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Effects of a default on the reflectivity of plane periodic media

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The influence of defect layers in a fluid-loaded plane multilayer composed of N periods is highlighted by analyzing the stop bands and pass bands of the reflection coefficient. The period is composed of two plates exhibiting a high impedance contrast, in aluminum and polyethylene for instance. No attenuation is taken into account. Two types of defect layers are considered: either the thickness of a plate in one period is varied, or a layer made up of another material is inserted between two adjacent periods. The reflection coefficient is obtained with a numerically stable transfer matrix method. The study consists in comparing the plots of the reflection coefficient of a structure with no defect to the ones of structures with defects inserted at different locations. The variations of the resonance frequencies in the pass bands according to the defect position, as well as the width of the stop bands, are studied. A particular attention has been paid to the resonances at low frequency, linked with the so-called vertical modes. In the case of a defect included between two periods, it may appear very narrow pass bands in the stop bands.

1 Introduction

We are interested in the influence of defect layers on the R reflection and T transmission coefficients by a plane multilayered fluid-loaded structure composed of N periods. In a periodic medium, it exists frequency ranges in which the reflection is unity. They correspond to stop bands in which no waves can propagate. Under certain conditions, the insertion of a defect may open thin pass bands in stop bands. In our study, a period contains two plates supposed perfectly bonded. Those ones, in aluminum (Al) and polyethylene (PE), exhibit a high impedance contrast. Two types of inserted flaws are considered, either inside a period by varying the polyethylene layer thickness, or by inserting a defect layer. The former case is based on experimental ascertainments which imply to take into account thickness variations. In the latter case, a layer whose acoustic impedance is different from those of the Al and PE plates plays the role of the defect. The defect layer is either in copper or in glass.

The R and T coefficients are obtained with a numerically stable transfer matrix method [1-2]. Alternative methods are valid only at normal incidence [3-4]. Our study consists in comparing the plots of those coefficients for a structure with no defect to the ones of structures with defects included at different positions. The healthy media of reference is composed of five periods and no attenuation is taken into account. A higher number of periods does not affect strongly the widths of the stop bands occurring for periodic media. Simulations are performed at normal incidence. In the case of a thickness variation of a PE layer in a period, we can observe, according to the location of the defect, variations of the frequency loci of the minimums of the reflection coefficients, compared to those of the healthy structure. We have paid particular attention to the R minimums at low frequency, these ones being linked to the so-called vertical modes [5-10].

In the first section, the expressions of the reflection and transmission coefficients of multilayered plane structures, periodic or not, are recalled.

In the second section, the reflection coefficients of a five period “healthy” structure and the ones of the two constitutive layers of a period are compared.

In the third section, the influence of a thickness variation of a PE layer at various locations in the whole media is investigated [11-12]. Plots of reflection coefficients are compared with the reference one, in different frequency ranges.

In the fourth section, the effects of inserting a defect layer either in a given period, or between two adjacent periods are studied. Finally, primary conclusions of this work are given, as well as further possible investigations.

2 Theory

We consider a plane monochromatic incident wave in the fluid at normal incidence (Fig. 1), on a periodic multilayered media composed of five periods. Each one contains two plates, the one in Al, the other in PE. The Al plates are ordered by an even number from 2 to 10, the PE ones by an odd number from 3 to 11. The structure is immersed in water indexed by 1 and 12, for the incidence and transmission medium, respectively. Even if the structure is not symmetric, the incidence side is indifferent.

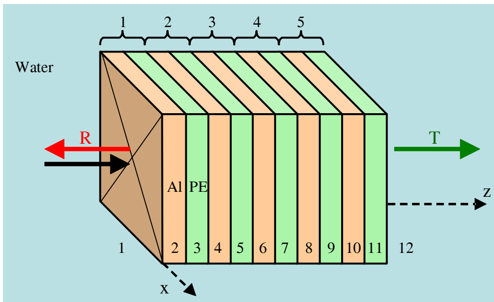

Figure 1. Geometry of the healthy 5×(Al/PE) periodic structure.

The reflection and transmission coefficients are obtained by using the stiffness matrix method. It defines the 4×4 compliance matrix for the period n, where s_{ij} are 2×2 submatrices:

$$s^n = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}_n \quad (1)$$

Rokhlin et al. [1-2] have implemented a stable recursive algorithm for the computation of the compliance matrix of multilayered media composed of N periods. The submatrices s_{ij} for N periods are deduced from the ones for (N−1) periods and those of an additional n^{th} period by the following recursive relations:

$$\begin{cases} s_{11}^N = s_{11}^{N-1} + s_{12}^{N-1}(s_{11}^n - s_{22}^{N-1})^{-1} s_{21}^{N-1} \\ s_{12}^N = -s_{12}^{N-1}(s_{11}^n - s_{22}^{N-1})^{-1} s_{12}^n \\ s_{21}^N = s_{21}^n (s_{11}^n - s_{22}^{N-1})^{-1} s_{21}^{N-1} \\ s_{22}^N = s_{22}^n - s_{21}^n (s_{11}^n - s_{22}^{N-1})^{-1} s_{21}^n \end{cases} \quad (2)$$

The elementary compliance matrix s^n of a period is obtained by the same recursive relation.

It can be demonstrated that the reflection coefficient R and the transmission coefficient T are expressed as functions of elements of the global compliance matrix s^N in the following forms:

$$R = \frac{(s_{11}^{22} - iy_F)(s_{22}^{22} - iy_F) + \left(s_{12}^{22}\right)^2}{(s_{11}^{22} + iy_F)(s_{22}^{22} - iy_F) + \left(s_{12}^{22}\right)^2} \qquad (3)$$

$$T = \frac{2iy_F s_{12}^{22}}{(s_{11}^{22} + iy_F)(s_{22}^{22} - iy_F) + \left(s_{12}^{22}\right)^2} \qquad (4)$$

where s_{ij}^{22} is the (2,2) element of the submatrix s_{ij},

and $y_F = -\dfrac{\cos\theta}{\rho_F c_F \omega}$, with θ the incidence angle, ρ_F and c_F are the density and the sound velocity in the fluid, respectively.

3 Simulation

3.1 Periodic structure

The physical and geometrical properties of the constitutive media are summarized in Table 1.

Table 1: Material properties.

Material	Z (MRa)	c_L (m/s)	c_T (m/s)	ρ (kg/m^3)	d (mm)
Al	17.9	6380	3100	2800	2
PE	2.22	2370	1200	940	2
Water	1.47	1470	-	1000	-

The reflection spectrum of the elementary stack (Al/PE) results from the contributions of Al and PE single layers (Figure 2). As illustrated, narrow pass bands are related to those of the PE layer, whereas the wider ones are due to the Al layer.

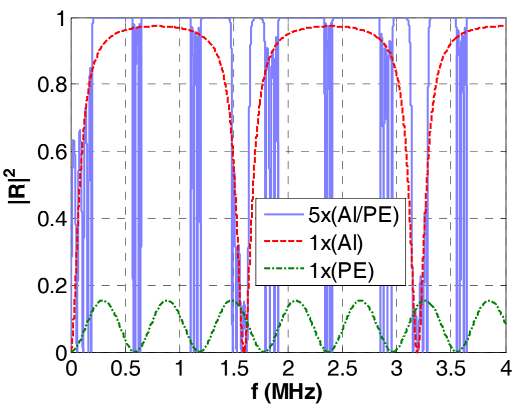

Figure 2: Reflection spectrums of the 5×(Al/PE) periodic structure, of an Al plate and of a PE plate.

In order to evaluate the vertical mode positions as a function of the number of periods, the reflection spectra are compared for N = 2 to 5 periods (Figure 3). One can observe (N−1) minimums, linked with vertical modes, i.e. they have vertical dispersion curves in the frequency/incidence angle plane [7-10]. Particularly, for the 5×(Al/PE) structure (dashed cyan curve), we observe four vertical modes denoted as VM_1 to VM_4.

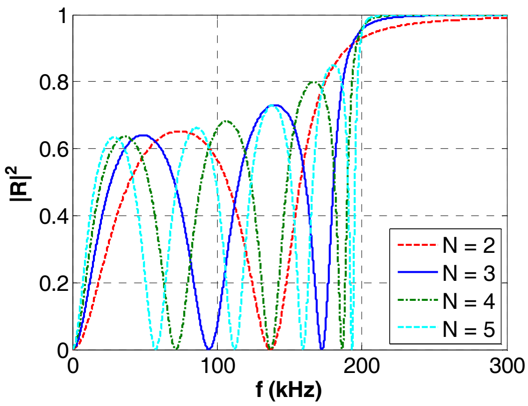

Figure 3: Comparison of the reflection spectrum of N×(Al/PE) periodic structures.

As illustrated by Figure 4, the low frequency vertical modes are studied when the thickness of a PE layer, at a given location, is varied of 10% around its reference value. Thus, the healthy structure is composed of PE layers of constant thickness d_{PE} = 2 mm (solid black line), whereas the defect layer is fixed either at $d_{PE,inf}$ = 1.8 mm (dashed blue line) or $d_{PE,sup}$ = 2.2 mm (dotted red line). These thickness variations involve frequency shifts of the minimums of the reflection spectrum $|R|^2$ linked with the four vertical modes of the 5×(Al/PE) periodic structure.

We consider a PE defect layer located in position 3 (PE layer 3). The thickness are either $d_{PE,3}$ = $d_{PE,inf}$ or $d_{PE,sup}$ (Figure 4 (a)). The first vertical mode VM_1 for the healthy structure is observed at f_1 = 57.6 kHz. For the two thickness variations, there is nearly no variations of the reflection spectrum up to 78 kHz. The shifts of the other minimums are observed towards the low frequencies for the 2.2 mm thick defect and towards the high frequencies for the 1.8 mm thick defect.

For a defect location in PE layer 7 (Figure 4 (b)), the frequencies of the vertical modes VM_3 and VM_4 are shifted from their reference values f_3 = 159 kHz and f_4 = 193 kHz. We can measure relative frequency variations whose values are $\Delta f_{3,inf}/f_3$ = +1.0%, $\Delta f_{3,sup}/f_3$ = −1.0%, and $\Delta f_{4,inf}/f_4$ = +1.7%, $\Delta f_{4,sup}/f_4$ = −2.2%, respectively.

For a defect location in PE layer 11 (Figure 4 (c)), the frequencies of the vertical modes are nearly the same. This could easily be explained by the small reflection contribution of this last PE layer. Indeed, at the interface with water, the acoustic impedances in these two media are close (Table 1), resulting in a reflection coefficient at the interface $|R_{PE/water}|^2$ = 0.04.

(a)

(b)

(c)

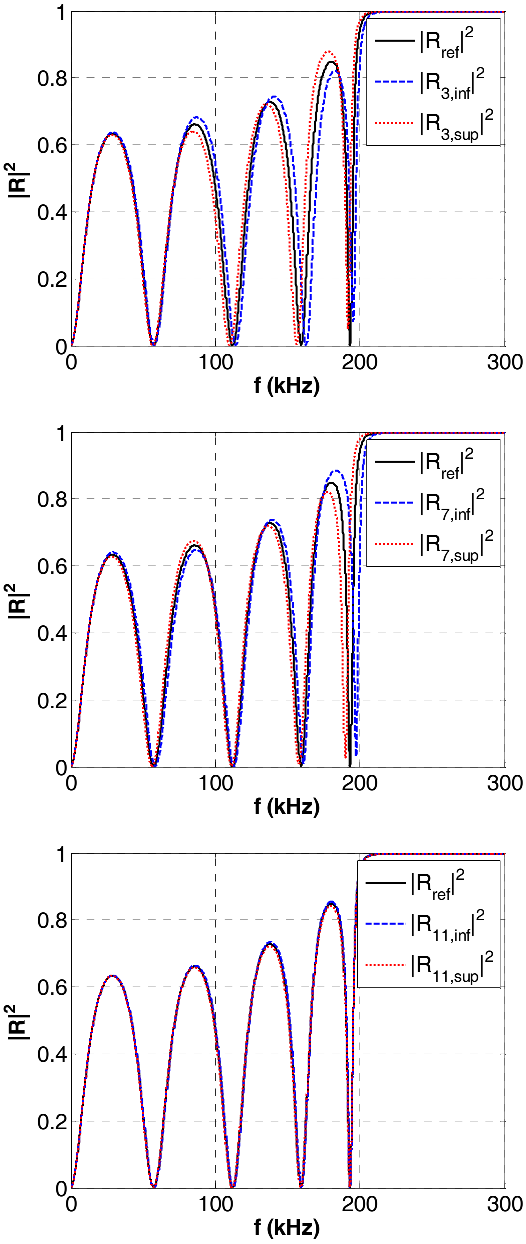

Figure 4: Reflection spectrums for the reference thickness d_{PE} = 2 mm (solid line) and the defect values $d_{PE,inf}$ (dashed line) and $d_{PE,sup}$ (dotted line), for PE layers (a) 3, (b) 7 and (c) 11.

The influence of the thickness of the PE layer 3 varying from $d_{PE,3}$ = 0 to 4 mm is illustrated in Figure 5. We can define a relative sensitivity S(m,n) of a thickness variation on PE layer n for the considered vertical mode VM_m, with m = 1, 2, 3, 4 and n = 3, 5, 7, 9, 11:

$$S(m,n) = \frac{\Delta f_m}{f_m} \Big/ \frac{\Delta d_{PE,n}}{d_{PE,n}} \qquad (5)$$

This sensitivity can be calculated from the minimums in Figure 5 (blue color level). It corresponds to the inverse of the slope of the blue curves, and it is clearly non linear, especially for the modes VM_3 and VM_4. The sensitivity of the thickness variation on $d_{PE,3}$ is greater for the mode VM_3 than for the mode VM_4. This result is confirmed by a more detailed study of the sensitivity of the modes VM_3 (Figure 6 (a)) and VM_4 (Figure 6 (b)).

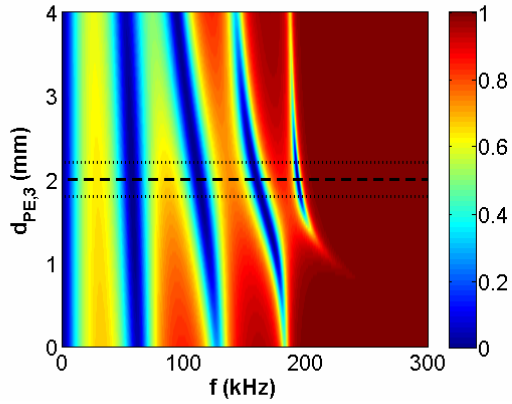

Figure 5: Vertical modes VM_1 to VM_4: PE layer 3 thickness $0 < d_{PE,3} < 4$ mm, reference value (dashed line) and defect values (dotted lines).

Consequently, a relative thickness variation $\Delta d_{PE,3}/d_{PE,3}$ of the PE layer 3 equal to +10% results in relative frequency variation $\Delta f_{3,sup}/f_3$ equal to −1.9%, and $\Delta f_{4,sup}/f_4$ equal to –0.8%. As illustrated by Figure 5 (between dotted lines) and Figure 6, the relative thickness sensitivities $\Delta d_{PE,n}/d_{PE,n}$ of PE layer n (n = 3, 5, 7, 9, 11) on the relative VM_m mode frequencies $\Delta f_m/f_m$ (m = 3, 4) are all negative and nearly linear.

(a)

(b)

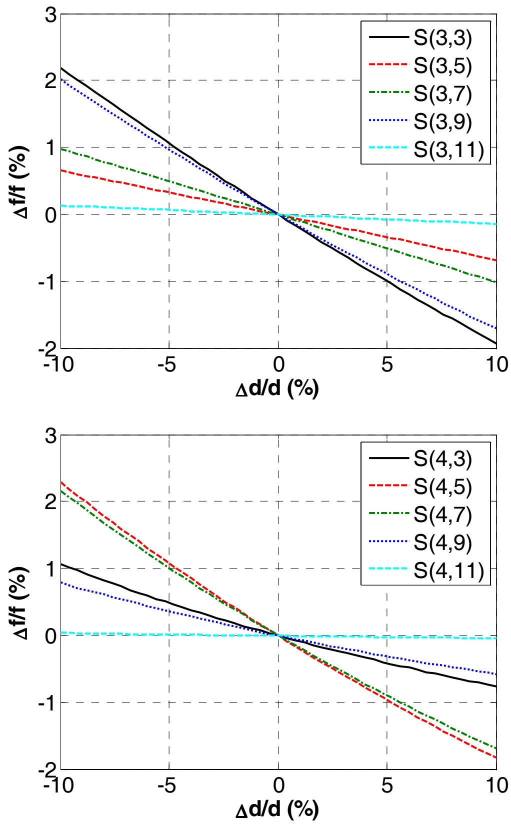

Figure 6: Sensitivities (a) S(3,n) for the mode VM_3 and (b) S(4,n) for the mode VM_4, depending on the PE layers n = 3, 5, 7, 9, 11.

A sensitivity matrix $[S_{mn}]$ (eq. (7)) is formed with elements defined by constants $S_{mn} = \min(S_{mn,inf}, -S_{mn,sup})$.

For instance, the sensitivity coefficient S_{43} implies a relative frequency variation $\Delta f_4/f_4 = S_{43}.(\Delta d_{PE,3}/d_{PE,3})$:

$$\left[\frac{\Delta f_m}{f_m}\right] = [S_{mn}]\left[\frac{\Delta d_{PE,n}}{d_{PE,n}}\right] \tag{6}$$

with $$[S_{mn}] = -\begin{bmatrix} 0.09 & 0.19 & 0.20 & 0.10 & 0.03 \\ 0.19 & 0.10 & 0.07 & 0.20 & 0.02 \\ 0.22 & 0.07 & 0.10 & 0.20 & 0.01 \\ 0.11 & 0.23 & 0.22 & 0.08 & 0.01 \end{bmatrix} \tag{7}$$

We observe a maximum sensitivity (red values) S_{17} = 0.20 for the thickness $d_{PE,7}$ on the mode VM_1, S_{45} = 0.23 for the thickness $d_{PE,5}$ on the mode VM_4, for instance. As already noted, the thickness $d_{PE,11}$ has nearly no sensitivity on the modes VM_m.

3.2 Insertion of a defect layer

In this section, we study the influence of the insertion of a defect layer, either in copper or in glass (Table 2).

Table 2: Defect material properties.

Material	Z (MRa)	c_L (m/s)	c_T (m/s)	ρ (kg/m^3)	d (mm)
Copper	44.7	5010	2270	8930	0.5
Glass	12.6	5640	3280	2240	0.5

Two configurations are investigated, either the defect layer is inserted in a given period (odd location), or between two adjacent periods (even location). The second case is illustrated in Figure 7.

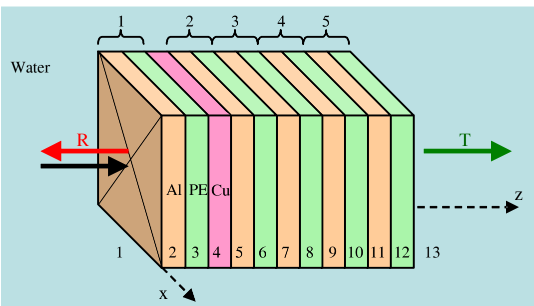

Figure 7: Geometry of the problem: defect at location 4.

The plots of the reflection coefficients for a defect in an odd location (3, 7, 11) are given in the frequency range 0-2 MHz in Figure 8 (a) for a copper defect and in Figure 8 (b) for a glass defect. They are compared to the reference reflection coefficient without defect $|R_{ref}|^2$. We can observe five pass bands in this frequency range. The frequency minimums in the pass bands, when there is a defect inserted in the periodic structure, are shifted compared to the reference.

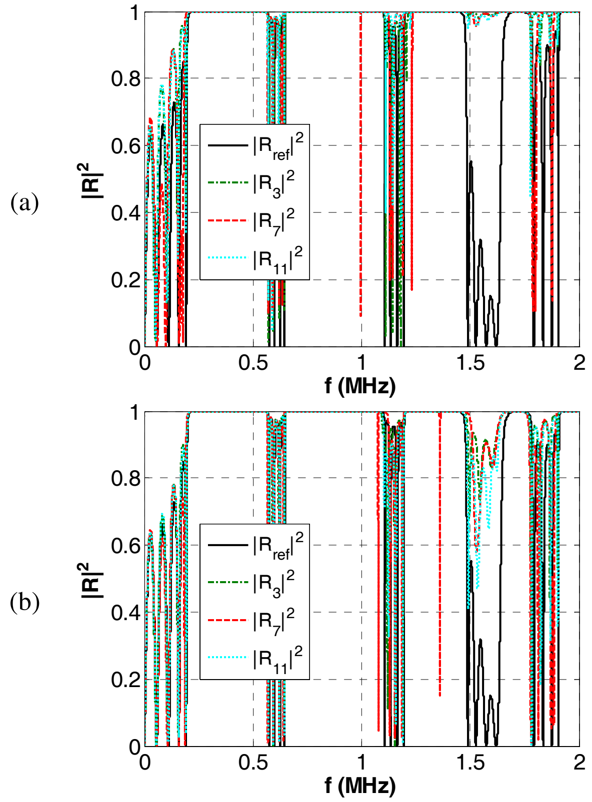

Figure 8: Reflection spectrum of the 5×(Al/PE) periodic structure with (a) copper and (b) glass defects inserted at locations 3, 7 and 11.

As illustrated by Figure 9, the defect layer position has nearly the same influence, either before the third period (location 6) or in the middle (location 7).

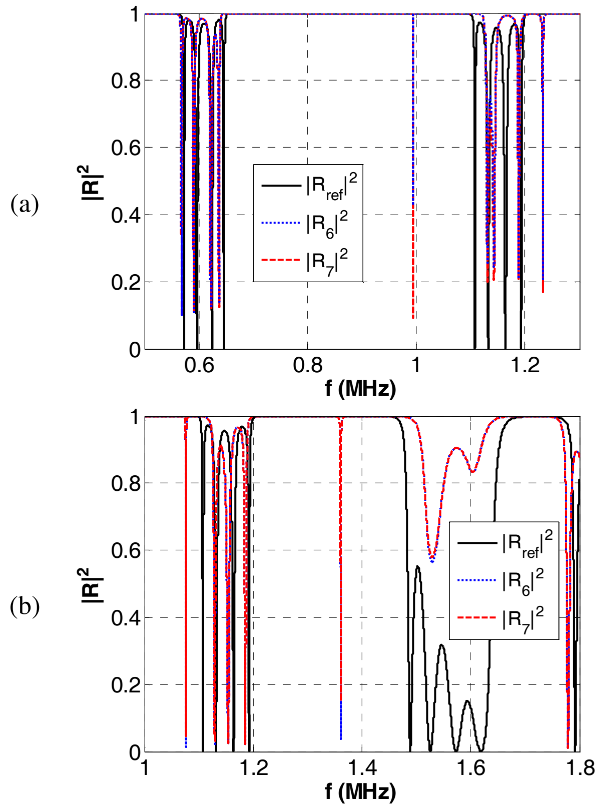

Figure 9: Reflection spectrum of the 5×(Al/PE) periodic structure with (a) copper and (b) glass defects inserted at locations 6 and 7.

In the case of a copper defect layer inserted at location 7 (Figure 9 (a)), we can observe a very narrow pass band at f_{copper} = 0.99 MHz, with a half width pass band Δf_{copper} = 50 Hz. This thin pass band reaches $|R|^2$ = 0.09 and is located in the stop band ranging from 0.65 to 1.12 MHz.

In the case of a glass defect layer inserted at location 6 (Figure 9 (b)), we can also observe a very narrow pass band at f_{glass} = 1.36 MHz, with a half width pass band Δf_{glass} = 750 Hz. This thin pass band reaches $|R|^2$ = 0.04 and is located in the stop band ranging from 1.21 to 1.45 MHz.

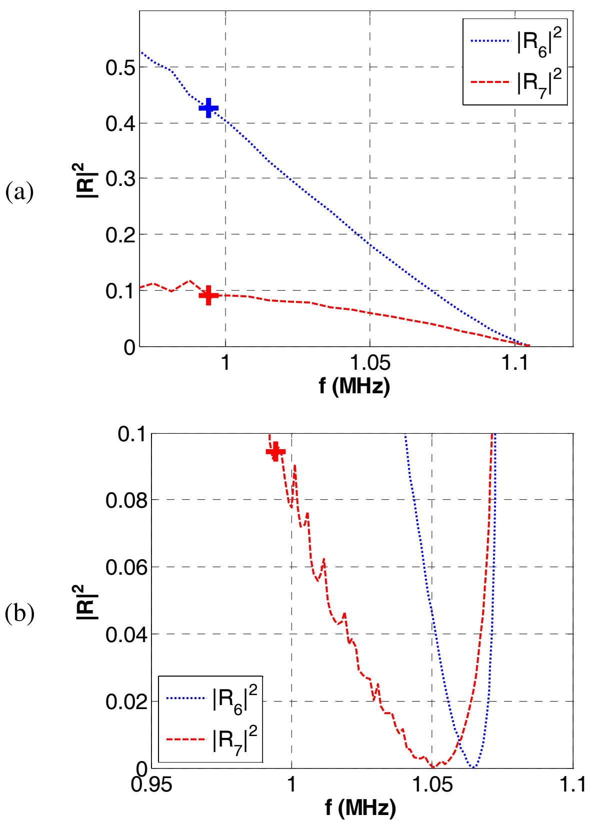

Figure 10: Copper defect layer insertion at locations 6 and 7 influence on the minimum of $|R|^2$ as a function of (a) thickness $0 < d_{copper} < 0.6$ mm ('+': $d_{copper} = 0.5$ mm), (b) density $1500 < \rho < 10000$ kg/m^3 ('+': $\rho = 8900$ kg/m^3).

As illustrated by Figure 9 (a), in the pass band ranging from 1.1 to 1.2 MHz, in the reference periodic structure the first resonance is located at 1.10 MHz, is shifted in the stop band at 0.99 MHz when a copper defect layer is inserted in the structure. The influence of the thickness of the copper layer on this frequency shift is illustrated in Figure 10 (a) and confirms the shift deduced from Figure 9 (a). In addition, it also depends on the density of the defect layer as shown in Figure 10 (b). More precisely, the minimum reflection is obtained for a virtual density ρ = 3100 kg/m^3 for $|R_6|^2$, and ρ = 4300 kg/m^3 for $|R_7|^2$.

4 Conclusion

In this study of periodic structures, the influence of small variations of the thickness layer was highlighted by the analysis of the reflection spectrum. The sensitivity of the vertical modes to thickness variations was evaluated and shown to be nearly linear. In the case of the insertion of a defect layer, a very narrow pass band emerge in a wide stop band, at a frequency depending on the thickness and acoustical properties of the defect layer.

The perspectives of this work are to carry out simulations at oblique incidence and to consider more than two layers in a period. We can also consider boundary conditions different from perfect ones between plates and periods. First experimental results have been presented in Refs. [11-12] and they are to be continued.

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