thickness Cantor (d).

same evolution. We can thus conclude that the manufacturing process is reliable. Curves dissemblance for Fibonacci stacks can be explained by a layer mis-orientation. Transverse waves are strongly influenced by orientation, and thus it can affect transmission coefficient after the second critic angle. Transmission coefficients for Fibonacci and Cantor stacks are very different from unidirectional's. An important decrease of transmission can be seen from an incidence of 10°. Nevertheless, this lost of transmission is compensated by an enlargement of the angle incidence range where waves can propagate: for unidirectional samples, transmission stops around 50° whereas other samples present transmitted energy up to 60°. The transmission ratio is computed for a narrow frequency band. It indicates that, on the whole incident angle domain, pre-fractal stacks present transmit less energy than unidirectional ones (table 2). The more orientations the stack contains, the lower the transmission ratio is: angular Cantor (n°3) and Fibonacci (n°2) stacks transmission ratio is lower than for thickness Cantor (n°4).

Sample	1	2	3	4
Transmission ratio (%)	64.08	39.78	39.20	43.34

Table 2: Transmission ratio in (%) computed on the whole incident angle for 5MHz

These results highlight a conclusion: it seems that the more the disorder there is, the more important the transmission is. This is going to be comforted by numerical results.

Practical samples were limited because they have to be made from 12 layers (or less) and because our transducers bandwidth are limited around 5MHz. But simulation allows us to investigate more deeply the effect of disorder considering a higher number of layers.

4 Numerical results: the importance of disorder to reduce sound transmission

In this section, stacks computed transmission coefficient and transmission ratio are compared with ordered and disordered stacks. Order is represented by periodic stacks while disordered ones are constituted by a succession of random oriented layer (orientation is chosen among angle composing the associated auto-similar stack). To highlight order or disorder effect, numerical simulations allow to work with a high number of layer (27) and to be exempt from mirror symmetry. A high number of layer allows to create an effective disorder. Moreover, with a lot of layer, the thickness is important enough compared to the wavelength, and bulk waves can perceive more the disorder during their propagation.

4.1 Waves propagation model in multi-layered anisotropic media

The stiffness matrix algorithm published by Rokhlin and Wang in 2002 [20] is used to compute the transmission coef-

ficient. The details of theoretical development can be found in [20] is detailed enough. We present a schematisation of the algorithm in figure 5.
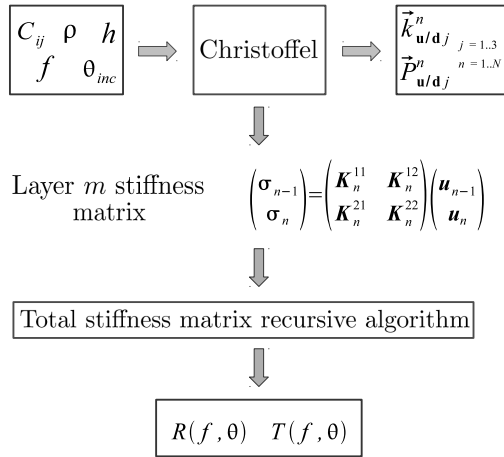

Figure 5: Stiffness matrix algorithm [20]

Transmission and reflexion coefficients come from the stack total stiffness matrix. This one is computed from a recursive algorithm that uses every layer stiffness matrix. Stiffness matrix connects stresses and displacements at the two boundaries of a layer, and derives from the calculation of polarizations and wave vectors by the resolution of Christoffel equation.

4.2 Comparison of ordered, fractal and disordered stacks

Unfortunately, rigidity matrix damping part is not well known. Thus, numerical simulations are made with the real elasticity tensor given by equation 1. Moreover, this allows to isolate attenuation power only by the structure geometry. Ordered, quasi-periodic and disordered stacks are compared thanks to the transmission ratio. Ordered stacks are an unidirectional and a periodic stacks constituted by the repetition of a bilayer with two different orientations $[\varphi_0 \; \varphi_1]_N$. Quasi-periodic stacks are what we named thickness Cantor in the first section and a stack based on the Fibonacci word sequence detailed below:

$$AB$$
$$ABA$$
$$ABA \; BA$$
$$ABA \; BA \; ABA$$
$$ABA \; BA \; ABA \; ABA \; BA$$
$$\cdots$$

A and B are two φ_0 and φ_1 oriented layers. Then, there are angle auto-similar samples: Cantor and Fibonacci. For each of these stacks, a disordered stack is built: it possesses the same orientations, and the layers are randomly oriented among these angles. Figure 6 presents the transmission ratio in percent for these diverse stacks. Stacks are sorted according to their disorder degree: the more angles a stack contains, the more disordered it is , if the stack is created according to a determined sequence, it is ordered. To maximize the velocity contrast for longitudinal waves, bi-layers are constituted by 0° and 90° oriented layer, and angle Cantor stack central layer is 90° oriented.
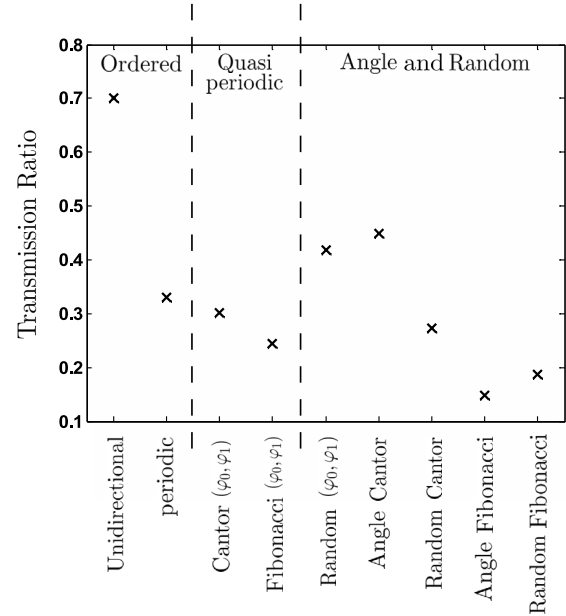

Figure 6: Transmission ratio (%) for 5MHz from ordered to disordered stacks.

Figure 6 leads to three major conclusions. First, the more disordered (containing numerous orientations) the more efficient the stacks are. This confirms the precedent section experimental results. Angular and random Fibonacci stacks possess the lowest transmission ratio because they are constituted by a very high number of different oriented layers. It is very interesting to note that angle Fibonacci stack achieves the same transmission ratio as random Fibonacci one. Thus, it introduces the same disorder degree. A second observation is the efficiency of quasi-periodic stacks (thickness Cantor and thickness Fibonacci) compared with ordered ones in term of energy loss. With few layers, effective disorder is difficult to create, it is thus more interesting to organise them according to a quasi-periodic sequence introducing a weak disorder. Finally, the angle Cantor and its associated random stack are the less efficient. For 27 layers, the external layer orientation becomes very close to 0° due to the building sequence (equation 2). Consequently, the stack is very similar to an unidirectional one except from the central layers. Their transmission ratio is almost twice as low as unidirectional stack's. Therefore, the use of very small defects can have a major consequence in term of acoustic transmission. Angle stacks can be qualified as quasi-periodic because they introduce defects not in the layer topological disposition, like thickness stacks, but in the contrast between layers: orientation difference is traduced by a periodic variation of waves velocity. As a conclusion, it can be said that in order to minimize the global transmission, disorder has to be privileged.

5 Conclusion and perspectives

Experimental and numerical works coincide to show that the more orientations and the less organized they are, the more the wave 5MHz global transmission decreases. The more angles the stack contains, the more complex transmission and reflexion conditions at the interfaces are, and the higher the chance is that waves can be localized in the stack. However, a lot of parameters have to be taken into account

in order to create transmission reducing composites. To create disorder, stacks have to be thick enough. Moreover, the choice of angle mismatch between two successive is determining for the stack efficiency because of anisotropy. Angle stacks can reach the same disorder and sound reducing degree than random stacks. The greatest advantage of quasi-periodic stacks advantage is their deterministic manufacturing process. It would be interesting to investigate the characterization in a lower frequency range to check if, like in other works, auto-similar sequence can impact a wavelength range upper than heterogeneities size.

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