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Source localization using a sparse representation of sensor measurements

J. Lardies, H. Ma and M. Berthillier

FEMTO-ST, Département de Mécanique Appliquée, 24 rue de l'Épitaphe, 25000 Besancon, France
joseph.lardies@univ-fcomte.fr

Abstract

We propose a non-parametric technique for source localization with passive sensor arrays, using the concept of a sparse representation of sensor measurements. We give an interpretation of sensor data by sparsely representing these data in an overcomplete basis. The estimation problem is put in a model-fitting framework in which source position is achieved by finding the sparsest representation of the data. The approach presented in the communication is based on the singular value decomposition (SVD) of multiple samples of the array output and the use of a second-order cone programming for optimization of a resulting objective function. We formulate the problem in a variational framework, where we minimize a regularized objective function for finding an estimate of the signal energy as a function of acoustical source position. The key is to use an appropriate non-quadratic regularizing functional which leads to sparsity constraints and superresolution. The acoustical sources will be correlated or uncorrelated, wideband or narrowband. Numerical and experimental results in an anechoic room are presented. Our algorithm is compared to traditional algorithms such as beamforming, Capon and MUSIC.

1 Introduction

We consider the problem of locating P radiating sources by using an array of N sensors. The emitted energy from the sources can be, for example, acoustic or electromagnetic, and the receiving sensors could be any transducers that convert the received energy to electrical signals. Examples of sensors include electromagnetic antennas, seismometers, hydrophones and microphones. This type of problem finds applications in sonar, radar, communication, aeroacoustic testing, seismology, non destructive testing, acoustic imaging, air acoustics, underwater acoustics, where a passive listening is applied, and many other fields. This problem basically consists of determining how the energy is distributed over space with the source positions representing points in space with high concentrations of energy. Hence, we have a spatial spectral estimation problem. The sources generate a wave field that travels through space and is sampled, in both space and time, by the sensor array. It is assumed that the sources are point emitters situated in the far field of the array. In addition, we assume that both the sources and the sensors are in the same plane and that the signals and noise are random processes with zero mean, stationary and statistically independent. The propagation medium is not dispersive and so the waves arriving at the array can be considered to be planar. Under these assumptions, the only parameter that characterizes the source locations is the angle of arrival or direction of arrival [1-3].

The goal of array processing is to estimate the locations of sources by combining the received data from multiple sensors so that the desired signal is enhanced, while the unwanted signals, such as interference and noise, are suppressed. The most basic approach to array processing is the classical delay-and-sum method, in which the received signal from each sensor is weighted and delayed so as to focus on different points in space. However, this method suffers from low resolution and high sidelobe levels. There is an important literature on methods that provide superior performance over the delay-and-sum approach. In first the standard Capon beamformer has been proposed followed by the multiple signal classification (MUSIC) method. These techniques provide superresolution when the sources are uncorrelated and the number of snapshots is high. Many extensions of these methods have been proposed to deal with modelling errors, such as steering vector mismatches. However, none of these methods is able to cope with very low snapshot numbers, coherent or highly correlated sources. Recently, sparse-representation-based source localization methods provide another interpretation of array data by sparsely representing array data in an overcomplete basis, stressing the fact that the position of sources are very sparse relative to the entire spatial domain. In this way, the localization problem is put in a model-fitting framework in which this localization is achieved by finding the sparsest representation of the data. The approach presented in the communication is based on the singular value decomposition (SVD) of multiple samples of the array output and the use of a second-order cone programming for optimization of a resulting objective function. The key is to use an appropriate non-quadratic regularizing functional which leads to sparsity constraints and superresolution. With the proposed method the source localization problem is transformed into a convex optimization problem that can be efficiently solved by efficient algorithms. In particular, the problem is formulated as an l_1-regularization problem; the l_1-norm is used to impose sparsity-constraint on the spatial spectrum. In this communication, the acoustical sources will be correlated or uncorrelated, wideband or narrowband.

The remainder of this paper is organized as follows. In section 2 the array signal model is presented. In section 3 the l_1-regurarisation method for source localization is described. Numerical and experimental results in an anechoic room are conducted to compare the performance of the studied algorithm with traditional algorithms such as beamforming, Capon and MUSIC for source localization. A conclusion is given in section4.

2 Array signal model

The receiving array considered in this communication has N omnidirectional sensors and is immersed in an acoustic noise field which consists of P independent discrete sources. Because of the geometric positions of the sensors, the total signal power incident on each sensor is the same, but the phase information is different on each receiver. The purpose of any estimator is to use the phase information in some way to infer which signals reached the receiving array and the goal of sensor array source localization is to find the locations of sources of wavefields that impinge on the acoustical array. The available information is the geometry of the array, the parameters of the medium where wavefields propagate and the time measurements or outputs of the sensors. For purposes of exposition, we first focus on the narrowband scenario. For a set of P sources, the signals observed at the outputs of the N sensors array are represented by the N-dimensional vector [1-3]

$$y(t) = \sum_{i=1}^{P} a(\theta_i)\, s_i(t) + n(t) \qquad (1)$$

where $s_i(t)$ is the complex amplitude of the i^{th} source. It is a zero-mean complex random variable: $E[s_i(t)]=0$. The signal power p_i of the i^{th} source which we wish to localize is represented by its variance $p_i = Var[s_i(t)]=E[s_i(t)s_i(t)^*]$. Here E[ ] denotes an ensemble average and the superscript * represents the complex conjugate. The direction of arrival of the i^{th} source is represented by the N-dimensional vector $a(\theta_i)=[a_1(\theta_i)\ a_2(\theta_i) .... a_N(\theta_i)]^T$, often called the array manifold vector or the i^{th} steering vector or the i^{th} source direction vector and b(t) is the additive noise. The noise is assumed to be spatially white (independent or uncorrelated from sensor to sensor) and the same power level of noise is present in each receiver. With these assumptions, the cross-spectral matrix for the noise alone is $R_b=E[b(t)b(t)^H]=p_bI$ where p_b is the noise power, I is the (NxN) identity matrix and the superscript H denotes the complex-conjugate transpose operation. Equation (1) may be rewritten in the matrix form

$$y(t) = A(\theta)\, s(t) + b(t) \quad t \in \{t_1, t_2, \dots t_T\} \qquad (2)$$

The (NxP) matrix $A(\theta)$, where each column is a source direction vector : $A(\theta) = [\, a(\theta_1)\ a(\theta_2) .... a(\theta_P)]$, is the so-called array manifold matrix. For any single plane wave arrival, the outputs from the N individual receivers will differ in phase by an amount determined by the geometry of the array and the arrival direction. In other words, the elements A_{qr} of the matrix $A(\theta)$ are functions of the signal arrival angles and the array elements locations. Thus, one has $A_{qr} = \exp(j\,\phi_{qr})$ where ϕ_{qr} is the phase of the signal at the q^{th} receiver from the r^{th} source, measured relative to some arbitrary reference point. That is, A_{qr} depends on the q^{th} array element, its position relative to the origin of the coordinate system, and its response to a signal incident from the direction of the r^{th} source. s(t) is the K-dimensional vector, the components of which are the complex amplitudes of the sources. Since the P arrivals are by assumption independent, the correlation matrix between the different signal sources is

$$R_s=E[s(t)s(t)^H] = \mathrm{diag}(p_1, p_2, \dots, p_P) \qquad (3)$$

and at the operating frequency, the spatial correlation matrix (or covariance matrix) of the receiver outputs may be expressed, for signals uncorrelated of each other and of noise, as

$$R = E[y(t)y(t)^H] = A(\theta)\, R_s\, A(\theta)^H + R_b \qquad (4)$$

In the case of wideband sources it is impossible to represent the delays by simple phase shifts. A way to deal with this issue is to separate the signal spectrum into several narrowband regions, each of which yields to narrowband processing. To separate the spectrum into narrowband regions it is possible to use a filter bank $h_1(t)$, $h_2(t), \dots, h_N(t)$ in which each filter $h_k(t)$ has a small spectral support around the central frequency f_k, satisfying the narrowband assumption. After filtering the outputs of each sensor with each filter the result is a set of W time-domain problems of the form

$$y_k(t) = A(f_k, \theta)\, s_k(t) + b_k(t) \qquad (5)$$

We can then solve each of the W problems using the narrowband methods described in the communication. Once we solve each of the narrowband sub-problems we get a spatio-frequency spectrum of the localized radiating sources. Unlike the narrowband case, in the wideband case the steering matrix is not the same for all snapshots since it depends on frequency. For simplicity, we consider in our presentation the narrowband case and the estimation of directions of arrival is a nonlinear parametric problem: these directions are present in the array manifold matrix $A(\theta)$ which is unknown. This matrix is composed of steering vectors corresponding to each source location. Our objective is to estimate the directions of arrival $\{\theta_i\}$, i=1,2…,P from the N sensors output data y(t) in the case of correlated or uncorrelated sources, narrowband or wideband sources.

3 Source localization by sparsity

3.1 Overcomplete representation

The problem solved by the sparse representation is to search for the most compact representation of a signal in terms of linear combination of atoms in an overcomplete dictionary. The problem of finding the sparse representation of a signal in a given overcomplete dictionary can be formulated as follows. Given a (NxL) matrix $A \in C^{NxL}$ containing the elements of an overcomplete dictionary in its columns, with L>N, and usually L>>N, and a (Nx1) signal $y \in C^N$, the problem of sparse representation is to find an (Lx1) coefficient vector x such as y = Ax and $\|x\|_0$ is minimized, where $\|x\|_0$ is the l_0 norm and is equivalent to the number of non-zero components in the vector x. We would like to find [4-6]

$$\min \|x\|_0 \ \text{ subject to } \ y=Ax \qquad (6)$$

Finding the solution to equation (6) is very hard due to its nature of combinational optimization. It can be shown that under certain conditions on A and x, the optimal value of this problem can be found exactly by replacing the l_0-norm in equation (6) with the l_1-norm and by solving a related problem

$$\min \|x\|_1 \ \text{ subject to } \ y=Ax \qquad (7)$$

A natural extension when we allow white Gaussian noise is

$$y(t) = A\, x(t) + b(t)\ t \in \{t_1, t_2, \dots t_T\} \qquad (8)$$

A generalized version of equation (7), which allows for certain degree of noise, is to find x such that the following objective function is minimized [4-6]

$$\min \|y - Ax\|_2^2 + \lambda \|x\|_1 \qquad (9)$$

where λ is a scalar regularization parameter that balances the tradeoff between reconstruction error and the sparsity of the solution. The l_2-term forces the residual y-Ax to be small, whereas the l_1-term enforces sparsity of the representation. The optimization criterion is a convex optimization problem and can be readily handled by the use of a second order cone programming (SOC) algorithm [7].

Note that equation (8) is different from equation (2). The (NxP) matrix $A(\theta)$ in (2) depends of the unknown source locations that we wish to estimate. The (NxL) matrix A is known and does not depend on the true source locations: following a spatial sparsity approach we can uniformly discretize the bearing space into L>>P possible angle of arrival and construct a redundant matrix of L atoms corresponding to the array responses of the respective angles of arrival. We introduce then a grid of possible source locations $\{\tilde{\theta}_1 \ a\,\tilde{\theta}_2 .... a\tilde{\theta}_L\}$ and construct the (NxL) matrix $A=[a(\tilde{\theta}_1)\ a(\tilde{\theta}_2) .... a(\tilde{\theta}_L)]$. This matrix is composed of steering vectors corresponding to each potential source location as its columns and form the overcomplete basis of the sparse representation. Also, let

$$x_i(t) = \begin{cases} s_k(t) \text{ if } \tilde{\theta}_i = \theta_k \\ 0 \quad \text{otherwise} \end{cases} \tag{10}$$

and the problem takes the form given in (8). The important point is that A is known and does not depend on the source location $\{\theta_i\}$, as $A(\theta)$ did. For one snapshot, the sparse representation of sensor measurements takes the following form

$$\begin{bmatrix} y_1(t_j) \\ y_2(t_j) \\ . \\ y_N(t_j) \end{bmatrix} = [a(\tilde{\theta}_1)\ \ a(\tilde{\theta}_2) .... a(\tilde{\theta}_P) .... a(\tilde{\theta}_L)] \begin{bmatrix} 0 \\ x_1(t_j) \\ 0 \\ . \\ . \\ x_P(t_j) \\ 0 \end{bmatrix} + \begin{bmatrix} b_1(t_j) \\ b_2(t_j) \\ . \\ b_N(t_j) \end{bmatrix} \tag{11}$$

This expression constitutes the overcomplete representation for a single time sample. The source locations are now encoded by the non-zero indices of x(t) which is a L-sparse vector. With this representation the recovery of the source locations is equivalent to the recovery of the support of the sparse vector x(t). In effect, we have transformed the problem from finding a point estimate of (θ), to estimating the spatial spectrum of x(t) which has to exhibit sharp peaks at the correct source locations. In principle, one can use the over complete basis methodology at each time t. This leads to an important computational load and to sensitivity to noise, since no advantage is taken of other time samples. We would like to use all the sensor data in synergy to obtain greater accuracy and robustness to noise.

3.2 *l*1-SVD based approach

We consider an approach that uses different time samples in synergy. Let Y the (NxT) matrix of time data, X the (LxT) matrix parameterized temporally and spatially and N the (NxT) matrix of noise

$$Y = [y(1)\ y(2)\ ....\ y(T)] \tag{12}$$

$$X = [x(1)\ x(2)....x(T)] \tag{13}$$

$$B = [b(1)\ b(2)\ ....b(T)] \tag{14}$$

Then, (8) becomes

$$Y = AX + B \tag{15}$$

The matrix X is parameterized temporally and spatially but sparsity only has to be enforced in space since the sources amplitude s(t) are in general not sparse in time. To reduce the computational complexity and sensitivity to noise, we use the SVD of the (NxT) data matrix Y; we obtain then the signal and noise subspaces, we keep the signal subspace and mold the problem with reduced dimensions into a multiple sample sparse spectrum estimation problem. We take the singular value decomposition of the observed data matrix Y

$$Y = WMV' \tag{16}$$

and we keep a reduced (NxP) dimensional matrix Y_{SV} (the reduced observation matrix) which contains most of the signal power (the signal subspace)

$$Y_{SV} = WMD_p = YVD_p \tag{17}$$

where $D_p = [I_P\ 0]^T$ with I_P the (PxP) identity matrix an 0 the Px(T-P) matrix of zeros. Similarly we get

$$X_{SV} = XVD_p \ \ ; B_{SV} = BVD_p \tag{18}$$

and then we have

$$Y_{SV} = AX_{SV} + B_{SV} \tag{19}$$

Note that we have transformed Y (NxT) into Y_{SV} (NxP), X (LxT) into X_{SV} (LxP) and B (NxT) into B_{SV} (NxP) which reduces the computational complexity. For typical situations where the number of sources is small and the number of time samples may be in the order of hundreds this reduction in complexity is very appreciable.

Now, let us consider each column of this equation separately, which corresponds to each singular vector of the subspace signal

$$y_{SV}(p) = A\ x_{SV}(p) + b_{SV}(p) \qquad p=1,2\ldots,P \tag{20}$$

This equation has the same form as (8), except that instead of indexing samples by time, we index them by the singular vector number. We combine the data with respect to the singular vector index p using an l_2 norm. We define

$$\tilde{x}_i^{(l_2)} = \sqrt{\sum_{p=1}^{P} (x_{SV\,i}(p))^2} \tag{21}$$

and want to impose sparsity in X_{sv} only spatially, (in terms of i) and not in terms the singular vector index p. The sparsity of the resulting (Lx1) vector $\tilde{x}^{(l_2)}$

$$\tilde{x}^{(l_2)} = [\,\tilde{x}_1^{(l_2)}\ \tilde{x}_2^{(l_2)} ....\tilde{x}_L^{(l_2)}\,]^T \tag{22}$$

corresponds to the sparsity of the spatial spectrum. Since one requires sparsity in the spatial dimension only, we compute in first the *l*2-norm of all singular vectors of a particular spatial index of x_{SV} following (21), we penalize the *l*1-norm of $\tilde{x}^{(l_2)}$

$$\left\| \tilde{x}^{(l_2)} \right\|_1 = \sum_{i=1}^{L} \sqrt{\sum_{p=1}^{P} (x_{SV\,i}(p))^2} = \tag{23}$$

We can find the spatial spectrum by minimizing the cost function

$$J(\tilde{x}) = \left\| Y_{SV} - A\ X_{SV} \right\|_f^2 + \lambda \left\| \tilde{x}^{(l_2)} \right\|_1 \tag{24}$$

Note that this method uses information about the number of sources P, however, an incorrect determination of the number of sources in our framework has no important consequences, since we are not relying on the structural assumptions of the orthogonality of the signal and noise subspaces, such as the algorithm MUSIC does.

We have to minimize the objective function (24). We use the second order cone programming (SOCP) which is a suitable framework for optimizing functions [7]. The quality of estimation is confined to the grid resolution given by the angular position (θ) . We cannot make the grid very fine uniformly, since this would increase the computational complexity significantly. We create an iterative process of refining the grid whit higher and higher resolution around the estimated regions. This requires an approximate knowledge of the locations of the sources, which can be obtained by using a coarse grid first.

It should be noted that the SOCP (or *l*1-SVD) algorithm presents often a bias in the localization of closely spaced sources. We can easily verify that this is not due to the grid: even if the sources are present in the grid, the angular deviation exists. In fact, it is a bias inherent to the algorithm itself and is considered as a cost of the sparsity. We can attenuate this bias by a simple approach: instead of scaling the singular vectors by the singular values, while forming Y_{SV}, we may scale them by the squares of singular values. A new reduced observation matrix is then obtained. A SOCP optimal algorithm is then obtained. We present in the next section several results for the optimal SOCP source localization scheme. We compare the spectra of the methodology presented in the communication to those of beamforming, MUSIC and Capon's method under various conditions.

4 Numerical and experimental tests

Numerical simulations and experimental tests were designed to evaluate the performances of the SOCP algorithm in the source localization context. We consider a uniform linear array with N=6 omnidirectional sensors separated by half a wavelength of the actual narrowband source signals. Two narrowband signals in the far-field impinge on this array from directions of arrival 13° and 18°, which are closer together than the Rayleigh limit. The SNR is 20 dB, the number of snapshots is T=1024 and the grid is uniform with 1° sampling. We have L=180. In Figure 1 we present the spectrum obtained by SOCP and SOCP optimal. A bias is present with the use of SOCP (Figure 1(a)) and this bias disappears with SOCP optimal. A zoom of the spatial spectrum has been realized to observe the presence and the strongly attenuation of the bias as shown in Figure 1 (b).

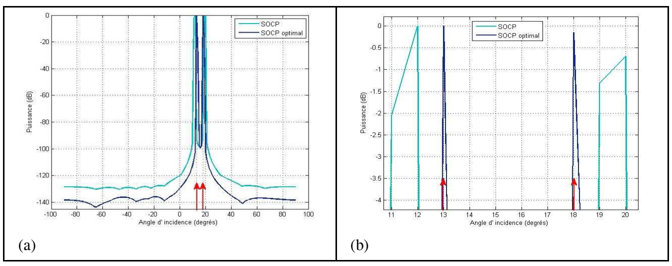

Figure1: (a) Localization of two uncorrelated sources by SOCP and optimal SOCP ; (b) zoom of the spatial spectrum

We consider now the case where the number of snapshots is T=20 and compare the spectrum obtained using the optimal SOCP method with those of conventional beamforming, Capon's method and MUSIC. We recall that the spectrum by beamforming is obtained by [1-3]

$$p_{CB} = \frac{1}{N^2} a(\theta)^H R\, a(\theta) \tag{25}$$

the spectrum by Capon is given by [1-3]

$$p_{Capon} = \frac{1}{a(\theta)^H R^{-1} a(\theta)} \tag{26}$$

the spectrum by MUSIC is given by [1-3]

$$p_{MUSIC} = \frac{1}{a^H(\theta)\, U_N U_N^H\, a(\theta)} \tag{27}$$

where U_N is the matrix of eigenvectors associated to the noise subspace.

Figure 2 (a) presents the localization of two uncorrelated sources situated at 13° and 18° using the four algorithms, with T = 20 samples. SNR is equal to 10 dB. The SOCP algorithm is able to resolve the two closely spaced sources, whereas MUSIC algorithm, Capon's method and beamforming method merge the two peaks.

The high resolution methods of spatial spectrum estimation frequency fail to work in a multipath environment if the multipath arrivals differ only in a constant carrier phase and amplitude. The magnitude of the correlation coefficient of such signals is equal to one and we refer to the signals as being totally correlated or coherent. Figure 2 (b) shows the spatial spectra for two correlated sources with T=20. SNR is 20 dB. Again, only the SOCP algorithm is able to resolve the two sources. This illustrates the power of our methodology in resolving closely spaced sources despite strong correlation between the sources.

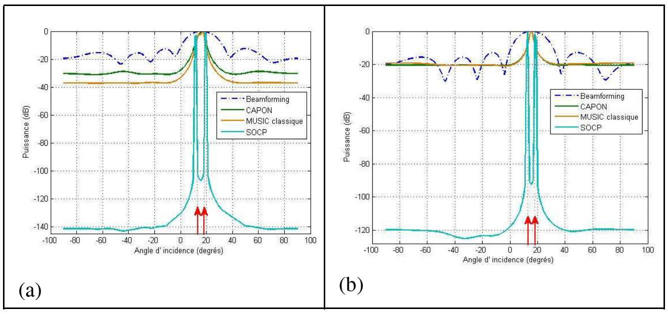

Figure 2 : (a) Localization of two uncorrelated sources ; (b) localization of two correlated sources

In Figure 3 we plot the spectra obtained using MUSIC and SOCP optimal for the same assumed number of sources which varies from 1 to 5. The exact number of sources is 2. The SOCP algorithm is more robust than MUSIC when the number of sources is unknown.

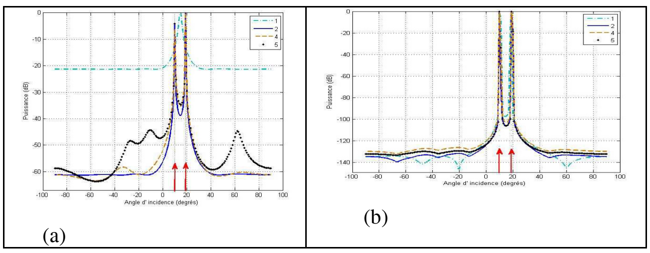

Figure 3 : Sensitivity of (a) MUSIC and (b) SOCP optimal to the assumed number of sources

In Figure 4 we present an example using the same six element uniform array but the signals are now wideband. We consider three chirps at 60°, 78° and 100°with frequency from 250 Hz to 500 H, T=1000 and SNR=20 dB. Using conventional beamforming, the chirps are merged and cannot be separated, whereas using SOCP they can be easily distinguished through their support. This shows that SOCP methodology is useful for wideband acoustical source localization.

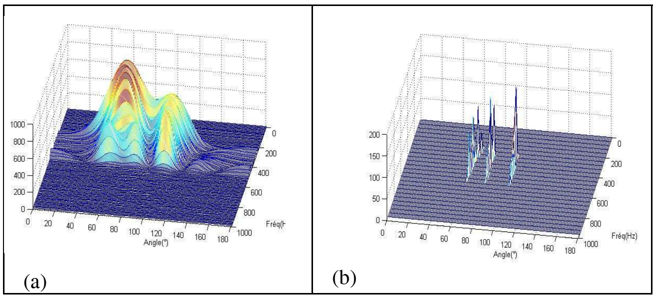

Figure 4 : Wideband source localization (a) Beamforming; (b) SOCP optimal

Experimental results are now presented, where our purpose is the localization of noise sources generated by two loudspeakers. The experimental setup is schematically shown in the block diagram of Figure 5 where an acoustical array and two sources (the loudspeakers) are placed in the anechoic chamber. The receiving acoustical array is linear and formed with six microphones equally spaced, with interelement spacing of d =4.5 cm. The two sources and the acoustical array are in the same horizontal plane. The transmitting loudspeakers generate two typical audio signals at a frequency of 3800 Hz corresponding to a microphone separation distance of one-half wavelength. The number of snapshots is T=4096. We are able to find the direction of the two sources by using the algorithms presented in the communication.

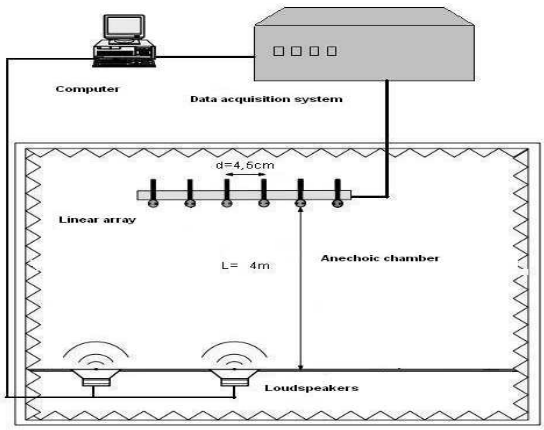

Figure 5: Block diagram experimental system

Figure 6 shows the experimental results of the spatial spectra for conventional beamforming, Capon, MUSIC and optimal SOCP algorithms. The two close acoustical sources are uncorrelated (Figure 6(a)) and correlated (Figure 6(b)). It is shown from these plots that the SOCP algorithm gives better results than conventional beamforming, Capon and MUSIC in particular for the correlated case. From these spatial-spectra plots we obtain the arrival angles of the two close sources $\theta_1 = 13°$ and $\theta_2 = 18°$.

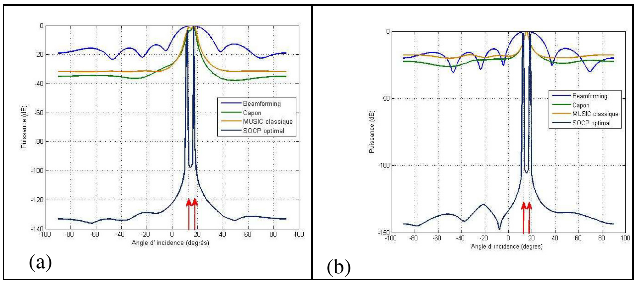

Figure 6 : Localization of acoustical sources in anechoic room. (a) Uncorrelated sources; (b) Correlated sources

5 Conclusion

We have presented a formulation of sensor array source localization problem in a sparse signal representation framework. The method presents several advantages over existing source localization methods such as beamforming, Capon and MUSIC including increased resolution, improved robustness to correlation of the sources and robustness in the localization of wideband sources. Numerical and experimental results have shown the effectiveness of the procedure. Further research include an investigation about the computation of the regularization parameter, a study of uniqueness and stability of sparse signal representation for the overcomplete bases that arise in source localization applications, an analysis of the number of sensors, the number of snapshots and the SNR in source localization.

These techniques have been developed for the localization of sources using an array of receivers. However, the principles can be applied to a wide range of other estimation problems, of which the spectrum analysis of a time series for eigenrequencies determination is an example

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