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Recent developments in Code_Aster to compute FRF and modes of VEM with frequency dependent properties

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This paper tackles the way how advanced capabilities are introduced in Code_Aster to take into account frequency dependent properties of viscoelastic materials. Computing of frequency response functions and modes of vibration (real or complex) is addressed. Having frequency dependent modes is a step forward for the modal projection method and for model updating with an experimental modal basis as a reference. An iterative method is proposed and implemented in Code_Aster in order to compute frequency dependent modes. The new capabilities are then compared with the standard approaches in the case of an automotive windshield, where the viscoelastic behavior of the polyvinylbutyral which composes the laminate glass is considered.

1 Introduction

ViscoElastic Materials (VEM) are intensively used to solve NVH issues. Damping pads and anti flutter products reduce the structure borne noise. Sealants and absorbing materials prevent the air borne transmission. Adhesives are common solutions to assemble parts. Whereas VEM are well known, the finite element analysis of their dynamic behavior is not straightforward. Most of the time, the use of a finite element software requires to simplify VEM as elastic materials. This simplification is not always the obvious thing to do and may lead to inaccurate results.

The first part of this paper addresses the finite element analysis of structures comprising VEM with frequency dependent dynamic properties. Conventional approaches are discussed and a method is proposed to overcome their limitations. The method computes frequency dependent modes using an iterative algorithm. The way how these modes can be used to improve the modal projection method is described. Implementation in Code_Aster is explained.

The second part describes the application of the method to the study of an automotive laminate windshield comprising a polyvinylbutyral material with a strong frequency dependent viscoelastic behavior.

2 Finite element analysis

2.1 State-of-the-art

From a simulation point of view, the frequency response of a structure comprising at least one VEM is obtained by solving the dynamic equilibrium equation:

$$\left([K^*(\omega)]-\omega^2[M]\right)\{u(\omega)\}=\{F(\omega)\}, \tag{1}$$

with $[K^*(\omega)]$, the complex stiffness matrix as:

$$[K^*(\omega)]=[K'(\omega)]+i[K''(\omega)]. \tag{2}$$

The frequency dependence of the matrix comes from the use of frequency dependent complex moduli to represent viscoelastic behaviors [1].

In many finite element codes, solving the system of Eq. (1) is not conventional because it needs to realize the stiffness matrix for each frequency step and to have computing procedures which are able to deal with frequency dependent matrices. For instance, NASTRAN-like codes propose only a direct response approach and it is not possible to define more than one frequency dependent behavior. This limitation is incompatible with today's structures comprising several different VEM. Concerning Code_Aster, there is neither possibility to define a frequency dependent behavior, nor to solve Eq. (1) without any extra-development.

The direct response approach consists in solving Eq. (1) for each frequency step as:

$$\{u(\omega)\}=\left([K^*(\omega)]-\omega^2[M]\right)^{-1}\{F(\omega)\} \quad \forall\omega\in[0,\omega_{\max}]. \tag{3}$$

This method has the advantage of computing the exact response of the system. But, as it is necessary to compute and inverse a complex matrix at each frequency step, the computing time can become prohibitive for industrial structures with several million degrees of freedom.

A few studies [2-4] have shown the interest to solve Eq. (1) with dedicated modal response methods in order to improve the computational efficiency while maintaining the accuracy of the results. Modal response methods compute frequency responses by projecting the system of Eq. (3) on a modal basis $[T]$, with the assumption:

$$\{u(\omega)\}\approx[T]\{q(\omega)\}. \tag{4}$$

The projection of the model on the considered basis leads to a low order model, that decreases significantly the number degrees of freedom and consequently the computing time of frequency responses:

$$\{q(\omega)\}=\left([T]^T[K^*(\omega)][T]-\omega^2[T]^T[M][T]\right)^{-1}[T]^T\{F(\omega)\}. \tag{5}$$

Using properties of elastic models (real and frequency independent stiffness matrix), the standard basis of the popular spectral decomposition method combines normal modes solving of the classical eigenvalue problem:

$$\left([K]-\omega_r^2[M]\right)\{\phi_r\}=\{0\} \tag{6}$$

and a static correction to ensure a correct representation of the low frequency contribution of truncated high frequency normal modes [5]. Classically, $[T]$ is composed by the normal modes between $0.$ and a minimum of $1.5\times\omega_{\max}$.

However, frequency dependence of the VEM dynamic properties prevents from using the spectral decomposition method, because the modal basis, $[\{\phi_{r=1,N}\}]$, is only valid at the frequency ω_{ref}, for which it has been computed. In practice, it has been demonstrated [3] the validity domain of the basis can be extended in a range around the frequency ω_{ref}. If the VEM of the model give increasing moduli with respect to the frequency, the use of $\omega_{\text{ref}}=\omega_{\max}$ will lead to a larger validity domain than for any other frequencies. Unfortunately, the validity domain of the basis may not extend over all the frequency range of interest. Inaccurate results may be obtained.

2.2 Computation of frequency dependent modes

For a frequency dependent real stiffness matrix, $[K(\omega)]$, the eigenvalue problem of Eq. (6) is written as:

$$\left([K(\omega)]-(\omega_r(\omega))^2[M]\right)\{\phi_r(\omega)\}=\{0\}, \tag{7}$$

where the eigenfrequency, $\omega_r(\omega)$, and the eigenvector, $\{\phi_r(\omega)\}$, are frequency dependent too. For a fixed, given frequency, $\omega=\omega_p$, such eigensolutions can be obtained by solving the standard problem of Eq. (6). Hence, the proposed method for solving the frequency dependent problem of Eq. (7) consists in using an iterative algorithm searching for:

$$\left|\omega_r(\omega_p)-\omega_p\right|<\varepsilon \quad \forall\omega_p\in[\omega_1,\omega_2], \tag{8}$$

where ω_1 and ω_2 are respectively the lower and the upper cut-off frequencies of the eigenproblem and ε is the convergence criterion of the iterative algorithm. First, for $\omega_p=\omega_1$, the stiffness matrix is realized and the eigenproblem of Eq. (6) is solved. Then, ω_p is updated by taking the value of the n^{th} eigenfrequency for which:

$$\left|\omega_n-\omega_p\right|=\min\left|\omega_{r=1,N}-\omega_p\right|. \tag{9}$$

Next, the stiffness matrix is realized for the new value of ω_p and the eigenproblem of Eq. (6) is solved again. The iterations will not stop while

$$\left|\omega_n-\omega_p\right|\geq\varepsilon. \tag{10}$$

When the procedure converges, ω_n and $\{\phi_n\}$ are extracted to form the frequency dependent eigensolutions of Eq. (7) and the iterative algorithm will continue using $\omega_p=\omega_{n+1}$ as the guess value to compute the next eigensolution. Finally, the algorithm will stop when $\omega_p>\omega_2$. It means all the frequency dependent eigensolutions have been computed between ω_1 and ω_2.

The computing time of the proposed method is classically driven by the number of frequency dependent modes to be computed, the number of degrees of freedom of the model and the value of the convergence criterion, ε. It is also dependent on the number of normal eigenvalues which are computed as solutions of Eq. (6) at each iteration. Having all the eigenvalues between ω_1 and ω_2 for each iteration is not necessary and could lead to prohibitive computing times. In theory, a minimum of two eigenvalues may be sufficient to run iterations: ω_n to update ω_p as described in Eq. (9) and ω_{n+1} to continue when the procedure converges. In practice, this minimum number of eigenvalues will be determined by the capabilities of the numerical solver for Eq. (6). It will be discussed for Code_Aster in section 2.4.

The proposed method can be naturally extended to the study of damped structures comprising VEM. The frequency dependent eigenvalues and eigenvectors are then computed as solutions of the following problem:

$$\left([K^*(\lambda_r)]+\lambda_r^2[M]\right)\{\psi_r\}=\{0\}, \tag{11}$$

with $\lambda_{r=1,N}^2$ the complex eigenvalues such that $\lambda_r^2=-\tilde{\omega}_r^2(1+i.\eta_r)$ with η_r the modal structural damping factor, and $\{\psi_{r=1,N}\}$, the associated complex modes. In this case, Eq. (8) is rewritten as:

$$\left|\tilde{\omega}_r(\omega_p)-\omega_p\right|<\varepsilon \quad \forall\omega_p\in[\omega_1,\omega_2], \tag{12}$$

with $\tilde{\omega}_r=\sqrt{-\mathrm{Re}(\lambda_r)}$, the corresponding eigenfrequency in a structural damping model [6]. ω_p will be updated by taking the value of the n^{th} eigenfrequency for which:

$$\left|\tilde{\omega}_n-\omega_p\right|=\min\left|\tilde{\omega}_{r=1,N}-\omega_p\right|. \tag{13}$$

Computing frequency dependent real or complex modes should be a step forward for comparison with experimental modal bases. Indeed, when structures comprising VEM are tested, their frequency dependent behaviors are physically measured. The proposed method could help to validate or to update finite element models with experimental references.

2.3 Improvement of the modal projection method

The frequency dependent modes can be used to form the basis of the modal projection method for response computations of VEM. They will extend the validity domain of the basis over all the frequency range of interest. In combination to a static correction, either real modes will be used for weakly damped structures, or complex modes for highly damped structures [3]. The static correction will be determined with the stiffness matrix realized for $\omega=0$. Real modes will be preferred to reduce computing times, since numerical solvers are much more efficient in this case. But for highly damped structures, the projection base composed with such modes may be insufficient to obtain accurate frequency responses. This can be improved using the modified Modal Strain Energy (MSE) method to reduce the errors [7]. So, frequency dependent real modes, $\{\hat{\phi}_r(\omega)\}$, become solutions of a modified form of Eq. (7) in order to take into account the damping matrix (imaginary part of the stiffness matrix) as:

$$\left([K'(\omega)]+\beta(\omega)[K''(\omega)]-(\hat{\omega}_r(\omega))^2[M]\right)\{\hat{\phi}_r(\omega)\}=\{0\}. \tag{14}$$

$[K'(\omega)]$ and $[K''(\omega)]$ are defined by Eq. (2) and $\beta(\omega)$ is calculated by the following empirical formula:

$$\beta(\omega) = \frac{\mathrm{trace}[K''(\omega)]}{\mathrm{trace}[K'(\omega)]}, \quad (15)$$

where the trace for an $N \times N$ matrix is defined as:

$$\mathrm{trace}[A] = \sum_{j=1}^{N} A_{jj} . \quad (16)$$

In the iterative algorithm, the use of the modified MSE method leads to calculate $\beta(\omega_p)$ for each iteration and to search for the solutions of the resulting eigenvalue problem:

$$\left([K'(\omega_p)] + \beta(\omega_p)[K''(\omega_p)] - \hat{\omega}_r^2[M]\right)\left\{\hat{\phi}_r\right\} = \{0\}. \quad (17)$$

Another method to improve the real modes could be the use of residual modes whose purpose is to represent the damping of VEM in the modal projection basis [8]. The coupling with frequency dependent modes has not yet been investigated.

2.4 Implementation in Code_Aster

The approach to frequency dependent modes described above has been implemented in Code_Aster via a macro command, namely "MACRO_DYNA_VISCO". This script uses standard commands existing in Code_Aster for the conventional modal method. Python codes have been added for the new developments:

- definition of a frequency dependent behavior,
- computation of frequency dependent modes by the iterative algorithm,
- frequency response computation with realization of the stiffness matrix at each frequency step.

In the iterations, modes are computed as solutions of Eq. (6) via the standard "MODE_ITER_SIMULT" command. Ten eigensolutions are searched around ω_p for each iteration. This number is a good compromise between securing the convergence of the iterative algorithm and limiting the total computing time.

The user can define several VEM with different behaviors and choose the method to compute the responses (direct, modal with constant real modes or modal with frequency dependent modes). The frequency dependent modes can be obtained using the modified MSE or not. At this time of the implementation, it is possible to compute frequency dependent complex modes, but they cannot be used as a modal basis for response computation because of existing programming locks in Code_Aster. This will be investigated in further works.

3 Industrial application

The frequency dependent modes approach described above and implemented in Code_Aster was applied to an automotive laminate windshield comprising a polyvinylbutyral material between two glass layers. The objectives were to validate the proposed approach and to compare with the standard computational methods for both modes and responses.

3.1 Description of the model

The thickness of the polyvinylbutyral was 0.76 mm. The thickness of each glass layer was 2.1 mm. The finite element mesh was realized with 1152 shell elements for the glass and 1728 solid elements for the polyvinylbutyral (three layers in the thickness). Shell and solid elements were linked with coincident nodes (compatible meshes) and an offset is introduced to define each shell mid layer. The aim was to compute velocity responses of the windshield for an unitary excitation up to 1000 Hz. The perimeter of the windshield was clamped. Excitation and responses were defined on the upper glass layer, normal to the windshield plane. Normal velocities were averaged using the responses for 18 nodes as shown in Figure 1.

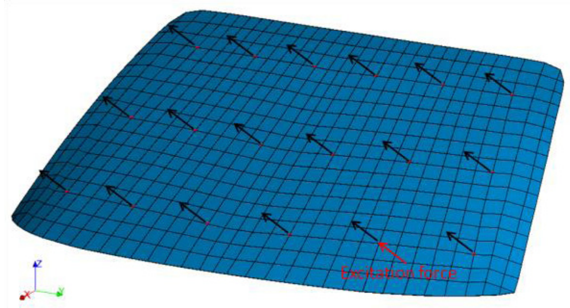

Figure 1: FE model of the windshield (red arrow for excitation force, black arrows for responses).

The polyvinylbutyral was considered as an isotropic VEM (Poisson's ratio of 0.45) and the glass as an isotropic elastic material with a constant loss factor of 1%. The dynamic behavior of the VEM was represented by the complex shear modulus extracted from the nomogram in Figure 2.

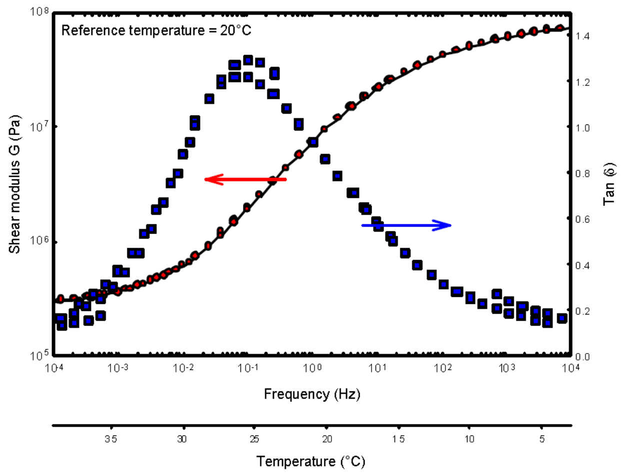

Figure 2: Reduced frequency nomogram of a standard polyvinylbutyral [9].

3.2 Comparison of modal bases

With the objective to compute responses up to 1000 Hz, modal bases were determined between 0 and 1500 Hz. Constant real modes were computed using the standard Sorensen's method implemented in Code_Aster [10], for a stiffness matrix realized at the frequency of 1000 Hz. Using the proposed developments, frequency dependent modes were searched with an accuracy of 0.01 Hz, i.e. $\varepsilon = 0.01$

in Eq. (8). Computing times are given in Table 1 as ratios to the time needed to have the constant real modes.

Table 1: Computing times of modal bases with Code_Aster.

Frequency dependent	Real/Complex	Modified MSE	Computing time
No	Real	No	1.0
Yes	Real	No	20.3
Yes	Real	Yes	22.5
Yes	Complex		28.7

Table 1 shows the extra cost of the modified MSE method is negligible compared to the total cost of the frequency dependent real modes. 119 constant real modes were computed between 0 and 1500 Hz, whereas “only” 115 frequency dependent modes (real or complex) were found. These four additional modes appear because of the approximation of the VEM as an elastic material.

Twelve frequency dependent real modes and the corresponding constant real modes are compared in Table 2 according to their frequencies.

Table 2: Comparison between twelve pairs of modes.

	Eigenfrequency (Hz)		
Mode order	Frequency dependent real modes	Constant real modes	Shift (Hz)
1	137.39	139.13	1.74
10	329.57	333.67	4.10
20	476.87	482.87	6.00
30	619.82	626.68	6.86
40	731.06	737.75	6.69
50	843.82	848.55	4.73
60	942.78	944.7	1.92
70	1053.29	1052.52	-0.77
80	1149.67	1147.13	-2.54
90	1248.65	1243.64	-5.01
100	1338.12	1330.49	-7.63
110	1443.61	1432.49	-11.12

Modes were chosen to be spread over the frequency range. Matching between modes was validated using a mass-weighted Normalised Cross Orthogonality (NCO) criterion [6].

Neglecting the frequency dependence leads to errors of several Hertz in the prediction of the eigenfrequencies of the windshield. The error is reduced around the frequency of 1000 Hz, i.e. the one used to realize the stiffness matrix for the computation of the constant real modes. The NCO values are higher than 0.98 for all paired modes in the frequency range of interest. Therefore, the stiffness of the polyvinylbutyral is not so influent on the modal shapes of the windshield.

3.3 Frequency response functions

A series of frequency response calculations using Code_Aster was performed via direct and modal methods involving the above developments. The reference was defined as the direct method using frequency dependent properties of the VEM.

The first study involved the comparison between the reference and the conventional modal method using constant real normal modes and constant properties. The modal method used constant values of the shear modulus and the loss factor of the polyvinylbutyral, calculated as averages between 0 and 1000 Hz. The resulting responses are plotted in Figure 3 and indicate severe errors of the modal approach.

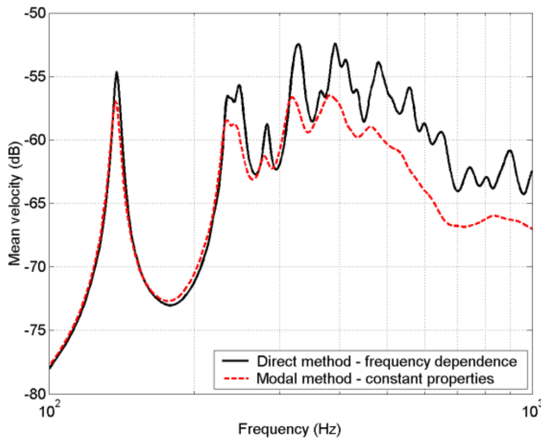

Figure 3: Comparison with the reference of the normal mode projection method using a constant stiffness matrix.

It is obvious the frequency dependence of the polyvinylbutyral must be taken into account in response computations of the windshield. In the following, all harmonic response computations have been performed with realization of the stiffness matrix at each frequency step for computing the response given by Eq. (5).

Hence, the second study is performed from a frequency dependent stiffness and a modal basis composed by constant real modes and a static correction. Static correction was computed for the stiffness matrix realized at 0 Hz, whereas modes were computed for the matrix realized at 1000 Hz. The resulting response is compared with the reference in Figure 4, showing a great improvement compared to the first study, which does not project the true frequency dependent stiffness matrix on the modal basis but a constant stiffness matrix.

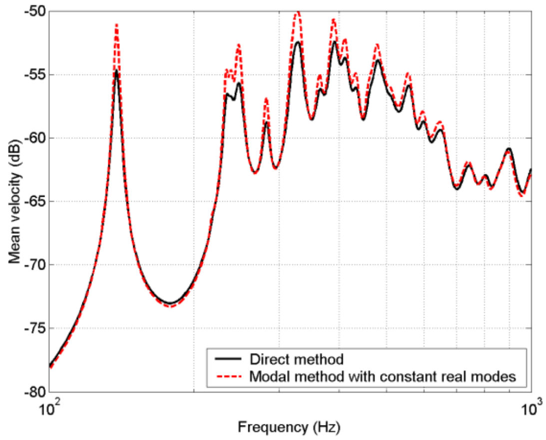

Figure 4: Comparison with the reference of the modal approach using true frequency dependent stiffness matrix, constant real modes and static correction.

Compared to the reference, large differences in amplitude are observed at all the resonant peaks up to 700 Hz with a maximum difference of 4 dB at the first resonant peak. This illustrates the constant real modes are not sufficient in modeling the polyvinylbutyral.

A third study was performed to assess the capability of the frequency dependent modes in representing the viscoelastic behavior of the polyvinylbutyral. Responses were computed by the proposed approach using two different modal bases. One was composed by frequency dependent real modes. The other one was composed by frequency dependent beta-modes which were obtained by the modified MSE. In both cases, modes were combined with a static correction which was computed again for the stiffness matrix realized at 0 Hz. The resulting responses are compared with the reference in Figure 5.

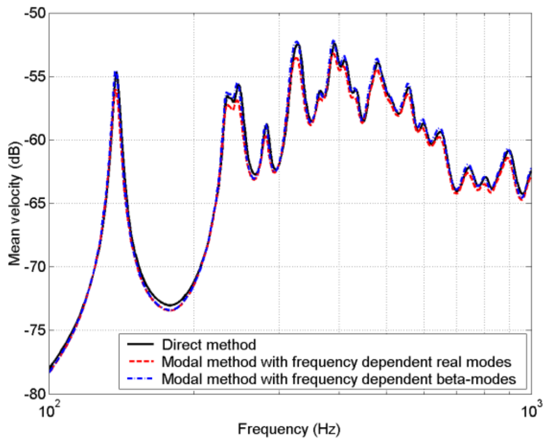

Figure 5: Comparison with the reference of the proposed approach using frequency dependent modes.

Both modal bases reduce strongly the differences with the reference. The differences are lower than 1.5 dB with the real modes, and lower than 0.5 dB with the beta-modes in all the frequency range. This confirms the importance of the frequency dependent modes in modeling viscoelastic materials with strong frequency dependent behaviors.

In terms of computational efficiency, the direct solution took 26 minutes whereas the modal approach required 10 minutes with the real modes and 12 minutes with the beta-modes. This improvement in computation time may be larger for models with a greater number of degrees of freedom.

4 Conclusion

An iterative method for computing frequency dependent modes of structures with VEM has been developed and implemented in Code_Aster. These modes can be used as a modal basis for frequency response calculations. They will extend the validity of the basis over all the frequency range by taking into account the frequency dependence of the stiffness matrix.

Capabilities of Code_Aster have been improved by implementing a macro command to define VEM with frequency dependent behaviors and compute resulting frequency responses via a direct or a modal approach.

A performance study was carried out to demonstrate the efficiency of the frequency dependent modes in modeling an automotive laminate windshield comprising a polyvinylbutyral material with a strong frequency dependent viscoelastic behavior.

References

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