Evaluation of an adjoint-based liner impedance eduction technique

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This paper describes a new method for acoustic liner impedance eduction. The eduction process aims at finding the liner impedance which minimizes the error between numerical simulations and a set of measured data. The method relies on the resolution of the two-dimensional Linearized Euler Equations in the frequency domain, spatially discretized by a Discontinuous Galerkin scheme. The minimization of the objective function is achieved by the resolution, at each iteration, of the direct and adjoint equations. This leads to an analytical expression of the objective function derivatives. The process is benchmarked on pressure measurements found in the literature, in a no-flow configuration. Results compare very favorably to the reference curves, except at the anti-resonance frequency of the material where a small discrepancy occurs. A numerical investigation is also made to show the possibility to educe liner impedance from acoustic velocity measurements.

1 Introduction

To reduce the noise generated by the turbofans, aircraft manufacturers usually mount acoustic treatments in the nacelle. These liners are typically made of honeycomb cells topped by a perforated sheet, working on the principle of Helmholtz resonators. Optimization of the resulting passive noise reduction requires an accurate knowledge of the acoustic liner properties in the conditions of use, which include the presence of a grazing flow. These properties are represented by the acoustic impedance, an homogenized quantity defined locally as the ratio between the acoustic pressure at the surface of the material over the normal acoustic velocity. Various methods exist for the measurement of liner acoustic impedance with grazing flow. Dean [1] introduced an in-situ technique relying on two microphone measurements respectively at the rear wall of the liner cavity and at the surface of the perforated sheet, associated to a propagation model inside the cavity. At ONERA, the acoustic impedance is deduced from Laser Doppler Velocimetry (LDV) measurements above the liner. To access the acoustic pressure at the surface of the perforated sheet, these velocity measurements are combined to either a Galbrun propagation model [2] or to acoustic pressure measurements [13].

Over the past few years, the limitations of existing techniques motivated the development of inverse methods for liner impedance eduction. The principle is to use a numerical or analytical propagation model to find the liner impedance which, in the best possible way, reproduces acoustic measurements made in a duct test rig. To achieve this, an objective function representative of the distance between measured and simulated quantities is minimized. NASA has brought valuable contribution with a robust method based on acoustic pressure measurements on the rigid wall opposite the test liner [3] and a finite element model. Indirect processes based on a 3-zones duct model (upstream, above and downstream of the liner) with a mode matching approach are also used [4, 5]. However these methods can suffer from the lack of information nearby the lining sample.

A new eduction method is currently being developed at ONERA. It is planned to be based on velocity measurements acquired by LDV above the liner. The propagation model relies on the two-dimensional Linearized Euler Equations (LEE) in the frequency domain to account for a sheared flow profile in further developments. The gradient of the objective function is calculated thanks to the adjoint system of equations. This offers the advantage to provide an analytical expression for the derivatives, while the finite difference method generally used introduces approximation errors and has a computational cost which increases with the number of search parameters. Both direct and adjoint system are spatially discretized by a Discontinuous Galerkin (DG) scheme, which particularly suits the considered problem in the way to express the boundary conditions.

In section 2, the governing equations and the discretization scheme are presented. The principle of the impedance eduction process is then exposed in section 3. Benchmarking is done in section 4 with data available in the literature, before introducing the velocity-based eduction process and giving some concluding remarks.

2 Governing equations and numerical method

2.1 Propagation model

The propagation model relies on the two-dimensional LEE. A steady incompressible mean flow, with velocity \( U_0 \) along the streamwise coordinate \( x_1 \) and \( V_0 \) along the transversal coordinate \( x_2 \), is subject to an acoustic perturbation \( \varphi = (u, v, p)^T \). The duct configuration typically considered is shown on figure 1.

The governing equations are written in a dimensionless matrix form, for an \( e^{i \omega t} \) time dependence:

\[
j \omega \varphi + A^1 \partial_i \varphi + B \varphi = 0 \quad (1)
\]

with

\[
A^1 = \begin{pmatrix}
U_0 & 0 & 1 \\
0 & U_0 & 0 \\
1 & 0 & U_0
\end{pmatrix}, \quad A^2 = \begin{pmatrix}
V_0 & 0 & 0 \\
0 & V_0 & 1 \\
0 & 1 & V_0
\end{pmatrix} \quad (2)
\]

and

\[
B = \begin{pmatrix}
\partial_1 U_0 & \partial_2 U_0 & 0 \\
\partial_1 V_0 & -\partial_1 U_0 & 0 \\
0 & 0 & 0
\end{pmatrix} \quad (3)
\]

The symbol \( \partial_i \) (\( i \in \{1, 2\} \)) represents the spatial derivative with respect to the coordinate \( x_i \). Velocities are nondimensionalized by \( c_0 \), pressures by \( \rho_0 c_0^2 \), lengths by the duct height \( H \) and time by \( H/c_0 \). The speed of sound \( c_0 \) and the density \( \rho_0 \) of the mean flow are considered uniform.

A Discontinuous Galerkin formulation is used to solve these equations numerically. The domain is partitioned into

![Figure 1: Generic geometry of the duct](image-url)
The use of a $\beta$-formulation instead of a $z$-formulation avoids singularities to arise in the case of rigid walls. Note that contrary to most of similar methods, the Ingard-Myers [6] condition is not required as the process is intended to be used with no flow or with sheared flow profiles.

The final formulation is easily obtained by expressing $\phi_h^*$ and $\psi_h^*$ as a linear combination of the basis functions and by summing over all the elements.

### 2.2 Adjoint system of equation

The eduction process requires the minimization of an objective function $\mathcal{J}$ describing the distance over an observation region between measured and calculated data. A general form of this objective function is:

$$
\mathcal{J} = \int_\Omega \| \varphi_{calc} - \varphi_{meas} \|^2 I_{\Omega_{obs}}(x) \tag{11}
$$

where $I_{\Omega_{obs}}(x)$ is the indicator function of the observation region $\Omega_{obs}$ such as

$$
I_{\Omega_{obs}}(x) = \begin{cases} 
1 & \text{if } x \in \Omega_{obs} \\
0 & \text{if } x \notin \Omega_{obs} 
\end{cases} \tag{12}
$$

At each iteration of the eduction process, the gradient of $\mathcal{J}$ is needed to find a new set of parameters which should lead to a smaller objective function. The gradient is often calculated by means of a finite difference method, but this leads to an approximate result and a calculation cost increasing with the number of parameters to be educated. Use of the adjoint equations allows on the other hand to obtain an analytical expression for the derivatives of $\mathcal{J}$.

If the linear impedance $z_i$ is obviously a search parameter to be considered, a choice has to be made regarding which other quantities should be included in the eduction process as parameters. Eversman & Gallman [7] propose an eduction method where termination impedance and effective Mach number are parameters of the search process. The method presented here includes termination impedance $z_i$ as well as a calibration coefficient $C \in \mathbb{C}$ to adjust the amplitude and phase of the source to the measurements.

The adjoint system of equations may be determined by the method of Lagrange multipliers, considering the direct equations (1) and the associated boundary conditions as constraints. The adjoint state is defined as the Lagrange multiplier $\varphi^*$ satisfying the following system, called “adjoint system”:

$$
\begin{align*}
-ju\omega \varphi^* - \partial_t \left( \mathbf{A}^\dagger \varphi^* \right) + \mathbf{B}^\dagger \varphi^* &= \partial \varphi \mathcal{J} \quad \text{on } \Omega \\
M_p^* \varphi^* &= 0 \quad \text{on } \Gamma_f \\
\mathbf{A}^\dagger n_i^* \varphi^* &= 0 \quad \text{on } \Gamma_i \cup \Gamma_s
\end{align*} \tag{13}
$$

The symbol $\dagger$ denotes the Hermitian conjugate. The adjoint impedance matrix is given by $M_p^* = \mathbf{A}^\dagger n_i + \mathbf{M}_p^\dagger$. Further details on the determination of the adjoint system can be found for example in [8] in the case of an optimal control problem.

The directional derivatives of the objective function can then be expressed as a function of the direct and adjoint states:

$$
\frac{\partial \mathcal{J}(\varphi, z, C)}{\partial z_i} = -\left( \frac{\partial M_p^*}{\partial \beta} \frac{\partial \beta}{\partial z_i} \varphi^* \right)_{\Gamma_f} \tag{14}
$$
3 Impedance eduction process

At each iteration on the search parameters, the direct and adjoint equations are solved. As the derivative of \( J \) with respect to \( \varphi \) appears in the right-hand side of the adjoint equation (13), the direct system must be considered before the adjoint system. Once the adjoint equations are solved, at a computational cost comparable to the resolution of the direct equations, the gradient of \( J \) is calculated with equations (14) and (15). A new set of search parameters is then defined by a BFGS algorithm [9] to reduce \( J \) until one of the stopping criteria is met.

The global eduction loop is synthesized on figure 3. Note that an interpolation of the raw test data is necessary since generally the nodes of the numerical mesh do not match the location of experimental points.

4 Benchmark with pressure measurements

The propagation model presented above has already been validated on academic test-cases and compared with experimental results in a previous paper [10]. Benchmarking of the eduction process is realized here on data published by the NASA Langley Research Center [11]. In this reference paper, the authors educe the acoustic impedance thanks to a finite element model of the convected Helmholtz equation and acoustic pressure measurements at the rigid wall opposite the lining sample.

Using these acoustic pressure measurements, in the case without flow, our eduction process has been applied with the following objective function:

\[
\mathcal{J}(\varphi, z, C) = \sum_{m=1}^{N} \| p_{m}^{calc} - p_{m}^{meas} \|^2 \tag{16}
\]

where \( N=31 \) is the total number of microphones.

Two analyses have been conducted: the “NRp” case uses a non-reflective boundary condition (equivalent to \( z_t = 1.00 + 0.00 j \)) at the termination plane, whereas in the “Elp” case the exit impedance is included as a search parameter. The configuration of the duct is represented on figure 4.

5 Numerical investigation on a velocity-based objective function

In further developments, the eduction method is planned to be used with acoustic velocity measurements obtained by LDV. Before testing the process with real measurements, the feasibility of impedance eduction relying on an objective
A new method for liner impedance eduction has been presented, with the specificity of involving the adjoint system.
to calculate the objective function derivatives. This allows to obtain an analytical expression for the gradient, and to include easily a number of parameters other than the liner impedance, at a computational cost which remains unchanged.

The method has been tested and validated against NASA results available in the literature. Only few results are presented in this paper for the sake of brevity, but they show very good agreement with the reference plots. The possible use of an objective function expressed in terms of acoustic velocity has been checked. It will allow the development of an eduction process based on LDV measurements, which give access to information very close to the liner.

A number of studies are currently being carried out, including a sensitivity study of the impedance educed values to the measurement noise generated by the instrumental system and the signal processing. LDV measurements with and without flow on microperforated liners have been performed and are under consideration at the moment. Finally, it could be interesting to consider the optimal control problem associated to our configuration to obtain an adjoint field giving indication on the best possible observation region.

References


