Active elastic metamaterials with applications in acoustics

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Elastic metamaterials provide a new approach to solving existing problems in acoustics. They have also been associated with novel concepts such as acoustic invisibility and subwavelength imaging. To be applied to many of the proposed applications a metamaterial would need to have the desired mass density and elastic moduli over a wide frequency band. To minimise scatter in acoustics applications the impedance of solid elastic metamaterials also need to be matched to the impedance of the surrounding medium. Previous work has looked at the trade-off between achieving the desired mass density and Youngs modulus, combined with an impedance which is matched to the surrounding medium. This paper will focus on the problems which arise when the previously developed theory and simulation is developed into an experimental demonstration. This includes the role which the control system dynamics play on the achievable performance. Extending the controllability of the bandwidth of the desired properties to an extent where the material could be applied to some of the novel applications will also be considered.

1 Introduction

The concept of a metamaterial was first established in the field of electromagnetics [15, 17]. Since this initial work the concept has broadened to include the field of acoustics and elastodynamics [13, 11, 19, 8]. Important properties of metamaterials include both a subwavelength structure and a capability of providing a material with negative values for its effective constitutive parameters, which in acoustics and elastodynamics is the mass density ρ and elastic moduli κ of the material. If pressure waves are being considered the elastic moduli is the bulk modulus, whereas it is the shear modulus when shear waves are being considered. When the mass density and elastic moduli have opposite signs the material acts to block the propagation of any wave within the material. When both parameters have the same sign wave motion is permitted. If both parameters are positive, the wave motion is as expected in a conventional material. If both parameters are negative then the refractive index of the material becomes negative real. Any wave moving from a material with a positive real, i.e. conventional, refractive index, to one with a negative real refractive index will experience negative refraction, whereby the refracted wave lies on the opposite side of the boundary normal to the conventional case. This is still in agreement with Snell's law, with the refractive index being given by $n = \pm\sqrt{\rho/\kappa}$ and is a concept which was first proposed in electromagnetics [18]. Realizing a material with a negative refractive index only became possible with the advent of metamaterials.

Elastic metamaterial designs have been previously proposed to achieve the objective of either a single negative effective parameter or both parameters simultaneously negative. A negative effective mass density can be realized through dipolar resonances contained within a host or attached to a transmission material. Examples include an array of lead spheres coated with a silicone rubber and embedded in an epoxy host [13], resonant masses connected through a spring element periodically along a transmission medium composed of series mass and spring elements [20] and mass-spring resonators periodically attached to a slender beam [19]. In these materials the negative effective mass density is associated with the fundamental resonant frequency of the embedded or attached elements. Alternatively a negative effective modulus can be realized through monopolar resonances contained within a host or attached to a transmission material. An example is a material composed of an array of split hollow spheres embedded within a sponge matrix [6]. Extensions to this study suggest that a multi-band and potentially broadband response can be achieved by varying the dimensions of the split hollow spheres [7].

Achieving a metamaterial with simultaneously negative mass density and elastic modulus is notably more difficult than achieving one with a single negative parameter. A recent study provides a design for a system with a negative effective mass density and modulus [12]. Locally resonant translational and rotational inertia coupled by an arrangement of springs provide a narrow frequency band in which both a negative effective modulus due to a monopole resonance associated with the rotational inertia and a negative effective mass due to a dipolar resonance associated with the translational inertia occur simultaneously. An important extension of this study is that the initial one-dimensional concept is expanded to two-dimensions and the response demonstrated in simulation. Another study also considers a two-dimensional approach in which a lumped parameter system is realized through a composite elastic structure which provides monopole, dipole and quadrupole resonances leading to dispersive properties for the effective bulk modulus, density and shear modulus respectively [10].

All of these passive solid elastic metamaterial designs rely on dynamic phenomena, usually in the form of spatially periodic resonant structures to realize the negative effective parameters. This leads to a fundamental problem, which is that the resulting effective mass density and elastic moduli are both inherently dispersive in nature and only negative over a limited frequency band. While some studies have shown that in solid elastic materials it is possible to create a wide frequency band in which one of the effective parameters is negative, the simultaneously double negative frequency band is restricted by the narrow band of the other negative effective parameter [12, 10]. It may be possible to extend these approaches such that the double negative frequency band broadens [7], but due to the dispersive nature of the effective parameters the resulting negative effective refractive index is dispersive. This may not be a problem for an application with a fixed narrow band response as the dispersion maybe limited over this band, but may limit the application of these designs to some applications, including the novel applications which have been proposed for metamaterials, such as invisibility cloaks [14] and subwavelength resolution lenses [9].

An alternative is a material in which the narrow double negative band can be adapted. This could be achieved through an active elastic metamaterial design, whereby a force applied to an array of single resonant units provides a system which emulates the monopole and dipole behavior required by an effective system to provide negative effective values for the bulk modulus and density [16]. The advantage of such an arrangement is that the control system can be tuned so that the double negative frequency band and transmission properties can be designed for a particular application, or potentially adapted online. Active acoustic metamaterials have also been investigated [1, 2]. In these metamaterials the transmission medium is a fluid as opposed to a solid elastic medium and have thus far been designed to control a single effective parameter with a positive value only.

The one-dimensional active elastic metamaterial previously proposed was purely theoretical and ignored important elements which need to be considered if the design is to be verified experimentally [16]. It assumed that the displacement and velocity of the lumped elements in the transmission material and of the resonators could be measured perfectly and that these measurements could be fed back and a force applied to the resonant masses which is purely proportional to the motion of the measured elements. In reality this could not be achieved experimentally as any sensor used to measure the motion and any actuator used to apply a force will inherently posses a dynamic response. In addition there will be a delay introduced into the feedback loop due to the sample time of the control system. The concept for an elastic metamaterial presented here remains broadly the same as that previously proposed. However, in this paper the dynamics of the actuator are introduced into the model as it is well known that introduction of the actuator dynamics in control systems can lead to issues with both stability and achieving the desired closed loop response. Following introduction of the actuator dynamics, the stability of the system, together with its ability to provide the desired effective mass and stiffness response with simultaneously negative bands is investigated.

2 Metamaterial Design

Figure (1) displays the metamaterial design which was previously presented, with the addition of the actuator dynamics represented by the Laplace domain transfer function $g_a(s, x_{r_n})$. The gray lines indicate the measurement/actuation (input/output) paths for the control system and g_c is the controller, which for the previous design for a double negative metamaterial was $(sc_c + k_c)(x_{n-1} + x_{n+1} - 2x_{r_n})$. When the actuator dynamics are ignored, i.e. $g_a = 1$, the effective mass m_e and combined effective damping and stiffness $b_e = c_e s + k_e$ are given by Eq. (1) and Eq. (2) respectively. This assumes the Kelvin-Voigt model of viscoelasticity such that the effective equation of motion for mass n is given by Eq. (3).

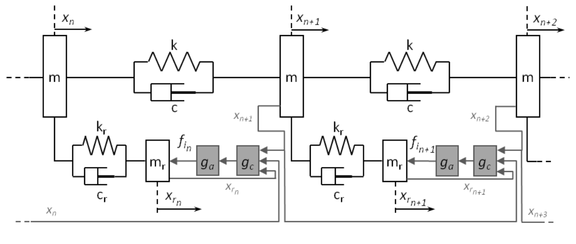

Figure 1: Design for a locally resonant metamaterial which consists of an array of transmission masses m, resonant masses m_r and feedback control system designated by the gray lines.

$$m_e = m + \frac{m_r\,(c_r s + k_r)}{m_r s^2 + (2c_c + c_r)\,s + 2k_c + k_r} \tag{1}$$

$$b_e = cs + k + \frac{(c_c s + k_c)\,(c_r s + k_r)}{m_r s^2 + (2c_c + c_r)\,s + 2k_c + k_r} \tag{2}$$

$$m_e x_n s^2 = (c_e s + k_e)\,(x_{n-1} + x_{n+1} - 2x_n) + f_n \tag{3}$$

The previously proposed control scheme requires an inertial force to be applied to the resonant masses in the metamaterial. In active vibration control this force can be provided by an inertial actuator, whereby a force generating mechanism is attached at one end to the structure to which the force is to be applied and at the other end to a mass which is not connected at any other point. As such the actuator can be modeled by a mass m_a connected to the supporting structure by a force generating element which provides force f_a and a spring k_a and damping element c_a in parallel [3, 5]. The force f_i provided by this type of actuator to the support structure is given by Eq. (4), where x_r is the displacement of the supporting structure, which for the proposed arrangement is the displacement of the resonant masses of the metamaterial structure.

$$\begin{aligned} f_i &= -\frac{m_a s^2\,(c_a s + k_a)}{m_a s^2 + c_a s + k_a} x_r - \frac{m_a s^2}{m_a s^2 + c_a s + k_a} f_a \\ &= g_x x_r + g_f f_a \end{aligned} \tag{4}$$

Thus the mechanical structure leads to an applied force which is a function of both the motion of the attached structure and of the force provided by the active element. The force provided by the active element depends on the generating mechanism. Two common mechanisms are electromagnetic actuation [5] and piezoelectric actuation [4]. Electromagnetic actuation is able to provide a significant force down to very low frequency, whereas piezoelectric actuation, due to the high inherent stiffness of the piezoelectric element can only provide small forces at low frequency. Since a metamaterial provides an effective response with a subwavelength structure, the focus is on controlling its response at low frequency, leading to the electromagnetic actuator providing the most suitable frequency response for this application. The force f_a generated by the electromagnetic element will be a function of the voltage applied to the element, which is typically a second order response due to electrical characteristics described by an LCR (Inductor-Capacitor-Resistor) circuit [5]. However, initially here we will assume that the electrical characteristics can be described by the transfer function g_v such that $f_a = g_v v_a$ where v_a is the applied voltage.

Introducing the mechanical model of the actuator into the equations of motion of the metamaterial elements leads to the effective mass m_e and complex stiffness b_e terms given in Eq. (5), Eq. (6) and (7). These differ from the previous ideal parameters in Eq. (1) and Eq. (2) in terms of the dynamics of the actuator. If at this stage the electrical characteristics of the actuator are ignored and it is assumed that the force f_a is linearly proportional to the applied voltage v_a, the focus is on the effect of the mechanical characteristics of the actuator. This simplification is reasonable when the force response of a typical actuator is measured. Figure 2 displays the measured force response (red) compared to the corresponding mechanical model (blue) for a commercial electromagnetic actuator (Data Physics Corporation SignalForce IV40) when connected to a fixed plate such that x_r in Eq. (4) is zero. It is clear that the response is dominated by the mechanical resonance of the actuator.

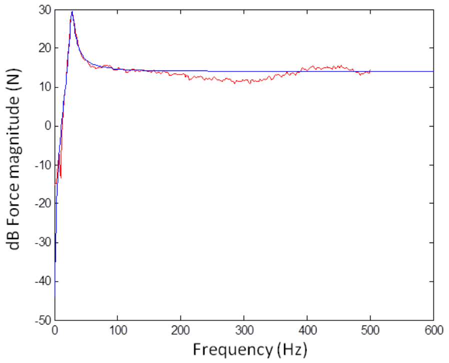

Figure 2: Force magnitude - frequency response of a typical commercial electromagnetic actuator. The red line is the measured force when the actuator is attached to a fixed plate and the blue line is the corresponding response of the model.

$$M_e = m + \frac{(c_r s + k_r)\left(m_r\left(m_a s^2 + c_a s + k_a\right) + m_a\left(c_a s + k_a\right)\right)}{d} \tag{5}$$

$$B_e = cs + k + \frac{m_a s^2 (c_r s + k_r) g_c}{d} \tag{6}$$

$$d = \left(m_r s^2 + c_r s + k_r\right)\left(m_a s^2 + c_a s + k_a\right) + m_a s^2 (2g_c + c_a s + k_a) \tag{7}$$

The desired terms required for analysis of the effective parameters are the real parts of Eq. (5), (6) and (7). Noting that the denominator of the transfer function on the right hand side of both terms is the same, the real parts of both parameters can be written as Eq. (8) and (9), in which d represents the denominator, n_m and n_b the numerators of the transfer function on the right hand side of the effective mass and stiffness respectively and * the complex conjugate in the frequency domain realized by the substitution $s = j\omega$.

$$Re\left(m_e\right) = m + \frac{n_m d\left(g_c\right)^*}{d\left(g_c\right) d\left(g_c\right)^*} \tag{8}$$

$$Re\left(b_e\right) = k + \frac{n_b\left(g_c\right) d\left(g_c\right)^*}{d\left(g_c\right) d\left(g_c\right)^*} \tag{9}$$

As the denominator of the term on the right hand side in both equations for the real part of the effective parameters is the complex denominator multiplied by the complex conjugate of the denominator of the original terms, it is positive semi-definite. As a result the objective of providing negative effective mass and stiffness requires the numerator of both terms to be negative over a specified frequency bandwidth. Assuming positive values for all of the parameters leads to alternating sign for increasing powers of ω^2 where ω is the frequency. To ensure a large negative value for the terms on the right hand side, the negative frequency band of the numerator must overlap with one of the resonant frequencies provided by the denominator.

Due to the increased complexity that the actuator dynamics have on the equations describing the effective mass and stiffness, it is interesting to consider what the simplest control scheme is which allows both the effective mass and stiffness to become negative. Subsequently it can be determined whether this control scheme can be extended to provide enhanced performance. Keeping the variables which are fed back the same allows the structure required for the effective mass and stiffness to be realized in such a way the denominator is the same for both. This is important as the roots of this denominator determine when the effective parameters are sufficiently large. The simplest controller which can be used in conjunction with this is direct displacement feedback, i.e. $c_c = 0$ in the previously proposed scheme which leads to $k_c(x_{n-1} + x_{n+1} - 2x_{r_n})$. Implementing this control structure significantly simplifies the expressions for the effective mass and stiffness when the actuator dynamics are introduced. The addition of the additional control parameter $c_c s$ may provide additional performance benefits, but this is the subject of further work.

A necessary, but not sufficient condition for the effective mass and stiffness to be negative can be derived by consideration of the region in which the numerators of the transfer functions in Eq. (8) and Eq. (9) are negative. This leads to two inequalities which are the conditions for the effective mass and stiffness respectively and are given in Eq. (10) and (12). In Eq. (10) for brevity the coefficients are represented by variables due to their length. All four of the coefficients are positive definite as they only contain positive summations with only e_4 and e_2 a function of the control parameter. As a result the condition E_1 for the effective mass will be negative in two regions: the first is a low frequency band and the second a high frequency band which stretches to infinite frequency. The low frequency region is dependent on the coefficient e_2 being sufficiently larger than e_0 and e_4. Whereas the condition E_2 for the effective stiffness will also be negative in two frequency bands: the first is a low frequency band which stretches down to zero frequency and the second is a high frequency band which stretches up to infinite frequency. To ensure negative effective parameters these negative regions must overlap with at least one of the resonant frequencies of the denominator in Eq. (8) and (9). To be simultaneously negative they must overlap with the same resonance. Technically they do not necessarily need to overlap with the center point of the resonant frequency as long as the magnitude induced by its effect is sufficient to be greater than the passive mass m and stiffness k.

$$E_1 = -e_6\omega^6 + e_4\left(k_c\right)\omega^4 - e_2\left(k_c\right)\omega^2 + e_0 < 0 \tag{10}$$

$$\begin{aligned} E_2 = \; & - \; \left(m_a m_r k_r - c_r\left(m_a\left(c_a + c_r\right) + m_r c_a\right)\right)\omega^4 \\ & + \; \left(k_r\left(m_a\left(k_a + k_r + 2k_c\right) + m_r k_a\right) - c_r^2 k_a\right)\omega^2 \\ & - \; \left(k_a k_r^2\right) < 0 \end{aligned} \tag{11}$$

As the ability to control both the numerator and denominator is coupled due to the single control coefficient, the sign and magnitude of the effective parameters cannot be controlled independently. Thus the design doesn't guarantee the ability to provide negative effective parameters for a given passive system. A further problem which arises is that the lower frequency negative band of E_1 lies immediately above the lower band for E_2 leading to non-simultaneously negative mass and stiffness in the frequency domain. By extending the control system to $k_c(x_{n-1} + x_{n+1} - 2a_c x_{r_n})$ the extra design freedom a_c allows the lower frequency band of E_1 to be moved to a lower frequency for $a_c > 1$ leading to a simultaneously negative band in the frequency domain. The stability and controllability of the active metamaterial using this extended control system is discussed in the next section.

3 Stability and Controllability

If it is assumed that the active metamaterial employs the previously described control scheme which can provide negative effective parameters and includes actuator dynamics, the controlled system can be represented as a standard multivariable transfer function with feedback. Assuming that $\vec{u} = [\vec{f}, \vec{f_r}]^T$ where $\vec{f}$ and $\vec{f_r}$ are the vectors of forces applied to the transmission and resonant masses respectively and that $\vec{y} = [\vec{x}, \vec{x_r}]^T$ where $\vec{x}$ and $\vec{x_r}$ are the vectors of displacements of the transmission and resonant masses respectively, the control G_c is expressed as in Eq. (12). Where the four sub matrices of K_c are all $n \times n$ matrices (n is the order of the system i.e. the number of transmission masses) with $K_{c_{1,1}}$ and $K_{c_{1,2}}$ both zero matrices, $K_{c_{2,1}}$ a zero matrix except for 1's on the two diagonals either side off the leading diagonal and the two end elements of the leading diagonal and $K_{c_{2,2}}$ is the zero matrix with -2 of the leading diagonal except for -1 on the elements at both ends of the diagonal.

$$\begin{aligned} G_c &= g_x \hat{I} - k_c g_f K_c \\ &= g_x \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix} - k_c g_f \begin{bmatrix} K_{c_{1,1}} & K_{c_{1,2}} \\ K_{c_{2,1}} & a_c K_{c_{2,2}} \end{bmatrix} \end{aligned} \quad (12)$$

The closed loop response of the active metamaterial can then be determined by $\vec{y} = (I - GG_c)^{-1} G\vec{u}$, where G is the open loop response. Analyzing the stability of the proposed metamaterial is not trivial due to its multivariable nature, coupled with the effect that an increasing number of layers has on the order of the response. Internal stability is guaranteed if $(I - GG_c)^{-1}$ is stable and there are no right half plane pole zero cancellations between G and G_c. As G is stable the concern is whether any right half plane zeros in G cancel with right half plane poles in G_c. The poles of the controller in Eq. (refeq12) are essentially the roots of the the denominator of the actuator model g_f given in Eq. (5) and so are guaranteed to be stable, leading to no right half plane pole-zero cancellations. Thus determining the stability of the proposed active metamaterial reduces to analysing the roots of $(I - GG_c)^{-1}$. The remaining problem lies with the increasing complexity associated with adding more transmission masses to the metamaterial.

The stability of this system subjected to a varying feedback gain can be analysed using well known stability analysis tools. Figure (3) shows the movement of the closed loop poles for a fourth order system $n = 4$ (i.e. four transmission masses) subject to the proposed extended control system with $a_c = 0.2$ and passive material parameters $m = 5kg$, $m_r = 2kg$, $c = 0.1Ns/m$, $c_r = 2 \times 10^3 Ns/m$, $k = 5 \times 10^4 N/m$ and $k_r = 8 \times 10^3 N/m$. These are not necessarily realistic parameters for a metamaterial, but are matched to the actuator dynamics displayed in Figure (2) and used from a proof of concept point of view. Nevertheless, these parameters could be combined with realistic dimensions to provide a metamaterial for very low frequency operation. The blue lines show the movement of the poles as the gain increases from zero (indicated by the red crosses). The trajectory of the closed loop poles indicates that the active metamaterial is stable from zero feedback gain up until an upper cut-off $k_{c_{stable}}$ when the complex conjugate pair of poles at the top and bottom of the locus move into the right half plane.

To provide a system which is both stable and has negative effective mass and stiffness, the desired region for the gain

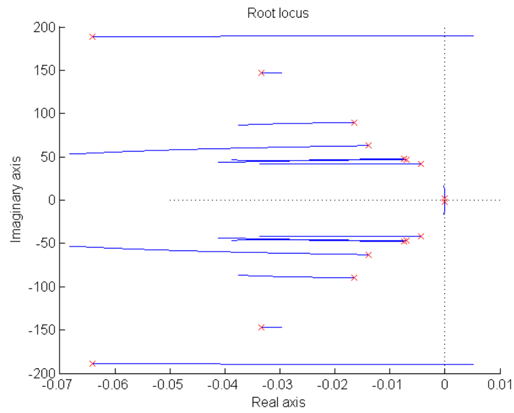

Figure 3: Plot of closed loop poles for increasing feedback gain k_c. The two poles at the top and bottom become unstable above $k_{c_{stable}}$

specified by these two performance objectives must overlap. This can be determined by solving for the range of gains analytically. However it is effective to present the influence of the gain in a graphical form. Figure (4) plots the real part of the potentially unstable pole in Figure (3) against the real part of the effective mass (blue) and stiffness (red) for a gain which varies from zero up to above the upper cut-off for stability. Interest lies in the lower left quadrant as this is where both the effective mass and stiffness will be if they are negative and the system is stable. It is clear that over a range of gains both the effective mass and stiffness can be negative for a stable system.

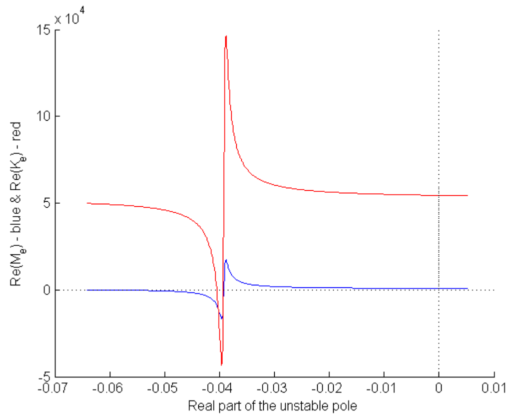

Figure 4: Plot of the real parts of the effective mass (blue) and stiffness (red) against the real part of the potentially unstable poles as the control parameter k_c increases.

The requirement for a negative refractive index is that both the frequency dependent mass and stiffness are simultaneously negative. Figure (5b) plots the real part of the effective mass and stiffness over a frequency range for a fixed and stable k_c. It is evident from this plot that the effective mass and stiffness become negative simultaneously (i.e. the plot enters the lower left quadrant) for this stable system over a range of frequencies. Figure (5a) takes a frequency in which the parameters are simultaneously negative and plot the change in these variables against a changing k_c. This plot indicates that there is also a range in feedback gains for a given frequency in which both parameters become simultaneously negative. Figure (5) thus indicates that the active metamaterial design which incorporates actuator dynamics and the modified control law $k_c(x_{n-1} + x_{n+1} - 2a_c x_{r_n})$ does provide a double negative region for the effective parameters and this region to some extent is tunable.

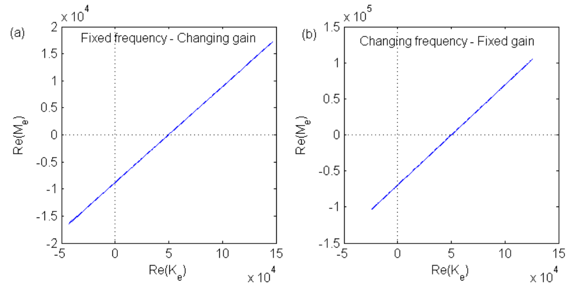

Figure 5: Real parts of the effective mass against stiffness for: (a) A fixed frequency and changing control parameter k_c; (b) A changing frequency and fixed control parameter k_c.

4 Conclusion

A previously proposed theoretical design for a one dimensional active elastic metamaterial has been extended to a design closer to that which will be implemented experimentally through the introduction of actuator dynamics. The analysis has shown that this design is capable of providing a negative effective mass and stiffness, but the actuator dynamics provide limitations to achieving the desired response, notably restrictions on achieving simultaneously negative mass and stiffness in a frequency band which was relatively easily achieved in the ideal design. By making a simple extension to the proposed control scheme it has been shown that the added control variable can help decouple the response of the effective mass and stiffness, leading to a double negative region being achieved. Subsequently it has been shown that the proposed active metamaterial is stable for a region of control parameters and in this stable region the double negative band exists and is controllable to some degree. Further extensions to this work will be to conduct further in-depth analysis of the stability and controllability, introduce sensor dynamics, assess the effect of model uncertainties and to extend the design to a full experimental realization.

Acknowledgments

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