Inverse method for porous material characterization using the constraint satisfaction problem approach

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The Constraint Satisfaction Problem (CSP) approach has been used successfully in optimization procedure for several engineering applications. In this paper this method is evaluated in an inverse procedure for recovering porous material parameters from acoustical data. The sought parameters are the five of the Johnson-Champoux-Allard model: porosiy, resistivity, tortuosity, viscous and thermal characteristic lengths. First, the CSP algorithm is presented: it is based on interval arithmetic and domain reducing algorithms. The procedure is applied to one virtual porous material to show its potential. Acoustical input data are the density and bulk modulus of the equivalent fluid to the material at two frequencies (50 Hz and 1000 Hz). The efficiency of the proposed method is finally discussed.

1 Introduction

The modeling of sound absorbing materials, like mineral fiber or open cell foams, requires the knowledge of several parameters like porosity and air flow resistivity [1]. Several experimental methods exist to determine these parameters. They can be divided in two categories. The first are based on non-acoustical experiments. For instance, the determination of porosity, i.e. ratio of air volume in the material, is possible by measuring air volumes [2] or by weighting the saturated sample in several conditions [3]. The disavantage of such methods is the cost and the number of experimental set-up required to determine all the parameters. To overcome these limitations, inverse methods have been proposed: they are based on the measurement of acoustical properties like reflection or transmission coefficients of a porous sample, using standard set-up [4, 5, 6, 7]. These methods may suffer from experimental limitations and ill conditionning of the inverse problem [7].

This paper focuses on the mathematical inverse procedure using the Constraint Satisfaction Problem (CSP) approach, which efficiency has been demonstrated for enginering applications[8]. The aim is to show the efficiency of the CSP approach for recovering porous material parameters from acoustical data.

First, the Johnson-Champoux-Allard [1] (JCA) model using 5 parameters is recalled. Then, the CSP approach is described and applied to one virtual sound absorbing material. The convergence of the method is finally discussed.

2 Porous material modeling

Johnson-Champoux-Allard [1] (JCA) model is based on a semi-empirical representation of the porous material accounting for viscosity of the air and thermal exchange between the air and the skeleton. The skeleton is assumed to be motionless and its temperature constant. Effect of viscosity is accounted for in the expression of the dynamic density $\tilde{\rho}$, given by

$$\tilde{\rho}(\omega) = \alpha_\infty \rho_0 \left[1 + \frac{1}{j\hat{\omega}} \tilde{G}(\omega) \right], \tag{1}$$

with

$$\tilde{G}(\omega) = \sqrt{1 + j\frac{M}{2}\hat{\omega}}, \tag{2}$$

and $\hat{\omega} = \frac{\omega \alpha_\infty \rho_0}{\phi \sigma}$ the dimensionless frequency, $M = \frac{8\alpha_\infty \mu}{\phi \Lambda^2 \sigma}$ the form factor. The porosity ϕ , the air flow resistivity σ, the tortuosity α_∞ and the viscous characteristic length Λ are 4 of the 5 parameters describing the porous material. The density ρ_0 and the viscosity μ are properties of the air. Note that the tilde indicates a frequency dependent and complex variable.

Thermal effects are included in the expression of the dynamic bulk modulus $\tilde{K}$ as:

$$\tilde{K}(\omega) = \frac{K_0}{\gamma - (\gamma - 1)\left[1 + \frac{8\mu}{j\omega\rho_0 B^2 \Lambda'^2}\tilde{G}'(\omega)\right]^{-1}}, \tag{3}$$

with

$$\tilde{G}'(\omega) = \sqrt{1 + j\frac{\omega \rho_0 B^2 \Lambda'^2}{16\mu}}. \tag{4}$$

The thermal characteristic length Λ' is the fifth parameter to be determined. K_0 is the adiabatic bulk modulus of the air, B^2 the Prandlt number and γ the ratio of specific heats of the air.

Finally, the properties of the equivalent fluid to the porous material are [1]: $\tilde{\rho}_{eq} = \tilde{\rho}/\phi$ and $\tilde{K}_{eq} = \tilde{K}/\phi$. These properties can be determined experimentally from acoustical measurements using an impedance tube [5, 9, 6]. In this work, they are computed for a given sound absorbing material (table 1).

Since the inverse procedure works with real literal expressions, imaginary and real parts of $\tilde{\rho}_{eq}$ and $\tilde{K}_{eq}$ have been explicited using Maxima symbolic code.

3 Theory of CSP

3.1 Constraint Satisfaction Problem

A Constraint Satisfaction Problem (CSP) is defined by a triplet (X, D, C) such that [10]:

- $X = \{x_1, ..., x_n\}$ is a finite set of variables which we call constraint variables with n being the integer number of variables in the problem to be solved;
- $D = \{d_1, ..., d_n\}$ is a finite set of variable value domains of X such that: $\forall i \in \{1, ..., n\}, x_i \in d_i$;
- $C = \{c_1, ..., c_p\}$ is a finite set of constraints, p being any integer number representing the number of constraints of the problem: $\forall i \in \{1, ..., p\}, \exists! X_i \subseteq X / c_i(X_i)$.

A constraint is any type of mathematical relation (linear, quadratic, non-linear, Boolean, ...) covering the values of a set of variables. The constraints considered in this paper are of the following kind: arithmetical, such that $x > y + 1; 3x + 2y < z$; explicit, in the form of n-tuples of possible values such that: $(x, y), (0, 1), (2, -1), (3, 3)$.

The variable domains can be discrete, in the form of sets of possible values, or continuous, in the form of intervals on real numbers. In our application, the latter description will be used.

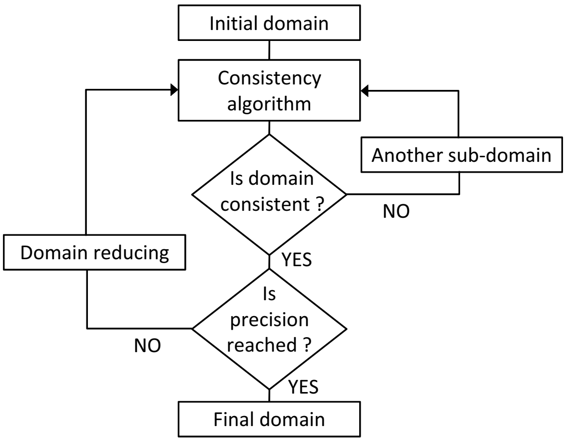

Figure 1: CSP organigram.

3.2 Solving numerical CSP

Solving a CSP boils down to instantiating each of the variables of X while meeting the set of problem constraints C. To do so, the domains d_i are reduced until the precision (target size of each domain) is reached.

The algorithm is depicted in figure 1. From the initial domain D, a consistency algorithm is applied. It aims at finding the largest domain for each variable within the initial domain so that each constraint is verified. If one constraint within the initial domains may not be satisfied, the domain D is called not consistent. If the domain D is consistent, it is reduced by the Branch and Prune algorithm [11]. This algorithm defines the strategy to choose which domain d_i to reduce first and how to reduce its size. In our case, the domain reduction keeps the lower half of the interval. Then, consistency algorithm is run. If the domain is not consistent, another reduced domain is considered (the upper half of the interval, for instance). The process is stopped when the precision is reached. In our case, the process will give the lower domain for each variable. It could be launched several times until all the solutions are found. In the following, interval arithmetic, consistency algorithms, and Branch and Prune algorithms are detailled.

3.3 Interval arithmetic

Consistency algorithms are based on interval arithmetic [11]. It is a method that can provide lower and upper bounds for a function with interval unknowns. One of its important advantages is that it allows computer round-off errors to be taken into account. The interval evaluation of a function determines an interval that guarantees the inclusion of the exact lower and upper bounds of this function. We denote $\underline{x}$ as the lower bound and $\overline{x}$ as the upper bound of the interval x. Basic operations used on floating-point intervals are for instance:

$[\underline{x}, \overline{x}] + [\underline{y}, \overline{y}] = [\underline{x} + \underline{y}, \overline{x} + \overline{y}]$,
$[\underline{x}, \overline{x}] - [\underline{y}, \overline{y}] = [\underline{x} - \overline{y}, \overline{x} - \underline{y}]$,
$[\underline{x}, \overline{x}] * [\underline{y}, \overline{y}] = [\min(\underline{xy}, \underline{x\overline{y}}, \underline{\overline{x}y}, \underline{\overline{xy}}), \max(\overline{\underline{xy}}, \overline{\underline{x}\overline{y}}, \overline{\overline{x}\underline{y}}, \overline{xy})]$.

Interval arithmetic has been extended to take into account other operators. For example, quadratic, trigonometric, logarithmic and other non linear constraints has been developed. For example if $f(x) = x + \sin(x)$ then, the interval evaluation of f for x in $[1.1, 2]$ can be computed as follow: $f([1.1, 2]) = [1.1, 2] + \sin[1.1, 2] = [1.1, 2] + [0.8912, 1] = [1.9912, 3]$.

3.4 Consistency algorithms

Several consistency techniques exist in the literature [12]. Let us present Hull consistency that will be used in our application.

Let (X, D, C) be a constraint satisfaction problem involving a vector X of n variables and let $[x_i]$ be the domain of x_i. (X, D, C) is said to be hull consistent if for every constraint c_j in C and $\forall i (1 \le i \le n)$, there exists two points in $[x_i]$ (lower and upper bounds) which satisfy c_j. As an example, let's start with the constraint between two variables: $y = x^2$ with $(x, y) \in [0, 1] \times [0, 6]$. $x^2 \in [0, 1]$ involves that $x \in [-1, 1] \cap [0, 1]$, and finally, $x \in [0, 1]$. Then $y \in [0, 1]^2$ involves that $y \in [0, 1] \cap [0, 6] = [0, 1]$. The new cartesian product $(x, y) \in [0, 1] \times [0, 1]$ is hull consistent with the constraint and the interval of y has been reduced.

3.5 Domain reducing

Domain reducing algorithms as Branch and Prune [11] start the process by selecting the variable to bisect, i.e. the first domain d_i to be reduced. The order in which this choice is done is referred as the variable ordering. A correct ordering decision can be crucial to perform an effective solving process in case of real-life problems. After selecting the variable to bisect, the algorithms have to select a subinterval from the variable domain. This selection is called the value ordering. It can also have an important impact on the duration of the solving process. In our application, the most efficient ordering has been found to be first Λ', then ϕ, α_∞ and finally either σ or Λ.

4 Results

4.1 Causality analysis

The causality analysis [13] allows to determine the minimal set of input parameters which is necessary to instantiate all the output variables of the problem. In our case, the output parameters are the 5 sought parameters of the porous material: ϕ, σ, α_∞, Λ and Λ'. The input parameters are the frequency dependent properties of the equivalent fluid: $\tilde{\rho}_{eq}(\omega)$ and $\tilde{K}_{eq}(\omega)$. They may be measured using an impedance tube [5, 9, 6]. The causality analysis consists of finding a path from the necessary input parameters to the output ones, by managing a causal graph, i.e. an oriented graph where variables are linked together via the relations of the problem (see figure 2).

In the JCA model, $\tilde{K}_{eq}$ is function of the 2 parameters (ϕ, Λ') (see section 2). Consequently, the causal graph (figure 2) shows a link between real and imaginary part of $\tilde{K}_{eq}$ at frequency 1 and the couple (ϕ, Λ'). Those last variables are linked together in the same block: this indicates that a subset

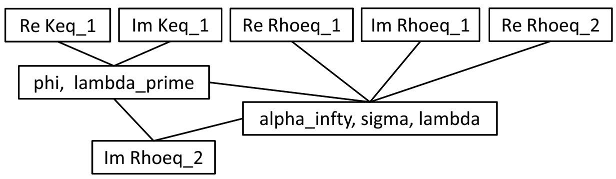

Figure 2: Causality analysis graph.

of (linear or non linear) equations makes those variables self dependent.

Moreover, $\tilde{\rho}_{eq}$ is function of the 4 parameters (ϕ, σ, α_∞, Λ). The triplet (σ, α_∞, Λ) may be determined by adding to the couple (ϕ, Λ'), the 3 following input values: real and imaginary part of $\tilde{\rho}_{eq}$ at frequency 1 and real part of $\tilde{\rho}_{eq}$ at frequency 2. Again, the triplet (σ, α_∞, Λ) are in the same block.

Finally, from the 5 output parameters, imaginary part of $\tilde{\rho}_{eq}$ at frequency 2 may be determined. This variables is not strictly necessary in the process but may increase its efficiency.

4.2 Application

In this section, the method is applied to one material representive of a standard polymer foam used for sound absorption. The characteristics are given in table 1 with the bounds of the initial domain. Each variable is considered to be real.

Table 1: Properties of porous material: targeted values, initial intervals, intervals obtained after first iteration of Hull consistency algorithm.

Parameter	Target	Initial		Hull (1st it.)	
ϕ	0.98	0.9	1	0.9	0.99
σ (Nsm^{-4})	15 000	5 000	50 000	9 850	22 055
α_∞	1.05	1	1.5	1	1.33
Λ (μm)	100	10	2 000	37	2 000
Λ' (μm)	250	10	2 000	102	1 900

The method is applied using the following input parameters: real and imaginary parts of $\tilde{K}_{eq}$ at 1000 Hz, real and imaginary parts of $\tilde{\rho}_{eq}$ at 50 Hz and 1000 Hz. These frequencies fall within the limits of classical impedance tube.

Figure 3 illustrates the evolution of the solution (lowest and upper bounds of interval) for each parameter as function of the precision. The precision is the relative size of the interval to be reached to stop the process. In our case, it is chosen the same for each sougth variables. A precision of 10^{-3} means that the size of the interval of each variable is less than 3 significant decimals. The precision for the input parameters, i.e. equivalent fluid properties, is 10^{-10}.

The convergence for all parameters can be considered to be reached for a precision of 10^{-2}. Note that only the first solution is plotted; since the domain reducing starts by keeping the lower half of the interval, the values are the most often found below the targeted values. It is noticeable that the intervals for the resistivity is far smaller. This means that the problem has no solution if σ is not accurately determined.

The previous results may be analysed in terms of relative error. The error is calculated using the lowest bound of the interval. For the precision of 10^{-2}, the error is less than 3% for each variable. For the precision of 10^{-1}, the error is less than 8% for porosity, tortuosity and resistivity, which is still satisfactory. However the error is far higher for viscous and thermal lengths, being 19% and 36% respectively.

5 Conclusion

The ability of the Constraint Satisfaction Problem (CSP) approach, as an inverse procedure for recovering porous material parameters from acoustical data has been evaluated.

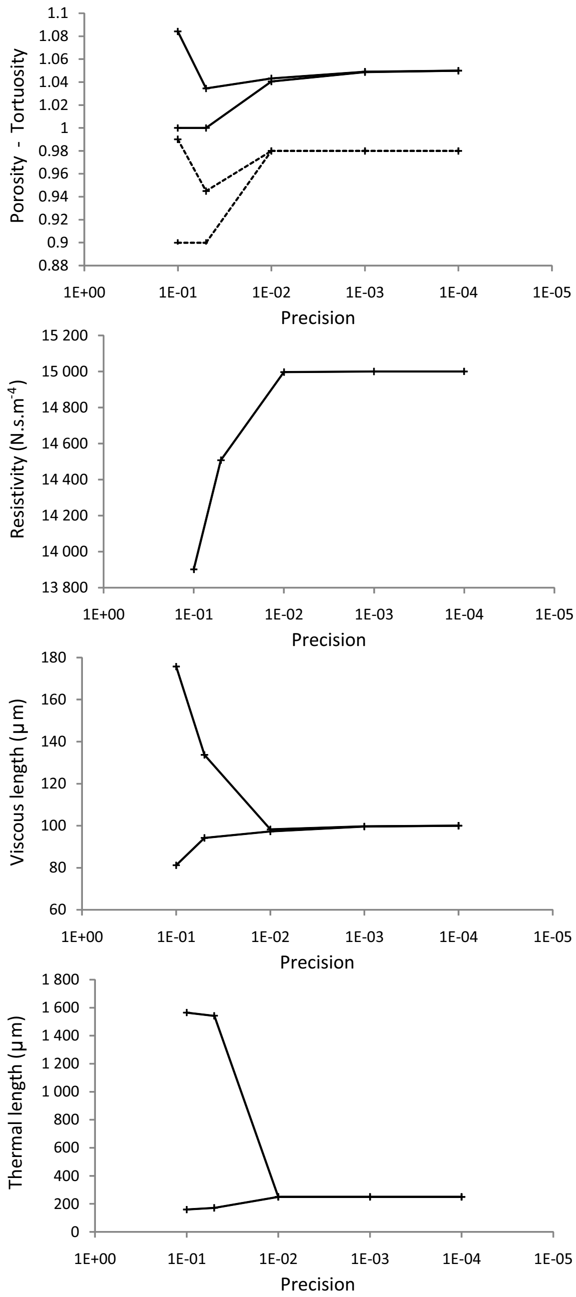

Figure 3: Convergence of parameters as function of given variable intervals: a) tortuosity and porosity (dashed line); b) resistivity; c) viscous length; d) thermal length.

The five parameters of the Johnson-Champoux-Allard model (porosiy, resistivity, tortuosity, viscous and thermal characteristic lengths) may be determined from the density and bulk modulus of the equivalent fluid known at two frequencies (50 Hz and 1000 Hz). The calculation time for the best precision is of the order of one second on a standard personnal computer. This makes the CSP approach a good candidate as an inverse method to recover the five JCA parameters from experimental data. Indeed, to manage the distance between the model and real-life data, another level of strategy must added. As an example, the consistency loop may include a frequency scan within the frequency range of the experimental data. Moreover, a global distance parameter may be introduced as a variable in the CSP algorithm. This procedure will be adressed in future works.

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