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Methods for transfer matrix evaluation applied to thermoacoustics

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The design of a thermoacoustic (TA) prime-mover partly relies on the knowledge of its *onset* conditions, i.e. the resonance frequency of the self-sustained oscillations and the minimum heat power supply which is necessary for the phenomenon outbreak. The onset conditions can be calculated once the transfer matrix of the TA core is determined, whichever by analytical modeling or acoustic measurements. The latter, however, consists of an interesting option to avoid thermophysical or geometrical considerations of complex structures, mainly with respect to the TA regenerator, as the TA core is treated as a *black box*. Two experimental methods are considered in this word: the Two-Loads Method and the Impedance Method. These methods are here presented, discussed and compared on several aspects, taking into account simulated and experimental results for a stack and a regenerator both submitted to a steep temperature gradient. Regarding these aspects of investigation, the Two-Loads Method was proved to be the best for the stack, meanwhile the Impedance Method for the regenerator. In the second case, important experimental difficulties arise due to the regenerator intrinsic properties. Therefore, the Impedance Method was conceived towards the investigation into this specific problematic and succeeded on its purpose.

1 Introduction

Designing a thermoacoutic (TA) machine leans on the knowledge of its *onset* conditions, i.e. heating power supply Q_H and acoustic frequency f, which generate the required thermoacoustic instability in its core to make it work. Such conditions can be calculated if the Transfer Matrix (**T**) of the TA core is available. The TA core is defined as the region where the temperature gradient is not null. Its **T** matrix can be obtained experimentally and, aiming at this goal, two methods have been investigated: the Two-Loads Method and the new Impedance Method. In both cases the TA core is treated as a *black box*. Thus, by means of acoustical measurements only, its **T** matrix can be obtained as an acoustical two-port system, no mattering how complex are the geometrical or thermophysical properties of the TA core. Therefore, it is avoided the need of analytical modeling.

The thermoacoustic phenomenon takes place within the porous material, which plays the main role inside the TA core. Depending on its average pore size and porosity, the material may be classified as a *stack* or *regenerator*, determining so its application, either standing-wave TA engine or traveling-wave one, respectively [3]. For the first case the average pore diameter is required to reach a few thicknesses of the fluid thermal boundary layer performed during the acoustic oscillation nearby the solid walls, in order to establish an essentially irreversible heat transfer in between them [5]. For the regenerator, meanwhile, the quest is for a reversible thermal interaction [4]. That demands pores as small as possible, regarding other combined constraints of the regenerator, as its porosity and axial length, which must allow the sound wave pass through.

The Two Loads Method [1] consists of a procedure of pressure measurements where a classical 4 microphones method is applied twice, each time for one acoustic load condition, in order to obtain the 4 coefficients of the **T** matrix from measured transfer functions between the microphones.

The Impedance Method, on the other hand, consists of obtaining **T** from the measurements of the Impedance Matrix **Z** of the TA core by simply applying the proper analytical expressions that interrelate them. The impedance measurements, for instance, are made by means of a specific device called Acoustic Impedance Sensor [2]. The TA core here considered, however, is an asymmetrical system; and this aspect presupposes two impedance measurements to fulfill the **Z** matrix. It can be applied a two acoustic loads technique, as in the previous method, but, in our case, these two measurements are differentiated on the experimental configuration by taking each one in a different axial sense, stated as direct sense and inverted one.

Independently on the method to be applied, in order to characterize the TA core thermal aspects related to **T**, several thermal gradient conditions shall be imposed along the TA core longitudinal axis, which may be provided by a system of arbitrary heat power supply Q_H combined with cooling heat exchangers. Once the steady state regime is reached for each Q_H, a corresponding $\mathbf{T}(Q_H, f)$ can be calculated afterwards, therefore establishing in the final resulting **T** the coupled thermal and acoustical behavior of the TA core, implicitly.

A stack made of ceramics and a regenerator constituted of a grid of steel pile are tested with both methods. The experimental results obtained are compared mutually and also with respect to theoretical results with simulated noise and bias. Besides, two foams are also tested with the Impedance Method: one made of a Ni-Cr alloy and the other made of Reticulated Vitreous Carbon (RVC).

2 The Two-Loads Method

A schematic drawing of the experimental apparatus used for the determination of the **T** matrix of the TA core is shown in the Figure (1a). The TA core under study is shown in Figure (1b).

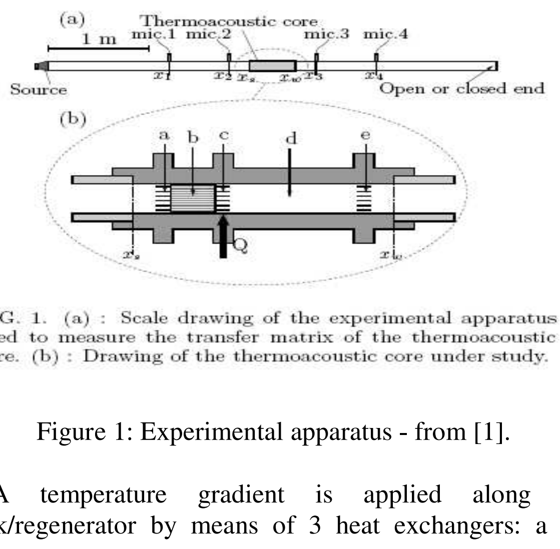


FIG. 1. (a) : Scale drawing of the experimental apparatus used to measure the transfer matrix of the thermoacoustic core. (b) : Drawing of the thermoacoustic core under study.

Figure 1: Experimental apparatus - from [1].

A temperature gradient is applied along the stack/regenerator by means of 3 heat exchangers: a hot exchanger (d) which supplies a heat power Q_H to the system, a cold exchanger (b) which ensures that the left side of the stack is maintained at room temperature, and a second cold exchanger (f) is used to control the temperature distribution along the thermal buffer tube (e).

In order to characterize the propagation of acoustic waves through the TA core, two straight ducts are connected to its ends. The duct on the left is connected to an electrodynamic loudspeaker, while the duct on the right is either open to free space or closed with a rigid wall. Two pairs of microphones are flush mounted along each duct at both sides of the TA core. Assuming that an harmonic plane wave is propagating in the device at the angular frequency ω, the acoustic pressure $p(x,t)$ and the acoustic volume velocity $U(x,t)$ are written as below:

$$\zeta(x,t) = \Re[\tilde{\zeta}(x)e^{-j\omega t}], \qquad (1)$$

where ζ represents either p or U, the accent ~ indicates the complex amplitude and $\Re[]$ denotes the real part of a complex number.

With this assumption, the complete apparatus can be used to measure the complex amplitudes of acoustic pressures $\tilde{p}(x_{l,r})$ and the acoustic volume velocities $\tilde{U}(x_{l,r})$ at the left side and right side of the two-port, respectively, and thus to deduce the **T** matrix using the following equation:

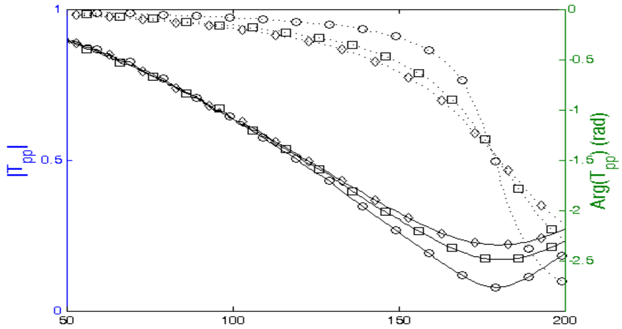
$$\begin{Bmatrix} \tilde{p}(x_r) \\ \tilde{U}(x_r) \end{Bmatrix} = \mathbf{T} \begin{Bmatrix} \tilde{p}(x_l) \\ \tilde{U}(x_l) \end{Bmatrix} = \begin{pmatrix} T_{pp} & T_{pu} \\ T_{up} & T_{uu} \end{pmatrix} \begin{Bmatrix} \tilde{p}(x_l) \\ \tilde{U}(x_l) \end{Bmatrix}. \qquad (2)$$

The measurement method used here to obtain **T** is a classical "two-loads", four-microphone method [6,7]. The basic principles of this method consists of calculating $\tilde{p}(x_{l,r})$ and $\tilde{U}(x_{l,r})$ from the measurements of acoustic pressures at positions x_i [i=1,2,3,4, see Fig.1(a)]. A set of two measurements is however required to obtain the four coefficients T_{pp}, T_{pu}, T_{up}, T_{uu} of the **T** matrix from the measured acoustic pressures $\tilde{p}(x_i)$, and this is realized here by using two different loads, the first with the open duct, and the second with the duct closed by a rigid wall.

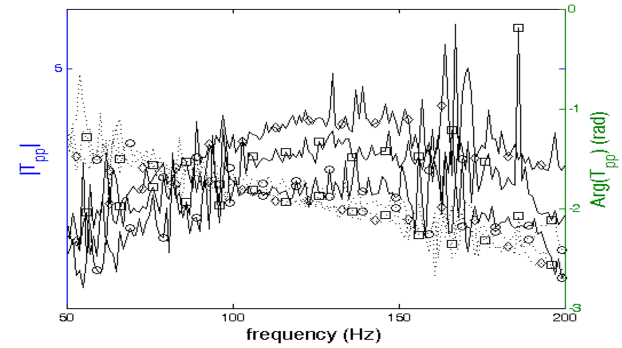
The **T** matrix measurements are taken in the frequency range from 50Hz to 200Hz, and they follow 28 different values of heat power supply, where Q_H varies from 0W to 81W with a constant increment of 3W.

2.1 Experimental Results

The Figure (2) presents the behavior of the T_{pp} coefficient, chosen as an arbitrary example, from 50 Hz to 200 Hz and for 3 different values of heat power supply Q_H : $0W$, $36W$ and $72W$, arbitrarily chosen as well.



a) Ceramic Stack.



b) Grid of Inox Pile

Figure 2: Module (solid line) and phase (dotted line) of T_{pp} in the frequency domain for $Q_H = 0W$ (o), $Q_H = 36W$ (square) and $Q_H = 72W$ (diamond) – Two-Loads Method.

When there is no heat supply into the acoustic system, it is expected to have an amplitude of the average **T** determinant being approximately 1, with average phase close to 0 rad. These conditions characterizes a reciprocal system and states for a good measurement quality.

The reciprocity for the Ceramic Stack resulted in the following values:

Amplitude average: 1.010; standard deviation: 3.3e-3.

Phase average(rad): 1.9e-3; standard deviation: 2.4e-3.

Therefore, the **T** matrix may be considered reciprocal, and these results reciprocity proves a quite good repeatability with respect to a previous work [1], contributing so to the consistency of the Two-Loads Method, for this kind of material (stack).

However, these reciprocity aspects resulted insufficient good for the Grid of Steel Pile:

Amplitude average: 0.9958; standard deviation: 0.1818.

Phase average(rad): 1.9e-3; standard deviation: 0.1265.

Even though very good on the averages, the standard deviations resulted poor. It is a consequence of a high level of noise in **T**, in spite of applying the same well controlled experimental procedures. This effect is confirmed in a theoretical sensitivity analysis for a regenerator, where arbitrary noise and bias are introduced to simulated transfer functions.

When applying the same analysis to the stack, the simulations resulted in low sensitivity to both parameters, coherently with the experimental behavior. This contributed hence towards the confidence on this analysis application to other possible solutions for this problem, before proceeding with another experimentation. That has leaded to the following method.

3 The Impedance Method

The Impedance Method has been developed as an attempt to experimentally obtain the **T** matrix of a regenerator TA core. This choice came out from our efforts to overcome the limitation of the previous method, which indeed became better understood after a theoretical sensitivity analysis. Hence, before starting with the second method engagement, it was done a sensitivity analysis for the Impedance Method as well, at the same arbitrary level of simulated noise and bias previously stated for the first method case, in order to evaluate the perspective predictions.

3.1 Method Definition

The **T** coefficients, stated just as in the definition (2), are calculated from the two-port equations that relate them to the **Z** coefficients, the latter which are actually measured using the Acoustic Impedance Sensor.

The Z matrix is defined as following:

$$\begin{Bmatrix} \tilde{p}_s \\ \tilde{p}_w \end{Bmatrix} = \mathbf{Z} \begin{Bmatrix} \tilde{U}_s \\ \tilde{U}_w \end{Bmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{Bmatrix} \tilde{U}_s \\ \tilde{U}_w \end{Bmatrix}. \quad (3)$$

The **Z** coefficients are obtained from experimental measurements of sound pressure using a specific device coupled to the TA core under test. This device is called Impedance Sensor and it comprises three microphones, a piezo-electric buzzer and the housing itself. The figure 1 shows the setup scheme of such Impedance Sensor when coupled to the TA core and the adaptive part.

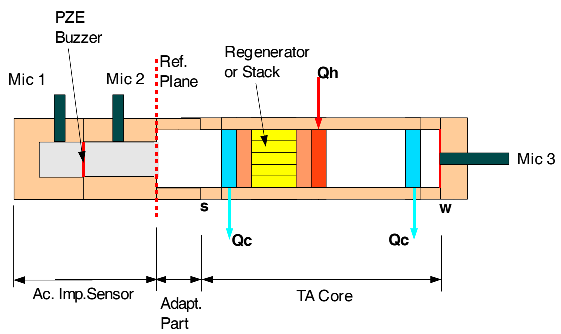


Figure 3: Impedance Sensor coupled with the TA core Direct Sense.

The piezo-electric buzzer is loaded by a small cavity on its rear face and it radiates towards a duct on its front face. The microphone 1 measures the pressure $p_1(x,t)$ inside the small cavity, while the microphone 2 measures the pressure $p_2(x,t)$ in the duct, before the entrance of the Adaptive Part. From p_1 and p_2, and taking into account geometrical considerations, the acoustic impedance Z_{11} can be calculated at the Reference Plane. On the opposite extremity, enclosing the system under test, the transfer impedance Z_{21} can be calculated when considering also the pressure measurements at the w position, where the microphone 3 is placed. Therefore, the system to be measured is defined as the region between s and w.

A set of 2 measurements is required to obtain the 4 coefficients of the **Z** matrix. In one of these it is realized inversion of TA core.

The *first measurement* is made in the direct sense, while the *second measurement* is made in the inverted one, as shown in the respective Figures 3 and 4.

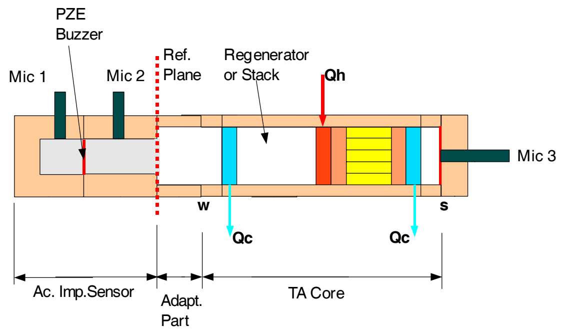


Figure 4: Impedance Sensor coupled with the TA core Inverse Sense.

3.2 Method Application

In order to take into account the microphones positions and the Reference Plane, the Z_{11} and Z_{21} coefficients are generally expressed as following [2]:

$$Z_{11} = \frac{(H_{21}/K) - \beta}{1 - (\delta H_{21}/K)}, \quad (4)$$

$$Z_{21} = \frac{H_{31}(1 + \delta Z_{11})}{\delta K_T}. \quad (5)$$

For Z_{22} and Z_{12} are applied the same equations (4) and (5), in correspondence.

The H terms are transfer functions that relate the sound pressure measurements after the calibration:

$$H_{21} = \frac{p_2}{p_1}\frac{s_2}{s_1} \quad \text{and} \quad H_{31} = \frac{p_3}{p_1}\frac{s_3}{s_1}, \quad (6)$$

where the s terms are the microphones corresponding sensitivities.

The terms β, δ, K and K_T are parameters related to the geometry and the calibration of the device (see ref. [2] for more details).

The measurement of **Z** demands a previous calibration for each experimentation.

All measurements followed the pattern adopted into the previous method, with an input power supplied ranging between 0W and 81W with a constant increment of 3W

3.3 Calibration

The procedure is, at first, closing the device with a rigid plate that ensures a good approximation to a infinite impedance. This rigid plate, nevertheless, has an orifice which encloses the third microphone. The transfer functions H_{21} and H_{31} are measured and here labeled with the subscript "cal" ever since, to be used later for the all set of remaining measurements.

As a consequence of such configuration, it results that the impedance at the Reference Plane is the same as the transfer impedance to the microphone 3, due to the fact that the position is coincident. Further developments lead to:

$$K = \delta H_{21cal} \text{ and } K_T = H_{31cal} \quad (7)$$

Therefore, as a final result of the calibration, the parameters K and K_T are obtained directly from the measurements of the transfer functions and both ratios of sensitivity s_2/s_1 and s_3/s_1 are implicitly taken into account. Once such parameters are available and the calibration transfer functions are registered, one can realize the following step, which concerns the impedance measurement itself.

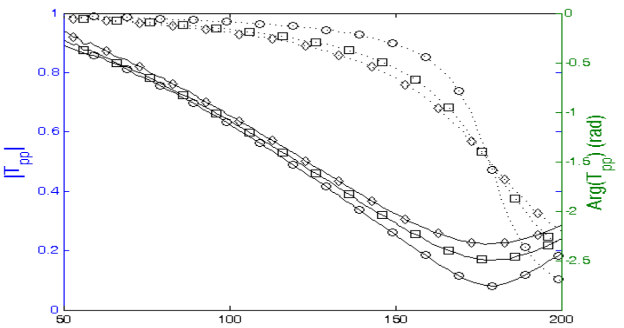
3.4 Impedance Measurements

The parameters K and K_T can be replaced by (7) into the equations (4) and (5). Therefore, the calibration is considered in the determination of **Z**. These equations provide a simple manner to calculate the matrix **Z** directly from the transfer functions, from the geometrical parameters, and from the gas thermophysical properties.

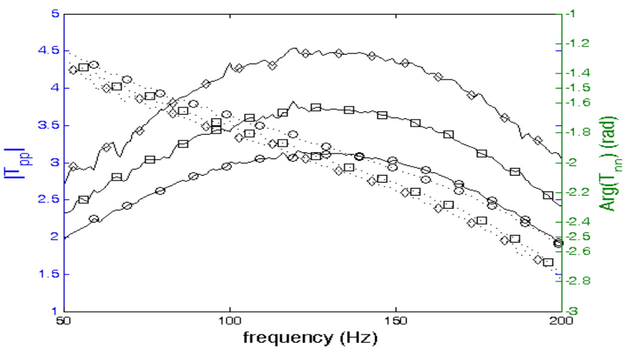
The samples here tested are exactly the same of the previous method. Besides, two other samples are tested for the Impedance Method: a foam made of a Ni-Cr alloy and another made of RVC.

3.5 Experimental Results

The Figure (5) presents the behavior of the T_{pp} coefficient in the same way as in the Figure (2), for the same 3 different values of heat power supply Q_H.



a) Ceramic Stack



b) Grid of Inox Pile

Figure 5: Module (solid line) and phase (dotted line) of T_{pp} in the frequency domain for $Q_H = 0W$ (o), $Q_H = 36W$ (square) and $Q_H = 72W$ (diamond) – Impedance Method.

In analogy with previous method, the reciprocities are here presented, for $Q_H = 0W$ either way.

For the Ceramic Stack it follows:

Amplitude average: 0.9815; standard deviation: 4.5e-3.

Phase average(rad): -1.4e-3; standard deviation: 3.6e-3.

These results approaches the reciprocity almost likely the Two-Loads Method, but not as much as it. The sensitivity to bias resulted slightly bigger in the sensitivity simulation, coherently.

For the Grid of Steel Pile regenerator good results are achieved, as seen below:

Amplitude average: 1.0037; standard deviation: 0.0107.

Phase average(rad): 9.0e-3; standard deviation: 7.2e-3.

With respect to the other samples, the values of reciprocity are presented too.

Ni-Cr alloy:

Amplitude average: 0.9288; standard deviation: 0.0154.

Phase average(rad): -0.0102; standard deviation: 0.0101.

RVC:

Amplitude average: 0.9893; standard deviation: 0.0317.

Phase average(rad): -0.0016; standard deviation: 0.0233.

Under an overview, these results indicates that the Impedance Method seams to be robust no matter the material to be characterized inside the TA core, excepting for the foam of Ni-Cr alloy, which calls for additional investigations.

4 Conclusion and Perspectives

We have presented a new method for the transfer matrix measurement of a thermoacoustic core, either regenerator or stack.

We have showed that this new method, contrarily to the Two-Loads Method, is accomplishing independently of the material under test, primarily if dealing with one that resembles a regenerator.

Future works will be devoted to confirm the validation of the Impedance Method, and to the exploration of its results on the goal of the dimensioning of thermoacoustic machines.

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