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Assessment of the validity of statistical energy analysis and transfer matrix method for the prediction of sound transmission loss through aircraft double-walls

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The prediction of sound transmission loss (TL) of aircraft double-walls using the transfer matrix method (TMM) and statistical energy analysis (SEA) is examined in this paper. The studied system is composed of: (1) a stiffened laminate composite skin panel whose critical frequency is around 4000 Hz, (2) an air gap partially filled with a fibrous layer and (3) a sandwich trim panel with critical frequency around 2500 Hz. The panels are mechanically decoupled. The structure is submitted to a diffuse acoustic field in the frequency range from 100 Hz to 10 kHz. The theoretical TL of the panels are compared with measurements in single- and double-wall configurations. Both approaches are able to predict the impact of modifying the parameters of the skin panel below its critical frequency. However, the modelling of the inner cavity in the SEA approach as a resonant subsystem has been found more adapted to this specific double-wall, leading to better agreement with measurements.

1 Introduction

Lightweight double-wall structures filled with air and absorbent materials have been extensively studied considering their wide range of industrial applications, such as building, automotive, railway and aircraft. A summary of the methods used to predict sound transmission through these structures is given in [1].

This paper aims at validating experimentally two approaches for predicting the sound transmission through mechanically decoupled lightweight structures representative of an aircraft sidewall. They are the Transfer Matrix Method (TMM) and Statistical Energy Analysis (SEA). In mid to high frequencies, these approaches are an alternative to finite element modelling, which is time-consuming and therefore not suitable for optimisation studies in the pre-design phase of conception.

2 Theory

This section gives a brief description of the transfer matrix method and the statistical energy analysis. Their main hypotheses and limits are also listed. The studied system is composed of: (1) a laminate unstiffened composite skin panel whose critical frequency is around 4000 Hz, (2) an air gap partially filled with a fibrous layer and (3) a sandwich trim panel with critical frequency around 2500 Hz. The structure is submitted to a diffuse acoustic field in the frequency range from 100 Hz to 10 kHz.

2.1 Transfer matrix method (TMM)

The Transfer matrix method is based on wave propagation through layers of materials represented by transfer matrices linking velocities and stresses at their boundaries [2, 3]. The layers are considered flat and of infinite extent but corrections of the acoustic radiation are applied to account for the finite size of the panels [4]. Fig. 1 represents the layers of the studied double-wall structure, immersed in air. Incident and reflected waves, with amplitudes I and R, are represented at excitation side (i). Transmitted wave, with amplitude T, is represented at receiver side (o).

The transmission within a layer is given by the following equation:

$$V^L(M_i) = \left[T^L\right] V^L(M_{i+1}), \tag{1}$$

where V^L is the vector of stresses and velocities and $\left[T^L\right]$ the transfer matrix of layer L.

For the fluid layer, $V^f = \left[p,\ v_3^f\right]^t$ is the transpose vector of the acoustical pressure p and the x_3 component of the fluid velocity, v_3^f. These variables are function of k_3, the x_3 component of the wavenumber in the fluid medium. It is defined as $k_3 = \sqrt{k_0^2 - k_1^2}$. The acoustic wavenumber is given by: $k_0 = \frac{\omega}{c_0}$), where c_0 m.s^{-1} is the speed of sound in the fluid. k_1 is the x_1 component of the wavenumber in the fluid medium. In addition, ρ_0 kg.m^{-3} is the density of the fluid medium.

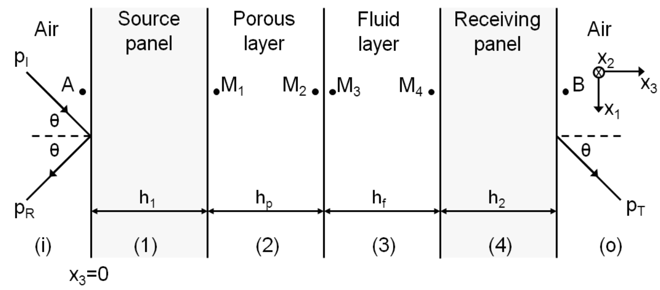

Figure 1: Transfer matrix representation of the transmission through the studied double-wall system.

The tested porous layer, being a fibrous material of low elastic modulus (below 5 kPa) and low density, is modelled using the limp approach [2]. Therefore, the same relations derived for the fluid are valid for the porous when ρ_0 and k_0 are modified to effective values ρ_l and k_l of the porous layer.

For a thin elastic layer, $V^s = \left[p\ v_3^s\right]^t$ and $Z_s(\omega)\, v_3^s = p(M_1) - p(M_2)$. Here, Z_s is the panel's impedance. In the case of a thin plate in bending, it is given by:

$$Z_s(\omega) = j\omega\rho_i h_i \left(1 - \frac{k_0^4}{k_b^4} sin^4(\theta)\right), \tag{2}$$

where ρ_i and h_i are the density and thickness of the panel and k_b its bending wavenumber.

For a laminate or a sandwich composite material, a general laminate model (GLM) is used [5]. It uses a hybrid displacement interface forces vector to define the dispersion equation. The obtained wavenumber is then inserted into Eq. (2) in order to compute the panel's impedance and resulting transfer matrix.

The transmission loss is then given by TL$= -10\,log(\overline{\tau})$, where $\overline{\tau}$ is the power transmission coefficient averaged between 0° and 78° (field incidence). It is obtained from the ratio of the amplitudes of the incident and transmitted waves, from each side of the multilayer.

2.2 Statistical Energy Analysis (SEA)

In the statistical energy analysis approach, the structure is represented by the energy level E_i, modal density n_i, coupling loss factors (CLF) η_{ij} and damping loss factors (DLF) η_{ii} of its components. The applied excitation is expressed in terms of input powers Π_i [6]. The following reciprocity relation links the CLF between subsystems: $\eta_{ij} n_i = \eta_{ji} n_j$.

The double-wall system is modelled using 5 interconnected subsystems (groups of similar resonant modes) [7], as shown in Fig. 2. A source, an inner and a receiver cavity are modelled. They are represented by subsystems 1, 3 and 5. The skin and trim panels, regrouping bending modes, are represented by subsystems (2 and 4). A 5×5 linear system is then solved to compute the energy of each subsystem.

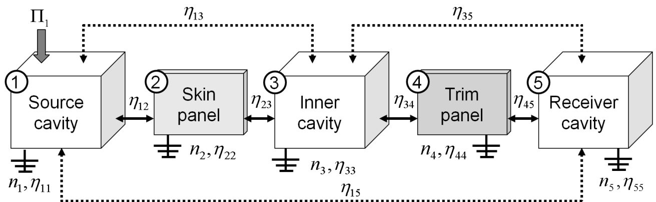

Figure 2: SEA representation of the double wall system.

The modal densities of the cavities (n_1, n_3 and n_5) are calculated using the high frequency approximation of room acoustics.

The damping loss factor of the cavities (η_{11}, η_{33} and η_{55}) are given by [7]:

$$\eta_{ii} = \frac{\alpha A_i c_0}{4\omega V_i}. \tag{3}$$

Here, α and A_i are the absorption coefficient of the cavity, assumed 0.01, and its total surface, respectively. The properties and dimensions of the source and receiving rooms are arbitrary, however, they are chosen so that their modal density and DLF are within the SEA limits of application.

The coupling loss factor between cavities (η_{13}, η_{35} and η_{15}) are calculated by using the mass-law transmission coefficient τ of the panel between cavities [8]:

$$\eta_{ij,cav} = \frac{\tau A_i c_0}{4\omega V_i}. \tag{4}$$

Coupling loss factor η_{15} represents the system, which behaves as an equivalent non-resonant single wall moving in phase, for frequencies lower than the double wall resonance of the system, approximated by [10]:

$$f_D = \frac{1}{2\pi cos(\theta)} \sqrt{\frac{\rho_0 c_0^2 (m_1 + m_2)}{\left(h_p + h_f\right) m_1 m_2}}. \tag{5}$$

In this equation m_1, m_2 h_p and h_f denote the mass of panels 1 and 2 and the thickness of porous and the fluid layers, respectively. When a layer of porous material is present in the cavity, the terms ρ_0 and c_0 are modified by the properties of the porous.

The damping loss factor of the panels are obtained experimentally for the panels installed in the measurement window, in order to account for damping added by the boundaries. The decay rate method (DRM) is used. It is described in section 3.

The modal density of the panels (n_2 and n_4) are obtained by integrating Eq. (6) over all heading directions, $n(\omega) = \int_0^\pi n(\varphi, \omega)\, d\varphi$ [8]:

$$n(\varphi, \omega) = \frac{A_p}{\pi^2} \frac{k(\varphi, \omega)}{\left|c_g(\varphi, \omega)\right|}, \tag{6}$$

where φ, A_p, k and c_g are the heading angle, the area of the panel, the wavenumber and the group velocity of the panel. The latter two are determined from the solution of the panel's dispersion relation. For the composite and sandwich panels, the general laminate model is used to compute the panel's wavenumber, which is directly used in Eq. (6).

The radiation coupling loss factor between the panel and the cavities, η_{21}, η_{23}, η_{43} and η_{45}, are computed using the following equation [8]:

$$\eta_{ij,rad} = \frac{\rho_0 c_0 \sigma_{rad}}{\omega \rho_i h_i}. \tag{7}$$

Here, ρ_i, h_i and σ_{rad} are the density, thickness and radiation efficiency of the panel. The radiation efficiency is obtained integrating $\sigma(k(\varphi, \omega))$ over all heading directions. The latter is calculated using Leppington's approach [9].

In the present modelling, a layer of porous is not treated as an individual subsystem but its influence is taken into account. For the configuration in which the skin panel is lined with a porous layer, the porous material has four main effects: (1) it increases the mass-law of the panel, (2) it acts as an added-damping, increasing the damping loss factor of the panel, (3) it attenuates the panel's radiation so that Eq. (7) is multiplied by the additional term $10^{\frac{-IL}{10}}$ in order to account for the Insertion loss (IL) of the porous material; (4) it increases the absorption of the cavity so that α in Eq. (3) becomes an average between the absorption of the cavity walls and the absorption of the porous layer.

Finally, no coupling exists between the source cavity and the trim panel $\eta_{14} = 0$, and between the skin panel and the receiver cavity $\eta_{25} = 0$. In addition, since the panels are not structurally connected, the coupling between panels is also neglected ($\eta_{24} = 0$).

The linear system is solved for an arbitrarily selected unit input power in the source room (diffuse acoustic field), the transmission loss is computed using the following equation [8]:

$$TL = NR + 10 \log_{10}\left(\frac{A_4}{\alpha A_5}\right). \tag{8}$$

Here, NR is the noise reduction given by:

$$NR = 10 \log_{10}\left(\frac{E_1}{E_5}\right) - 10 \log_{10}\left(\frac{V_1}{V_5}\right). \tag{9}$$

The term $\frac{E_1}{E_5}$ is the energy ratio between source and receiving cavities and $\frac{V_1}{V_5}$ is the ratio bewteen the volumes of the source and receiving cavities.

3 Description of the measurements

This section describes the measurement of the panels' DLF as well as the transmission loss experiments on the double-wall configuration. The panels are placed between a reverberant source room and a semi-anechoic receiver room. They have a surface area equal to 1.5 m^2. Contrary to the simulated skin panel, the tested composite panel is unidirectionally stiffened by 6 omega-shaped stiffeners. In a double-wall configuration, a 4-in cavity separates the two panels. It is filled with a 2-in aerospace grade fiberglass attached (but not bonded) to the source panel.

The damping loss factor of the panels placed on the measurement window are measured using the decay rate method

(DRM) [6]. Tests were conducted with the panels mounted in the TL window. In consequence edge damping is accounted for. The excitation is performed using an electro-mechanical shaker and results are averaged over 3 random excitation locations and 15 randomly located points over the panel surface. Two assumptions are made: damping follows an exponential decay and all modes in a third-octave band present the same damping loss factor.

The TL measurement follows ISO 15186-1 standard [11]. The structure is fixed between a reverberant and an anechoic room using a mounting frame. Joints between the panels and the frame are sealed using silicon and aluminium tapes. The edges of the panels are sandwiched between two flat bars with a neoprene decoupler. A white-noise in the frequency range of 100 Hz to 10 kHz is generated in the reverberant room. The transmission loss of the structure is given by:

$$TL = (L_p - L_I - 6). \tag{10}$$

L_p is the average sound pressure level in the source room, measured by a rotating microphone. L_I is the averaged intensity level over the measurement surface in the receiving room. The measurement is done by manually scanning the surface of the sample, A_i, with the intensimetry probe in order to obtain a spatial and temporal average. In the following discussion, the results are presented in one third-octave bands.

4 Results and discussion

The accuracy of the transmission loss predictions depends directly on the hypotheses of each modelling approach as well as on the accuracy of the input parameters. The input parameters needed for the transmission loss computation using the TMM method are mainly the damping loss factor and the mechanical properties of the panels, the acoustical properties of the fluid and porous material, and the thickness of each layer of the system. The damping loss factor of the panels are obtained experimentally as described in section 3. The other above parameters are used in the computation of the modal density, the damping and the coupling loss factors of each subsystem of the SEA model.

It should be recalled that present theoretical results neglect stiffeners effect. Their influence is expected mainly in the low frequency range, in which the panel behaves as an equivalent orthotropic stiff panel. The modal density of a stiffened panel reaches that of an unstiffened panel when the panel's bending wavelength becomes lower than the spacing between stiffeners. In the case of the studied composite panel, for frequencies higher than 500 Hz.

Fig. 3 compares theoretical and experimental results of the transmission loss of the composite panel. The models agree well with measurements up to 1 kHz. At higher frequencies the differences are due to the presence of stiffeners, which are not accounted for in the modelling. Radiation from stiffeners reduce the TL of the measured panel for frequencies lower than its critical frequency. In addition, for a stiffened panel, its response is driven by the radiation of the panel sections delimited by the stiffeners. Since the radiation of the smaller sub-panels is higher than that of larger panels, the TL decreases. At and above the critical frequency region the theoretical TL is lower than measurements, indicating an underestimation of the panel's damping loss factor. Indeed, the measurement of the panel's DLF is subjected to higher uncertainties in this frequency region.

The measured TL of the composite panel lined with a porous material is also compared with the transfer matrix method (represented by the two curves of Fig. 3 having the highest transmission loss). Similar conclusions are observed as in the case of the bare panel, except that the damping loss factor of the panel lined with the porous is well estimated, leading to a good agreement between theoretical and measured TL at and above its critical frequency. It should be noted that part of the radiation from stiffeners is attenuated by the porous layer, however the modelling of stiffened panels is still required to improve agreement with measurements.

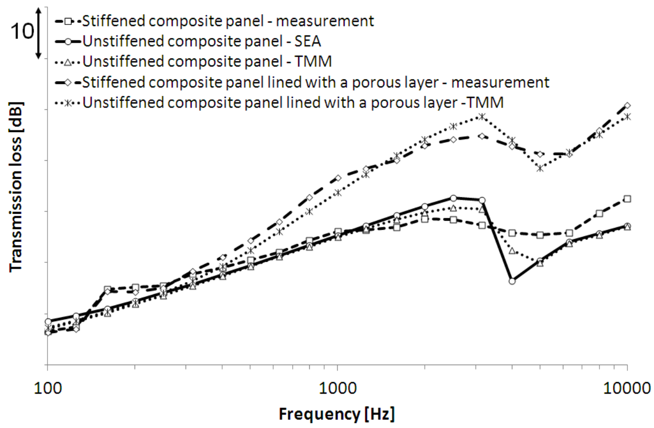

Figure 3: Transmission Loss of the composite panel: comparison between transfer matrix method (TMM), statistical energy analysis (SEA) and measurements for a bare panel and a panel with porous layer.

Theoretical and experimental transmission loss for the sandwich panel are shown in Fig. 4. Both models are in good agreement with measurements in the mass-law region. In the critical frequency region (around 2500 Hz), differences are due to uncertainties in the measurement of the panel's damping loss factor, particularly for SEA computations. This difference can be also caused by uncertainties in the properties of the panel, notably the shear moduli of the core, resulting in a shift of its critical frequency. Prediction using a Finite element/Boundary element model leads to similar results thus corroborating this assumption.

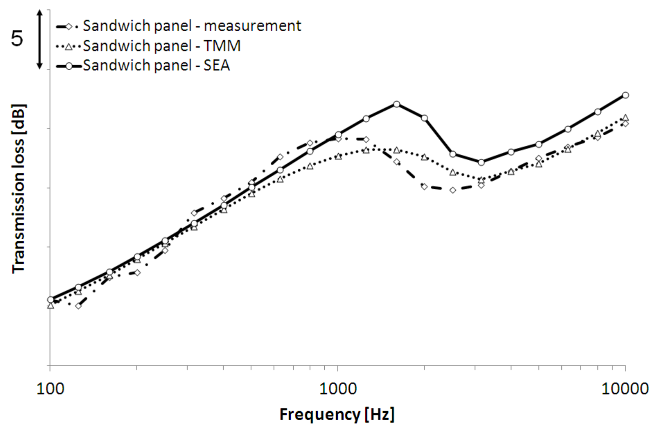

Figure 4: Transmission Loss of the sandwich panel: comparison between transfer matrix method (TMM), statistical energy analysis (SEA) and measurement.

Double-wall transmission loss results are shown in Fig. 5.

SEA approach presents a better agreement with experiments compared to TMM. Differences at frequencies lower than 500 Hz are due to the stiffeners, which are not modeled. The decoupling frequency, given by eq. (5), is around 125 Hz as observed in predicted curves. It is not observed in the experimental curve due to the high damping and limitations of the used test facility (cutoff frequency of 200 Hz for the reverberation room as well as stiffeners effects increasing the TL). At higher frequencies TMM results overestimate the double-wall transmission loss. This happens since the dominant transmission path includes the resonant behaviour of the inner cavity. This path is not modelled in the TMM, which only assumes 1D propagation (infinite lateral dimensions). Moreover, both models overestimate the measured results in the critical frequency region of the trim panel (around 2.5 kHz), which is mainly a consequence of the overestimation observed in Fig. 4 for the TL of the trim panel.

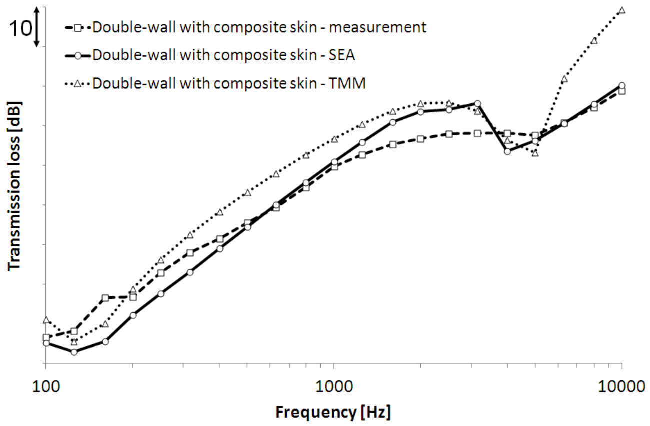

Figure 5: Transmission Loss of the double wall: comparison between transfer matrix method (TMM), statistical energy analysis (SEA) and measurement.

Another comparison of practical importance for aircraft applications is the impact on the double-wall transmission loss of the main structure; aluminium vs. composite skin. The accuracy of theoretical models in predicting this impact is analysed in Fig. 6 as a difference between the double-wall transmission loss with the composite skin and with an aluminium skin panel of critical frequency around 6 kHz. Between 400 Hz and 3 kHz the agreement between models and measurements is good. The influence of modifying the skin panel is well predicted by both models. At lower frequencies both models give the same trends but since stiffeners are not modelled, results do not agree with measurements. Between 3 kHz and 6 kHz, in the critical frequency region of the panels, the models predict the dip found in the experimental curve but it is highly overestimated due to uncertainties in the DLF of the panels. At higher frequencies, good agreement with measurements is only observed with the SEA approach.

5 Conclusion

This paper aimed at comparing two methods for the prediction of the transmission loss of complex lightweight double-walls, including a stiffened composite panel and a highly-radiating sandwich panel, with critical region around 2.5 kHz. For the studied configuration, it is shown that SEA is more suitable for the assessment of the TL of such structures compared to the TMM approach. Indeed, TMM overestimates

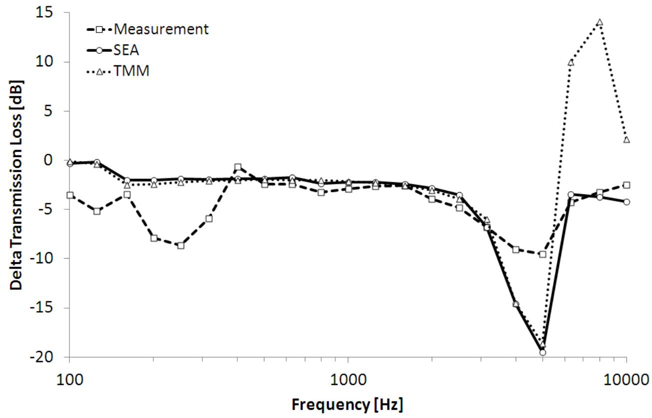

Figure 6: Difference between the transmission Loss of a double-wall with a composite skin panel and the transmission Loss of a double-wall with an aluminium skin panel: comparison between transfer matrix method (TMM), statistical energy analysis (SEA) and measurement.

the measured TL in the frequency range of analysis and diverges at frequencies higher than the critical frequency of the skin panel. It is found that the modelling of the inner cavity in the SEA approach as a resonant subsystem is more adapted to this specific double-wall. In the TMM approach, it is modelled as a layer of infinite lateral dimensions, neglecting thus the contribution of some sets of modes. Other comparisons done for double-wall systems composed of thin metallic panels have shown that both methods predict well the TL. The difficulties of the TMM in the presented example are traced to the high radiation of the sandwich panel (low coincidence region) and thus to the importance of accounting for the resonant behaviour of the inner cavity. However, for design purposes, both approaches are able to predict the influence in the transmission loss of modifying the skin panel material from aluminium to composite in the mass-law region.

Acknowledgments

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