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Analysis of a sandwich elastic plate structure by means of its transition terms

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In order to analyze coupled resonances, the formalism of the transition terms is used because it easily brings more information than a study restricted to the reflection and transmission coefficients. We measure at normal incidence R and T, the reflection and transmission coefficients of two aluminum plates separated by a very thin water layer. The transition terms are linear combinations of R and T and the coupled resonances appear in separate terms. Under the condition that the previous coefficients have been recorded with a common phase reference, it is possible to isolate the resonances well. The vertical mode and the resonances of the water layer can be easily located on the spectra of the experimental transition terms. The shifts between the resonances of the structure and the resonance frequencies of the symmetric and antisymmetric modes of a unique plate are exhibited as well. The technique presented here could be of great interest in the area of the non destructive testing (NDT).

1 Introduction

We consider two water-immersed identical elastic plates coupled by a film of water. The acoustic characterization of this structure needs the measurement of its eigenfrequencies. The literature is quite abundant about these measurements [see e.g. 1]. Usually, the structure is insonified by a pulse and the scattered signal is recorded. A Fourier transform is applied and the different frequencies appear on a spectrum (reflection or transmission coefficient). If the peaks related to the resonance frequencies do not overlap, the determination of the frequency values is easily made on the previous spectra. But the coupling gives rise to several resonances whose peaks more or less overlap on the plots of the reflection and transmission coefficients (R and T). These latter, then, give limited information about resonance frequencies. Here, we address the case where the elastic system exhibits a plane of symmetry. In this case the coupling of modes implies two modes with different symmetry: one mode is symmetric whereas the other is antisymmetric.

Issued from S-matrix theory, the two transition terms of the structure are linear combination of R and T. As they split the resonance frequencies in two separate plots, related to the symmetry of the modes, the determination of the resonance frequencies becomes easy. There is a good agreement between the experimental values and the theoretical ones.

2 Theoretical background

The scattering matrix denoted $\mathbf{S}$ is a useful tool devoted to analyse the modes of vibration of elastic structures [2-4]. It involves both the reflection (R) and the transmission (T) coefficients of the structure. For a structure with a plane of symmetry such as plates or stacks of plates and loaded with the same fluid at each external face, the $\mathbf{S}$ matrix takes the simple form:

$$\mathbf{S} = \begin{pmatrix} R & T \\ T & R \end{pmatrix}. \tag{1}$$

The transition matrix denoted $\mathbf{T}$ issued from the $\mathbf{S}$ matrix by

$$\mathbf{S} = \mathbf{I} + 2\,\mathbf{I}\,\mathbf{T}, \tag{2}$$

where $\mathbf{I}$ is the matrix identity, can be diagonalized in two terms corresponding to the symmetric T_S and the antisymmetric modes T_A [4]. These terms obey the relationships

$$T_S = \frac{1}{2i}\left(R + T - 1\right), \tag{3}$$

$$T_A = \frac{1}{2i}\left(R - T - 1\right). \tag{4}$$

At a resonance frequency the module of these terms reaches 1 which is their maximum value. As a preliminary conclusion, it can be said that the separation between symmetric and antisymmetric modes, and further, the separation of the coupled modes becomes possible if the complex values of R and T are properly measured.

The Resonance Scattering Theory (R. S. T.) gives the theoretical reflection and transmission coefficients of an elastic plate [2]:

$$R_1^{th} = \frac{CaCs - \tau^2}{Ca + Cs}\left(\frac{1}{Ca + i\tau} + \frac{1}{Cs - i\tau}\right), \tag{5}$$

$$T_1^{th} = i\tau\left(\frac{1}{Ca + i\tau} + \frac{1}{Cs - i\tau}\right). \tag{6}$$

The terms Ca, Cs and τ are given in appendix. They depend on the Lamé coefficients and density of the plate. The reflection and transmission coefficients of two identical plates coupled by a film of fluid of thickness d_F are calculated via the latter relations and using the classical Fabry-Pérot technique. It comes for an incident wave of circular frequency ω:

$$R_2^{th} = R_1^{th} + \frac{\left(T_1^{th}\right)^2 R_1^{th}\,\Phi(\omega)^2}{1 - \left(R_1^{th}\,\Phi(\omega)\right)^2}, \tag{7}$$

$$T_2^{th} = \frac{\left(T_1^{th}\right)^2 \Phi(\omega)}{1 - \left(R_1^{th}\,\Phi(\omega)\right)^2}, \tag{8}$$

in which $\Phi(\omega)$ is the phase lag related to the travel of the wave in the fluid layer where sound propagates with the velocity c_F. It obeys the relationship:

$$\Phi(\omega) = \exp\left(-i\,\omega\frac{d_F}{c_F}\right). \tag{9}$$

We propose the experimental determination of the transition terms as a means for obtaining the water thickness and the coupled resonance frequencies. Such a coupled structure has yet been discussed in an early theoretical and experimental study [5]. This pioneering paper showed that the coupled modes can be separated in symmetric or antisymmetric, but it was restricted to the study of the reflected and transmitted coefficients and determination of the frequencies of the modes could not be achieved.

3 Experimental technique

The experiment is carried out in two steps using the same setup limited to normal incidence (Fig. 1). First, an aluminum plate ($300\,\text{mm} \times 200\,\text{mm} \times 5\,\text{mm}$) and then two identical ones separated by a film of water (0.26 mm thick) are immersed vertically in a 2000 liter water tank.

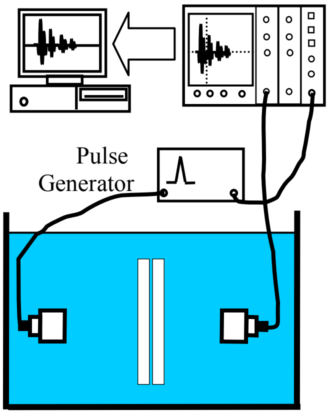

Figure 1: Experimental setup.

The distance between the transducers is about 1 m and the distance between the emitter and the target is about 50 cm. The targets are insonified with a normal incident pulse repeatedly launched by the emitter and produced by the 300 V discharge of the internal capacitor of the generator which also triggers the launching. Using broadband transducers (Panametrics® non-focused, diameter of the active element 4 cm, central frequency 2.25 MHz), the frequency domain investigated runs from 140 kHz to 3.5 MHz. The reflected and transmitted signals by the systems are stored after the electronic perturbations have been eliminated thanks to an average of 200 acquisitions. The sampling frequency is 100 MHz and the recorded signals have 80,000 samples.

4 Results

The complete determination of the reflection and transmission coefficients of our targets needs the recording with the same electronic system of the reflected and transmitted signals issued from "calibration targets", that is, whose reflection and transmission coefficients are already known. For the measuring of the transmission coefficients of one plate and of the coupled structure, the transmitted wave through a water layer of the same thickness of each of the elastic targets is recorded (transmission coefficient: 1). In that case, the direct signal from the emitter to the receiver is recorded (the plates are simply removed).

To obtain the experimental reflection coefficients of the previous targets, the water/air interface whose reflection coefficient is −1 within the frequency range is used. A mechanical device, not represented in Fig. 1, allows us to rotate the emitter and so the normally reflected signal onto this interface is recorded. Particular attention is paid to keep identical distances between a target and the active face of the emitter and the water/air interface and this face.

4.1 Temporal signals

The recorded reflected and transmitted signals are presented in Fig. 2, 3 for a unique aluminum plate and for the two plate structure. Those signals exhibit numerous echoes onto the faces of the targets; that is strongly enhanced in case of coupling which entails an increasing of the duration of the signals.

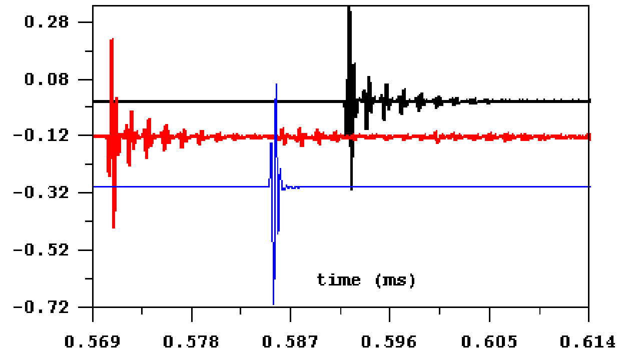

Figure 2: Reflected signals (shifted for clarity): onto a unique plate (black), onto the two plate structure (red) and onto the water/air interface (blue). Amplitude in Volts (same amplification).

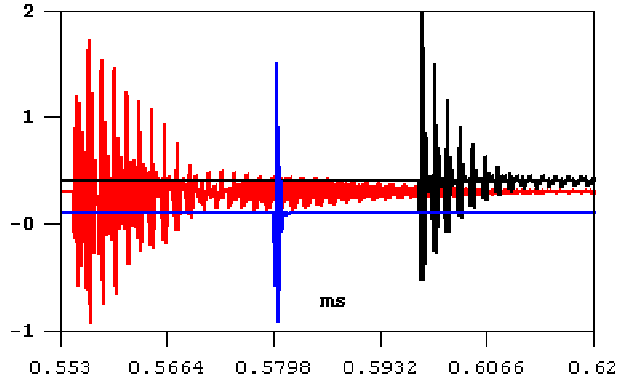

Figure 3: Transmitted signals (shifted for clarity): through a unique plate (black), through the two plate structure (red) and direct (blue). Amplitude in Volts (same amplification).

4.2 Reflection, transmission coefficients

The experimental transmission coefficient of a target of thickness d is the ratio of the Fourier transform of its transmitted signal to the Fourier transform of the incident signal onto it. This latter is obtained by shifting the recorded direct signal of the delay time $-d/c_F$. The experimental reflection coefficient is the ratio of the Fourier transform of the reflected signal onto the structure to the Fourier transform of the reflected signal onto water/air interface (its complex value is then multiplied by –1). Diffraction is neglected because those ratios of Fourier transforms are issued from temporal signals recorded under identical geometrical configurations regarding the transducers and the targets. This procedure allows us to compare the experimental coefficients to the theoretical ones obtained from a plane wave model.

The transition terms of the aluminum plate are plotted in Figs. 4, 5. The agreement is good regarding the location of the resonance frequencies. But as the normally incident acoustic beam onto the target is not perfectly cylindrical, some transverse waves are generated and their resonance frequencies superimpose to the normal spectra. This superposition mainly occurs on the symmetric transition term.

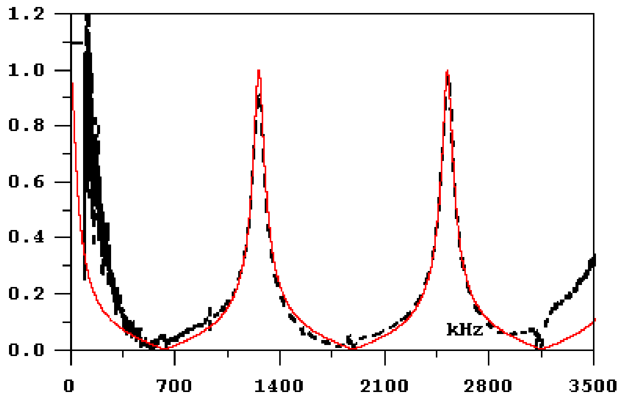

Figure 4: Modulus of the normal antisymmetric transition term of the aluminium plate: experimental (dotted black) and calculated (red).

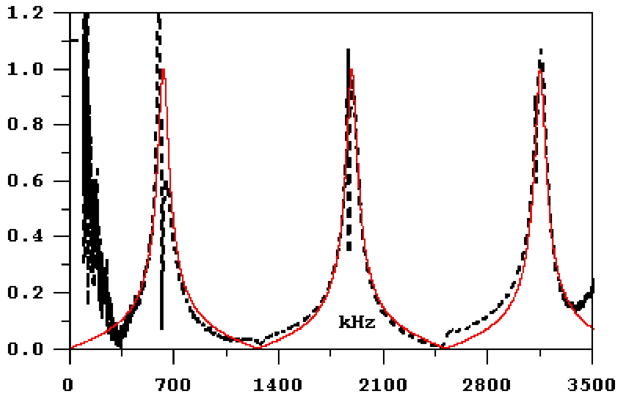

Figure 5: Modulus of the normal symmetric transition term of the aluminium plate: experimental (dotted black) and calculated (red).

Within the experimental frequency range, the aluminum plate exhibits two antisymmetric and three symmetric resonances clearly located in the plots above.

4.3 Experimental values

The previous study of the aluminum plate used here allowed us to determine its Lamé coefficients: $\lambda = 54.5$ GPa and $\mu = 26.5$ GPa with a density of $\rho = 2700\ \mathrm{kgm^{-3}}$ leading to $c_L = 6310\ \mathrm{ms^{-1}}$ and $c_T = 3133\ \mathrm{ms^{-1}}$ for the velocity of the longitudinal and transverse sound wave in the material. The velocity of sound in water is $c_F = 1478\ \mathrm{ms^{-1}}$.

It is well known that coupling two identical elastic systems at least provides twice as many resonances as a unique one does [6]. This is shown in Fig. 6 where the transmission coefficient of two coupled plates is plotted.

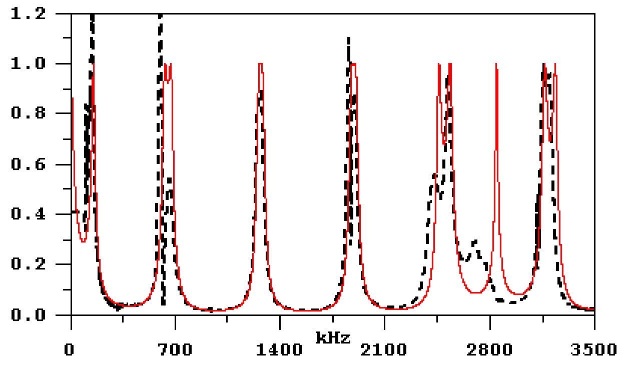

Figure 6: Modulus of the normal transmission coefficient of the two plate structure: experimental (dotted black) and calculated (red).

The resonances are more numerous than in the case of a unique plate. One can locate the resonances already present on the spectra of one of the aluminium plate. Peaks of some resonances overlap, while two new ones appear: at a very low frequency (≈150 kHz) and at 2845 kHz. On the whole there is a good experimental/theoretical agreement, but without any separation between symmetric and antisymmetric, little information about resonances can be extracted from these spectra.

Three reasons contribute to the augmentation of the number of resonances. First, each of the resonances (symmetric and antisymmetric) of one plate gives rise to two resonances: an antisymmetric one and a symmetric one, located at the vicinity of the given frequency of the plate. Second, the resonances of the fluid layer are superimposed to those latter. The frequencies obey the relationship $f_F = p\dfrac{c_F}{2d_F}$ where p is an integer. This relationship is used to determine the thickness of the film of fluid. The third kind of resonance is the vertical mode presented in Ref. [5]. It is located at a frequency lower than the frequency of the first Lamb mode of one plate and strongly related to d_F but weakly related to the incidence angle (Fig. 6, 8).

The spectra of the experimental and calculated transition terms of the two plate structure are presented in Figs. 7, 8.

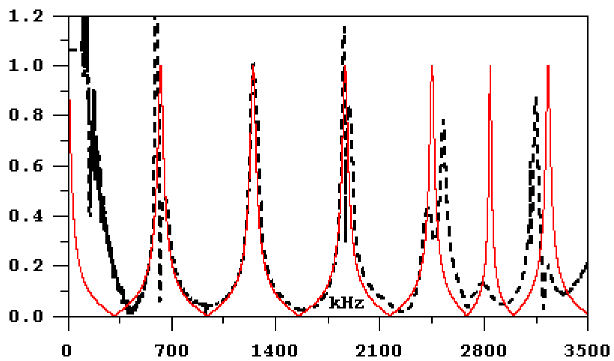

Figure 7: Modulus of the normal antisymmetric transition term of the two plate structure: experimental (dotted black) and calculated (red).

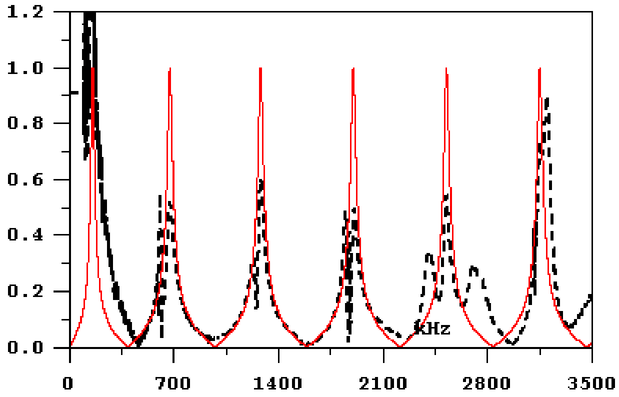

Figure 8: Modulus of the normal symmetric transition term of the two plate structure: experimental (dotted black) and calculated (red).

On the whole, the agreement is good regarding the location of the resonance frequencies. Here too, transverse waves superimpose to the normal spectra. This superposition comes from the value of the ratio $c_L / c_T \approx 2$ and mainly occurs on the symmetric transition term. Fig. 9 shows a close-up of the symmetric transition term. As it was also pointed out in the theoretical parts of Ref. [5, 7], the vertical mode corresponds to a symmetric mode of the structure. The experimental vertical mode is located at the expected frequency but its amplitude is larger than 1 because the vertical modes connected to the different oblique incidence waves superimpose at the same frequency; (it must be recalled that this mode was named vertical to emphasize the fact that unlike Lamb wave resonances, its resonance frequencies keep an identical value whatever the incident angle of the wave) [5, 7]. In Table I, the experimental and theoretical values of the resonance frequencies of the studied coupled structure are compared. The theoretical values are obtained following two ways: directly from the plot of the transition terms (Fig. 7, 8) and from the dispersion relations numbered (4), (5) in Ref. [5] which are recalled in the appendix.

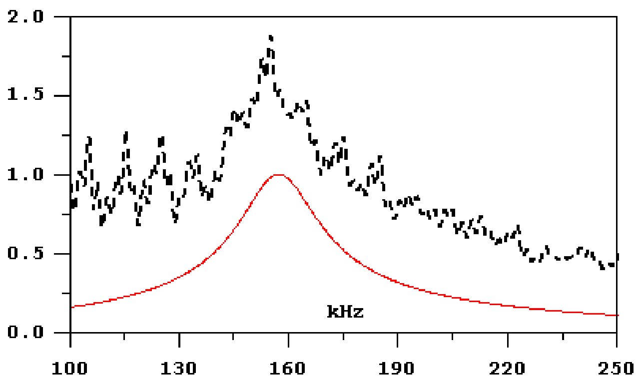

Figure 9: Vertical mode. Modulus of the normal symmetric transition term of the two plate structure: experimental (dotted black) and calculated (red).

Table I

Symmetric modes (kHz)			
Transition term	Theory	Exp.	Nature and origin related to one plate.
157.3	157.4	155	Vertical mode
674.8	677.4	678	Issued from S2
1282.3	1287.6	1287	Issued from A3
1902.9	1901.3	1913	Issued from S5
2527.0	2535.1	2531	Issued from A6
3151.9	3168.8	3159	Issued from S8
Antisymmetric modes (kHz)			
624.6	633.7	602	Issued from S2
1247.6	1249.7	1252	Issued from A3
1864.2	1869.6	1860	Issued from S5
2448.8	2448.8	2427; 2528	Issued from A6
2841.0	2841.2	2781	Resonance of the water layer.
3231.0	3242.0	3153	Issued from S8

5 Appendix

This part follows the results of Ref. [5] and recalls the dispersion relationships giving ω, the resonance frequencies of the studied trilayer (Solid/Fluid/Solid) at normal incidence. The thicknesses are respectively d_S for the plate and d_F for the film of fluid. The density of water is written ρ_F. It comes successively for the symmetric and antisymmetric modes:

$$2\,\mathrm{CaCs} + \tau\left(\mathrm{Ca} - \mathrm{Cs}\right)\cot\left(\frac{\omega}{c_F} d_F/2\right) = 0\,, \qquad \text{(A1)}$$

$$2\,\mathrm{CaCs} - \tau\left(\mathrm{Ca} - \mathrm{Cs}\right)\tan\left(\frac{\omega}{c_F} d_F/2\right) = 0\,. \qquad \text{(A2)}$$

With

$$\mathrm{Ca} = \left(\frac{\omega}{c_T}\right)^4 \tan\left(\frac{\omega d_S}{2 c_L}\right), \qquad \text{(A3)}$$

$$Cs = \left(\frac{\omega}{c_T}\right)^4 \cot\left(\frac{\omega d_S}{2c_L}\right), \qquad \text{(A4)}$$

$$\tau = \frac{\rho_F}{\rho}\frac{c_F}{c_L}\left(\frac{\omega}{c_T}\right)^4 . \qquad \text{(A5)}$$

6 Conclusion

The implementation of the S matrix theory leads to separate two close resonances via the amplitude of transition. This fact was already pointed out in the literature but the experimental evidence was not already achieved. In order to show the interest of the theoretical formalism, an experimental system composed by two plates coupled by a film of water is investigated. The experimental separation of resonances in symmetric and antisymmetric modes is performed in accordance with theoretical results. The method can be used for other systems provided that the transmission and reflection coefficients can be properly obtained.

References

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