THIS PRESENTATION IS CANCELLED (PAPER IS AVAILABLE IN PROCEEDINGS). Acoustic characterization of air saturated porous materials at audible frequencies

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An acoustic method based on sound transmission is proposed for measuring the viscous and thermal permeability, viscous and thermal tortuosity, and porosity of porous materials having a rigid frame at low frequencies. The proposed method is based on a temporal model of the direct and inverse scattering problems for the propagation of transient audible frequency waves in a homogeneous isotropic slab of porous material having a rigid frame. The acoustic parameters are determined from the solution of the inverse problem. The minimization between experiment and theory is made in the time domain. Tests are performed using industrial plastic foams.

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1 Introduction

The acoustic characterization of porous materials saturated by air [1] such as plastic foams, fibrous, or granular materials is of great interest for a wide range of industrial applications. These materials are frequently used in the automotive and aeronautics industries and in the building trade.

Acoustic damping in air-saturated porous materials is described by the inertial, viscous, and thermal interactions between the fluid and the structure [2]. Depending on the temporal component of the acoustic excitation (pulse duration), the relaxation times describing the fluid structure interactions are different. In the high frequency domain [1], the inertial, viscous, and thermal interactions are modeled by causal integro-differential operators acting as fractional derivatives [3] in the time domain. These interactions are taken into account, by the high approximation of the tortuosity for the inertial effects, and by the viscous characteristic length and thermal characteristic length for the viscous and thermal losses. In the low frequency domain, the inertial, viscous, and thermal interactions are described, respectively, by the viscous and thermal tortuosity, the viscous and thermal permeability [4]. In addition to these parameters, the porosity is a key parameter playing an important role for all relaxation times. The determination of these parameters is crucial for the prediction of the sound damping in these materials.

At present, the experimental and theoretical methods developed for the acoustic characterization of porous materials saturated by air are based on solving the direct and inverse scattering problems at the asymptotic domain (corresponding to the high frequency range). In this domain, the porosity, tortuosity, and characteristic lengths are estimated using reflected or transmitted waves. The very low frequency range has been investigated for measuring the viscous permeability [5]. In this range of frequencies, the propagation phenomenon is replaced by a diffusion process (Darcy regime) of the acoustic wave. More recently [6] the viscous tortuosity (inertial factor) and the thermal permeability have been measured by solving the inverse problem. However, we can note that the direct and inverse problems are not solved for determining the thermal tortuosity and porosity. These two parameters are important for describing the acoustic propagation at the viscous regime corresponding to the low frequency range. This domain corresponds to the range of frequencies such that viscous skin thickness $\delta = (2\eta/\omega\rho)^{1/2}$ is much larger than the radius of the pores, (η and ρ) are, respectively, the viscosity and density of the saturated fluid and ω represents the pulsation frequency).

The viscous tortuosity α_0 [7] corresponds to the low frequency approximation of the dynamic tortuosity given by Norris and also by Lafarge $\alpha_0 = \langle v(r)^2 \rangle / \langle v(r) \rangle^2$ where $\langle v(r) \rangle$ is the average velocity of the viscous fluid for direct current flow within a volume element, small compared to the relevant wavelength, but large compared to the individual grains/pores of the solid. The high frequency tortuosity α_∞ has a similar definition than α_0, but the difference is that for α_∞ v(r) corresponds to the velocity of the perfect incompressible fluid. $\alpha_0^{'}$ [2] is the thermal counterpart of α_0.

The static thermal permeability $k_0^{'}$ [2,4] of the porous material is a geometrical parameter equal to the inverse trapping constant of the solid frame. This parameter plays a role similar to the viscous permeability k_0 in the description of the viscous forces. Finally, another important parameter which appears in theories of sound propagation in porous materials is the porosity. Porosity is the relative fraction, by volume, of the air contained within the material. Unlike other parameters included in the description of different various phenomena occurring in the acoustical propagation of porous media at high and low frequency range, porosity is a key parameter that plays an important role in propagation at all frequencies.

In this work, we present a simple acoustical method for evaluating viscous and thermal tortuosity, viscous and thermal permeability, and porosity by measuring a propagating wave transmitted by a slab of air-saturated porous material in a guide (pipe). The advantage of the proposed method is that these four parameters can be obtained simultaneously by solving the inverse problem.

The outline of this paper is as follows. Section 2 recalls the equivalent fluid model and the low frequency approximation of the dynamic tortuosity and compressibility. Section 3 is devoted to the time domain formulation of the problem and to the propagation equation. Section 4 contains the solution of the inverse problem and the experimental estimation of the physical parameters.

2 Model

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory [8]. In air-saturated porous media, the vibrations of the structure can be neglected when the excitation is not very important, and the waves can be considered to propagate only in fluid. This case is described by the model of equivalent fluid, which is a particular case of the Biot model in which fluid-structure interactions are taken into account in two frequency response factors: dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al.* [9] and revisited by Pride [10] and Lafarge [2,4], the dynamic compressibility of the air in the porous material $\beta(\omega)$ given by Allard [1] and Lafarge [2,4]. In the frequency domain, these factors multiply the fluid density and compressibility, respectively, and show the deviation from fluid behavior in free space as frequency changes. Their expressions are given by:

$$\alpha(\omega) = \alpha_\infty \left(1 - \frac{1}{jx}\left(1 - p + p\sqrt{1 - \frac{M}{2p^2} jx}\right)\right),$$

$$\beta(\omega) = \gamma - (\gamma-1) \Big/ \left(1 - \frac{1}{jx'}\left(1 - p' + p'\sqrt{1 - \frac{M'}{2p'^2} jx'}\right)\right),$$

Where:

$$x = \frac{\omega\alpha_\infty \rho k_0}{\eta\phi},\ M = \frac{8k_0\alpha_\infty}{\phi\Lambda^2},\ x' = \frac{\omega\rho k_0' P_r}{\eta\phi},\ M' = \frac{8k_0'}{\phi\Lambda'^2}.$$

In these expressions $j^2 = -1$, P_r the Prandtl number, p and p' are the geometrical Pride parameters, given by:

$$p = \frac{M}{4\left(\frac{\alpha_0}{\alpha_\infty} - 1\right)} \quad \text{and} \quad p' = \frac{M'}{4\left(\alpha_0' - 1\right)}.$$

The functions $\alpha(\omega)$ and $\beta(\omega)$ express the viscous and thermal exchanges between the air and the structure which are responsible for the sound damping in acoustic materials. These exchanges are due on the one hand to the fluid-structure relative motion and on the other hand to the air compressions-dilatations produced by the wave motion. The parts of the fluid affected by these exchanges can be estimated by the ratio of a microscopic characteristic length of the media, as, for example, the sizes of the pores, to the viscous and thermal skin depth thickness $\delta = \left(2\eta/\omega\rho\right)^{1/2}$ and $\delta' = \left(2\eta/\omega\rho P_r\right)^{1/2}$. For the viscous effects this domain corresponds to the region of the fluid in which the velocity distribution is perturbed by the frictional forces at the interface between the viscous fluid and the motionless structure. For the thermal effects, it is the fluid volume affected by the heat exchanges between the two phases of the porous medium.

2.1 Viscous domain (low frequency range)

In this domain, the viscous forces are important everywhere in the fluid, and the compression dilatation cycle in the porous material is slow enough to favor the thermal interactions. At the same time the temperature of the frame is practically unchanged by the passage of the sound wave because of the high value of its specific heat: the frame acts as a thermostat. This domain corresponds to the range of frequencies such that viscous skin thickness $\delta = \left(2\eta/\omega\rho\right)^{1/2}$ is much larger than the radius of the pores r. In the low frequency domain, Lafarge [2,4], and Norris [7] have extended the low frequency development of $\alpha(\omega)$ and $\beta(\omega)$:

$$\alpha(\omega) = -\frac{\eta\phi}{j\omega\rho k_0} + \alpha_0 + \frac{2\alpha_\infty^4 k_0^3 \rho}{\eta\Lambda^4\phi^3 p^3} j\omega + \ldots (1)$$

$$\beta(\omega) = \gamma + \frac{(\gamma-1)k_0' P_r \rho}{\eta\phi} j\omega - \frac{\alpha_0'(\gamma-1)k_0' P_r^2 \rho^2}{\eta^2\phi^2}\omega^2 + \ldots (2)$$

To note that third term in does not result from the real expansion of $\alpha(\omega)$, this would imply a new geometric parameter that has not been identified. We content ourselves for now to use the model as for the characterization, even though we know that the development of alpha is incomplete.

3 Wave equation

Consider a homogeneous porous material that occupies the region $0 \le x \le L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field p and an acoustic velocity field t within the material. In the low frequency range, the acoustic fields satisfy the Euler equation and the constitutive equation (along the x axis). The basic equations of the equivalent fluid model are:

$$\rho\tilde{\alpha}(t) \otimes \frac{\partial v_i}{\partial t} = -\nabla_i p, \quad \text{and} \quad \frac{\tilde{\beta}(t)}{K_a} \otimes \frac{\partial p}{\partial t} = -\nabla . v, \ (1)$$

Where $\otimes$ denotes the time convolution operation, p is the acoustic pressure, v is the particle velocity, and K_a is the bulk modulus of the air. The first equation is the Euler equation, and the second one is a constitutive equation obtained from the equation of mass conservation associated with the behavior (or adiabatic) equation. $\tilde{\alpha}(t)$ and $\tilde{\beta}(t)$ are the tortuosity and compressibility operators. Their expressions are obtained by taking the inverse Fourier transform of $\alpha(\omega)$ and $\beta(\omega)$ as follows:

$$\tilde{\alpha}(t) = \frac{\eta\phi}{\rho k_0}\partial_t^{-1} + \alpha_0\delta(t) + \frac{2\alpha_\infty^4 k_0^3 \rho}{\eta\Lambda^4\phi^3 p^3}\partial_t + \ldots$$

$$\tilde{\beta}(t) = \gamma\delta(t) - \frac{(\gamma-1)k_0' P_r \rho}{\eta\phi}\partial_t + \frac{\alpha_0'(\gamma-1)k_0' P_r^2 \rho^2}{\eta^2\phi^2}\partial_t^2 + \ldots$$

In these equations, $\delta(t)$ is the Dirac operator, ∂_t the time partial derivative, ∂_t^{-1} is the integral operator $\partial_t^{-1} g(t) = \int_0^t g(t')dt'$.

The medium varies with depth x only and the incident wave is planar and normally incident. With no lack of generality, the pressure and particle velocity acoustic fields can be assumed to have only one component, denoted $p(x,t)$ and $v(x,t)$. It is assumed that the pressure and velocity field in the medium are zero prior to $t = 0$. The constitutive equations (1) become:

$$\rho\alpha_0 \frac{\partial v(x,t)}{\partial t} + \frac{\eta\phi}{k_0} v(x,t) + \frac{2\alpha_\infty^4 k_0^3 \rho^2}{\eta\Lambda^4\phi^3 p^3}\frac{\partial^2 v(x,t)}{\partial t^2} = -\frac{\partial p(x,t)}{\partial x}, \quad (2)$$

$$\frac{\gamma}{K_a}\frac{\partial p(x,t)}{\partial t} - \left(\frac{(\gamma-1)k_0' P_r \rho}{K_a \eta\phi}\right)\frac{\partial^2 p(x,t)}{\partial t^2} + \left(\frac{\alpha_0'(\gamma-1)k_0' P_r^2 \rho^2}{\eta^2\phi^2}\right)\frac{\partial^3 p(x,t)}{\partial t^3} = -\frac{\partial v(x,t)}{\partial x}$$

Constitutive relations in the time domain result from arguments based on invariance under time translation and causality. The parameter α_0 reflects the instantaneous response of the porous medium and describes the inertial coupling between fluid and structure. Instantaneous therefore means that the response time is much shorter than the typical time scale for acoustic field variation. The Euler Eq. (2) expresses the balance between the driving forces of the wave, the drag forces $\eta\phi/K_a$ due to the flow resistance of the material, and the inertial forces. The wave equation is derived from these two relations by elementary manipulations as follows:

$$\frac{\partial^2 p(x,t)}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2} - A\frac{\partial p(x,t)}{\partial t} + B\frac{\partial^3 p(x,t)}{\partial t^3} = 0,$$

$$\frac{1}{c^2} = \frac{\rho}{K_a}\left(\alpha_0\gamma - \frac{(\gamma-1)P_r k_0^{'}}{k_0}\right), \quad \text{A}=\frac{\eta\phi\gamma}{k_0 K_a},$$

$$B = \frac{\alpha_0(\gamma-1)k_0^{'}P_r\rho^2}{K_a\eta\phi} - \frac{\alpha_0^{'}(\gamma-1)k_0^{'2}P_r^2\rho^2}{\eta\phi k_0 K_a} + \frac{2\alpha_\infty^4 k_0^3\rho^2\gamma}{K_a\eta\Lambda^4\phi^3 p^3}.$$

The solution of this propagation equation gives the Green function of the porous material [11].

4 Inverse Problem

The inverse problem is to find the parameters $\alpha_0, \alpha_0^{'}, k_0^{'}$, $k^{'}$ and ϕ which minimize numerically the discrepancy function

$$U\left(\alpha_0, \alpha_0^{'}, k_0, k_0^{'}, \phi\right) = \sum_{i=1}^{i=N}\left(p_{\exp}^t(x,t_i) - p^t(x,t_i)\right)^2$$ wherein

$p_{\exp}^t(x,t_i)_{i=1,2...n}$ is the discrete set of values of the experimental transmitted signal and $p^t(x,t_i)_{i=1,2...n}$ the discrete set of values of the simulated transmitted signal predicted from theory. The theoretical expression of p^t is given by: $p^t(x,t) = \int_0^t T(\tau)p^i\left(t-\tau-\frac{(x-L)}{c_0}\right)d\tau$, p^i is the incident field. The expression of the transmission operator is given in Ref. The inverse problem is solved numerically by the least square method. For its iterative solution, we used the simplex search method (Nedler Mead) [11], which does not require numerical or analytic gradients. Experiments are performed in a guide (pipe) having a diameter of 5 cm. The experimental set up is given in Fig.1.

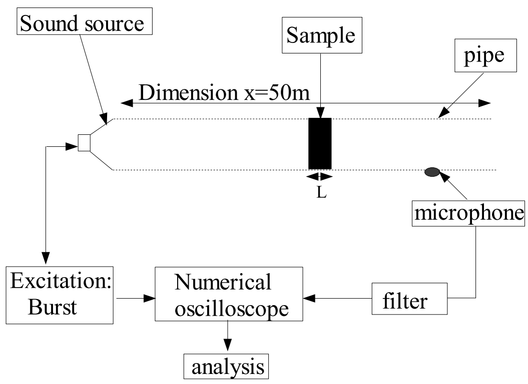

Figure 1: Experimental setup of acoustic measurements.

For measuring the viscous and thermal tortuosity, porosity and thermal permeability, a length of 3m of the pipe is enough because the frequencies used in the experiment are higher than 1 kHz. The cut-off frequency of the tube is f_c □ $4kHz$. A sound source Driver unit "Brand" constituted by loudspeaker Realistic 40-9000 is used. Bursts are provided by synthesized function generator Standford Research Systems model DS345-30MHz. The signals are amplified and filtered using model SR 650-Dual channel filter, Standford Research Systems. The signals (incident and transmitted) are measured using the same microphone (Bruel and Kjaer, 4190) in the same position in the tube. The incident signal is measured without porous sample, however, the transmitted signal is measured with the porous sample. Consider a cylindrical sample of plastic foam M of diameter and thickness of 5 cm. The viscous and thermal permeability of the porous material are measured using classic Kundt tube (continuous frequency) obtaining the value of $k_0 = 6\times10^{-9}m^2$ and $k_0^{'} = 18\times10^{-9}m^2$ with a ratio of 3 between $k_0^{'}$ and k_0. The tortuosity α_∞, and the viscous characteristic length Λ are measured in the high frequency range obtaining the values $\alpha_\infty = 1.04$ and $\Lambda = 300\mu m$. After solving the inverse problem numerically for the viscous and thermal permeability, viscous and thermal tortuosity and porosity, we find the following optimized values $k_0 = 5\times10^{-9}$, $k_0^{'} = 14.5\times10^{-9}$, $\alpha_0 = 1.42$, $\alpha_0^{'} = 1.09$, $\phi = 0.91$. We present in Figs. 2-5 the variation of the minimization function U with the inverted parameters.

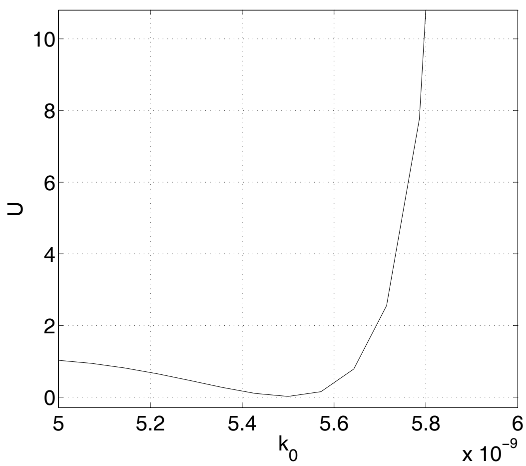

Figure 2: Variation of the minimization function *U* with the viscous permeability

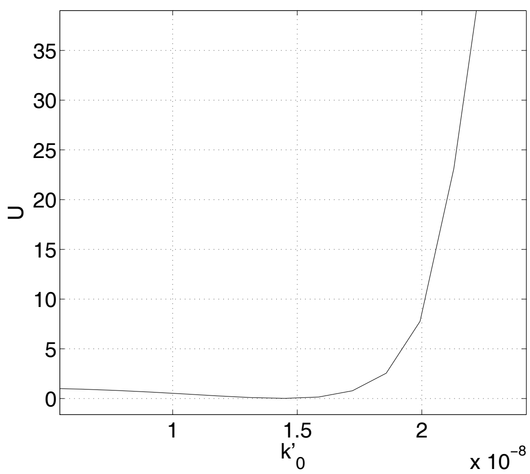

Figure 3: Variation of the minimization function *U* with the thermal permeability

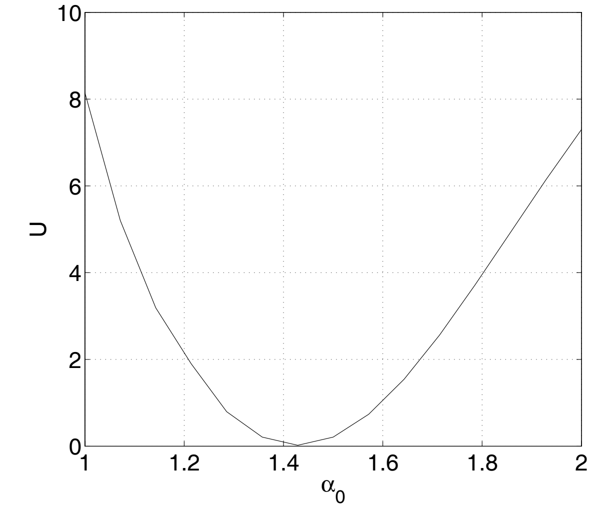

Figure 4: Variation of the minimization function *U* with the viscous tortuosity.

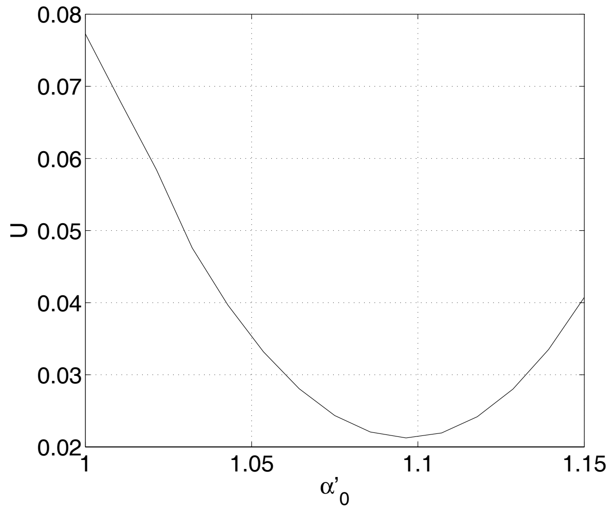

Figure 5: Variation of the minimization function *U* with the thermal tortuosity.

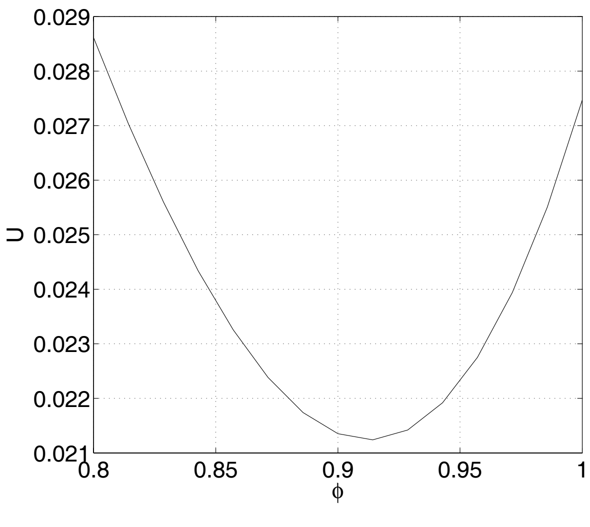

Figure 6: Variation of the minimization function *U* with the porosity.

The obtained ratio of 2.63 between the inverted $k_0^{'}$ and k_0 is consistent with the ratio given in literature that is found generally to be between 2 and 4 for plastic foams. In Fig. 6, we show a comparison between an experimental transmitted signal and simulated transmitted signal for the optimized values of viscous and thermal permeability, viscous and thermal tortuosity, and porosity.

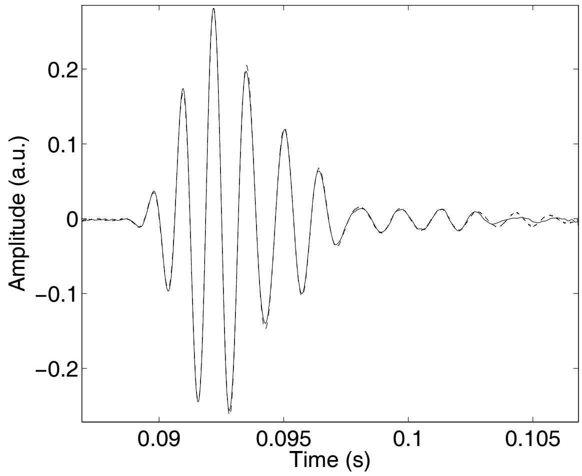

Figure 7: Comparison between the experimental transmitted signal (solid line) and the simulated transmitted signals (dashed line).

The correlation between the two curves is excellent. These results lead us to conclude that the optimized values of the thermal permeability and inertial factor are correct. The Biot's vibrations, which induced structural disturbance resulting from elasticity, prevent applying the method for the resistive porous materials having a low viscous permeability value.

5 Conclusion

In this letter, an inverse scattering estimate of the viscous and thermal permeability, the viscous and thermal tortuosity and the porosity was given by solving the inverse problem in time domain for waves transmitted by material a slab of air-saturated porous. The inverse problem is solved numerically by the least-square method. The reconstructed values of acoustical parameters are in agreement with those obtained using classical methods. The proposed experimental method has the advantage of being simple, rapid, and efficient for estimating those parameters and further characterizing porous materials.

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