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Ultrasonic modeling of a viscoelastic homogeneous plate

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An ultrasonic method for determination of acoustical and geometrical properties of a viscoelastic plate immersed in a known fluid (water) is presented. The method utilizes plane longitudinal waves that are normally incident upon a homogeneous plate with attenuation obeying a frequency power law and dispersion described by Szabo's model. It requires only the knowledge of the fluid properties and recording of two chirps: one without and one with the specimen inserted between the transmitting and receiving transducers. The transmission coefficient of the plate is measured and compared to the predicted one, in which the diffraction of waves emitted by the transducer is taken into account. An inverse scheme based on a nonlinear least squares algorithm allows then simultaneous determination of the plate thickness, density, attenuation (longitudinal) and phase velocity at a given frequency. The technique involves an estimation of standard errors of parameter estimates. An example is given and discussed for a Plexiglas plate. It is shown that thickness and velocity are estimated with a good accuracy (10^{-3}). Density is estimated also with a reasonable accuracy (10^{-2}), whereas estimated attenuation is less precise.

1 Introduction

The use of ultrasonic transmission coefficient in elastic constant determination of materials is well known [1] and valid for dispersive as well as nondispersive media. A detailed analysis of application of the through-transmission method for measurements of thin plate properties was given by Kinra [2], in which one of the four properties (longitudinal wave velocity, attenuation, thickness or density) of an immersed thin plate, was determined assuming the three others are known. In this paper, we propose a new method [3] for simultaneous determination of all of these properties using ultrasonic through-transmission immersion measurements at normal incidence. We assume the water properties are known, and the plate is viscoelastic and homogeneous.

Unlike the traditional method that utilizes only the amplitude information, our new method uses both the real and imaginary parts of the transmission coefficient of the plate and requires no phase-unwrapping. According to this method, parameters to be estimated are obtained by performing a least squares fit between the theoretical and experimental data points.

The paper is organized as follows. In Sec. 2, the theoretical transmission coefficient is first reviewed for the plane wave propagation, with and without diffraction correction. The models for attenuation and dispersion are also presented. The experimental procedure is described in Sec. 3 and a result for a specimen of Plexiglas is given using three different central frequencies of transducers. In the fourth section, an algorithm based upon an inverse search in a six-dimensional space is elaborated for the simultaneous determination of the model parameters and. A good agreement between theory and experiment is demonstrated. The sensitivity of the algorithm to the plate parameters is studied. Finally, conclusions are given in Sec. 5.

2 Theoretical background

2.1 Transmission through a viscoelastic plate at normal incidence

It is proposed to determine thickness, density, phase velocity and attenuation of a viscoelastic plate using only transmission measurement at normal incidence. To illustrate this let us consider a parallel face plate immersed in water. The harmonic plane wave transmission coefficient for this plate, can be expressed, in complex form, by

$$T = \frac{2e^{\gamma_w(f)E}}{\left[\frac{Z_w(f)}{Z_p(f)} + \frac{Z_p(f)}{Z_w(f)}\right] \sinh\left[\gamma_p(f)E\right] + 2\cosh\left[\gamma_p(f)E\right]} \tag{1}$$

where γ_w, γ_p are the complex propagation constants for the longitudinal plane wave, in the water and the plate, respectively, f is the frequency and E is the thickness of the plate. Here $Z_w = \frac{i\rho_w 2\pi f}{\gamma_w}$ and $Z_p = \frac{i\rho_p 2\pi f}{\gamma_p}$ are the complex acoustic impedances of water and the plate, where ρ_w and ρ_p denote the densities of the two media.

2.2 The models for attenuation and dispersion

Inside the plate, the complex propagation constant can be written as

$$\gamma_p(f) = \alpha_p(f) + i\frac{2\pi f}{v_p(f)} \tag{2}$$

where $v_p(f)$ and $\alpha_p(f)$ are the phase velocity and attenuation. For a wide variety of materials, frequency-dependent attenuation is typically modeled by a power-law relation involving two constants α_{0p} and n.

$$\alpha_p(f) = \alpha_{0p} f^n \tag{3}$$

where the power-law exponent n ranges from 0 to 2 for most liquids and solids. To model the dispersion, a time causal model developed by Szabo [4] is adapted. Szabo's model is a time-domain expression of causality analogous to the Kramers-Kronig relations in the frequency domain that link together the attenuation and dispersion. Using this model, we have for $n \neq 1$:

$$\frac{1}{v_p(f_0)} - \frac{1}{v_p(f)} = -\frac{\alpha_{0p}}{2\pi}\tan\left(\frac{n\pi}{2}\right)\left(f^{n-1} - f_0^{n-1}\right) \tag{4}$$

where f_0 is a reference frequency at which a reference phase velocity $V_{0p} = v_p(f_0)$ is defined. For the special cases of $n = 0$ (frequency-independent media) and $n = 2$ (viscous media), the model predicts no dispersion. Based on Szabo's model and on the attenuation power law, the measurement of the acoustic attenuation and velocity in the viscoelastic material reduces to the estimation of the three parameters: V_{0p}, α_{0p} and n. Attenuation in water is taken to be a quadratic power of the frequency: $\alpha_w(f) = \alpha_{0w} f^2$. In this case, the dispersion vanishes according to the model of Szabo: $v_w(f) = V_w$.

Using Eqs. (3) and (4), the three terms involved in the transmission coefficient (see Eq. (1)) can be expressed by

$$Z_{pw}(f) = \frac{Z_p(f)}{Z_w(f)} = \frac{\rho_p V_{0p}}{\rho_w V_w} \times \frac{\alpha_{0w} V_w f + i2\pi}{\alpha_{0p} V_{0p} \xi(f) + i2\pi} \quad (5)$$

$$\gamma_p(f)E = \frac{E}{V_{0p}} f \{\alpha_{0p} V_{0p} \xi(f) + i2\pi\} \quad (6)$$

$$\gamma_w(f)E = \frac{E}{V_w} f (\alpha_{0w} V_w + i2\pi) \quad (7)$$

where

$$\xi(f) = f^{n-1} - i\left(f_0^{n-1} - f^{n-1}\right)\tan\left(\frac{n\pi}{2}\right) \quad (8)$$

One can see that Eqs. (5) to (8) and therefore the transmission coefficient T depend on five plate properties: velocity V_{0p}, attenuation coefficient α_{0p}, attenuation exponent n, thickness E, and density ρ_p, and on three properties of water: velocity V_w, attenuation coefficient α_{0w} and density ρ_w. Eqs. (5) to (8) can be rewritten so that they depend only on six parameters:

$$Z_{pw}(f) = p_4 \times \frac{p_6 f + i2\pi}{p_3 \xi(f) + i2\pi} \quad (9)$$

$$\gamma_p(f)E = p_2 f \{p_3 \xi(f) + i2\pi\} \quad (10)$$

$$\gamma(f)E = p_1 f (p_6 + i2\pi) \quad (11)$$

$$\xi(f) = f^{p_5} - i\left(f_0^{p_5} - f^{p_5}\right)\tan\left(\frac{(p_5+1)\pi}{2}\right) \quad (12)$$

where the parameters p are given by

$$\begin{aligned} &p_1 = \tfrac{E}{V_w},\ p_2 = \tfrac{E}{V_{0p}},\ p_3 = \alpha_{0p} V_{0p}, \\ &p_4 = \tfrac{\rho_p V_{0p}}{\rho_w V_w},\ p_5 = n-1,\ p_6 = \alpha_{0w} V_w \end{aligned} \quad (13)$$

2.3 Transmission coefficient with diffraction

Investigations of the effect of diffraction on wave propagation have been made by a number of authors in the case of two identical circular transducers of radius a. The transducer is usually treated as a finite piston source. The acoustic field is found at each point in the propagation medium and an integration is performed over the receiving transducer surface. For the propagation in a fluid medium, the diffraction correction expression can be defined as the sum $\langle\phi\rangle$ (with diffraction) divided by $\langle\phi_0\rangle$ (plane wave)

$$D(s) = \frac{\langle\phi\rangle}{\langle\phi_0\rangle} = 1 - e^{-i\frac{2\pi}{s}}\left[J_0\left(\frac{2\pi}{s}\right) + iJ_1\left(\frac{2\pi}{s}\right)\right] \quad (14)$$

where $s = \frac{2\pi z}{ka^2}$ is the Fresnel parameter, k is the wavenumber and z is the propagation distance. Here J_0 and J_1 are the zeroth and first order Bessel functions of the first kind, respectively.

The objective here is to extend this analytical expression to the multiple transmission through a viscoelastic plate [6]. A schematic of the problem is shown in Figure 1. Two transducers separated by a distance L are placed in a water tank and aligned properly. If we use

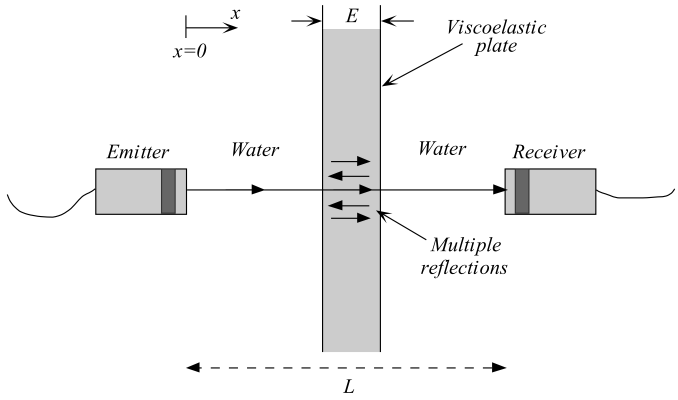


Figure 1: Propagation at normal incidence.

$s_0(t)$, $s_w(t)$ and $s_p(t)$ to designate the transmitted signal, the received signal with the water path only, and the received signal with the plate inserted between the two transducers, respectively, the Fourier transforms of $s_w(t)$ and $s_p(t)$ can be found as following:

$$S_w(f) = S_0(f)\, D(s_w)\, e^{-L\gamma_w(f)} S_r(f) \quad (15)$$

and

$$\begin{aligned} S_p(f) &= S_0(f)\, T_{wp} T_{pw} e^{-(L-E)\gamma_w(f)} S_r(f) \\ &\times \sum_{m=0}^{\infty} D(s_{2m+1})\, R_{ww}^{2m} e^{-E(2m+1)\gamma_p(f)} \end{aligned} \quad (16)$$

where $S_0(f)$ is the Fourier transform of $s_0(t)$ and $S_r(f)$ is the frequency response of the receiving transducer. $T_{wp} = \frac{2Z_w}{Z_w+Z_p}$ and $T_{pw} = \frac{2Z_p}{Z_w+Z_p}$ are the transmission coefficients at the two water-plate interfaces. In Eq. (16), the reflection coefficient $R_{wp} = \frac{Z_w-Z_p}{Z_w+Z_p}$, at the water-plate interface, is used since multiple transmissions are taken into account.

The diffraction corrections $D(s_w)$ and $D(s_{2m+1})$ are calculated according to Eq. (14) and using the appropriate values of the Fresnel parameter s, as follows

$$s_w = \frac{v_w(f)L}{fa^2} \quad (17)$$

$$s_{2m+1} = \frac{1}{fa^2}\left[(L-E)\, v_w(f) + (2m+1)\, E v_p(f)\right]$$
$$\text{for } m \in \{0,1,2,\infty\} \quad (18)$$

From Eqs. (15) and (16), the transmission coefficient with diffraction is given by:

$$\begin{aligned} T &= \frac{4Z_{pw}(f)}{[Z_{pw}(\omega)+1]^2} e^{\gamma_w(f)E} e^{-\gamma_p(f)E} \\ &\times \sum_{m=0}^{\infty} \frac{D(s_{2m+1})}{D(s_w)} \left(\left[\frac{Z_{pw}(\omega)-1}{Z_{pw}(\omega)+1}\right]^2 e^{-2\gamma_p(f)E}\right)^m \end{aligned} \quad (19)$$

As shown in the previous section, the transmission coefficient without diffraction correction depends on six dependent variables given by the set (13). It is convenient to use the same parameters also to describe the transmission coefficient with diffraction corrections. In addition, we define two more parameters

$$q_1 = \frac{a^2}{(L-E)V_w},\ q_2 = \frac{E}{L-E} \quad (20)$$

so that Eqs. (17) and (18) can be rewritten as:

$$s_w = \frac{1+q_2}{fq_1} \quad (21)$$

$$s_{2m+1} = \frac{1}{f q_1}\left[1 + \frac{2\pi q_2 p_1 (2m+1)}{p_2\left[2\pi - p_3\left(f_0^{p_5} - f^{p_5}\right)\tan\left(\frac{(p_5+1)\pi}{2}\right)\right]}\right] \quad (22)$$

$$\text{for } m \in \{0, 1, 2, \infty)$$

Note that q_1 and q_2 are specially related to the transducers separation distance L and radius a.

3 Experiments

Figure. 2 exhibits a schematic of the experimental set-up used for the trough-transmission measurements. Three pairs of transducers are used, a set of 1 MHz transducers (Panametrics V152), a set of 2.25 MHz transducers (Panametrics V389) transducers, all having a radius a = 19.05 mm, and a set of 5 MHz transducers (Panametrics V307) with a radius of 12.7 mm. The pair of transducers and the plate are immersed in a tank containing running water. The distance between each transducer and the plate is fixed at 150 mm. The two transducers are coaxially aligned to obtain maximum transmission. The transmitting transducer is excited with a 150 μs, hanning windowed chirp produced with an arbitrary function generator (Agilent 33220A).

Two measurements are made: in the first, with the plate in place, the received signal is acquired at the temperature θ_1, and then sampled and averaged using a digital oscilloscope (Yokogawa DL9240L). This averaged signal is called the plate signal, $s_1(t)$. Next, when the specimen is removed, the received signal is recorded as the reference signal, $s_2(t)$ at the temperature θ_2. For each signal, a digital thermometer (Traçable®) measures the temperature of water to the accuracy of 0.05 $°C$.

In our experimental system, a signal drift in time may be caused by the change of sound speed in water due to the temperature change. This variation in the immersion fluid temperature from the instant of the through-sample measurement to the the instant of reference measurement will result in significant errors in parameters determination. When temperature control is not available in the experimental set-up, it is necessary to compensate for the temperature changes. One obvious way is to measure the temperature for both the through-sample and through-water signals and use them to calibrate the reference signal with the temperature change. In this way, the correct experimental transmission of the plate is given by

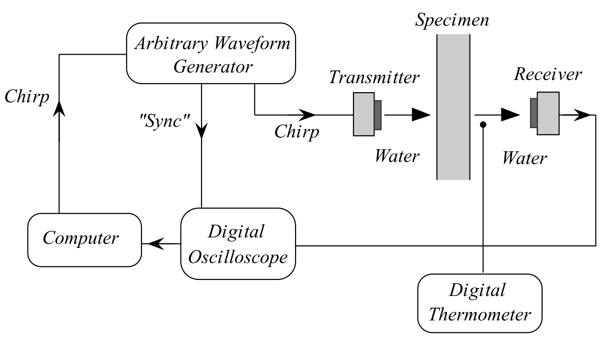
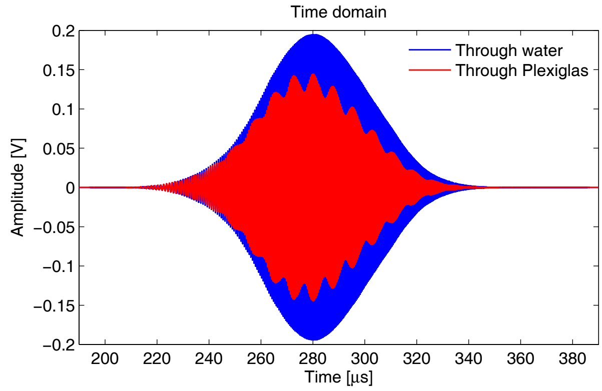
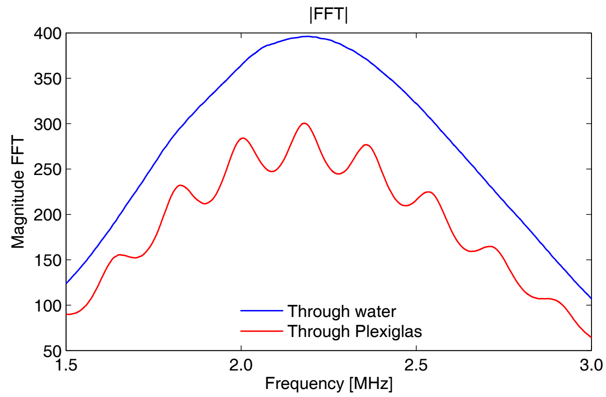


Figure 2: A schematic of the through-transmission set-up.



(a)



(b)

Figure 3: (a) Waveforms for the whole acoustic field transmitted through water and a Plexiglas sample. (b) associated FFT magnitude.

$$T_{exp} = \frac{S_1(f)}{S_2(f)} e^{i2\pi f L\left(\frac{1}{V_w(\theta_1)} - \frac{1}{V_w(\theta_2)}\right)} \quad (23)$$

where $S_1(f)$ and $S_2(f)$ are the FFT of $s_1(t)$ and $s_2(t)$, respectively.

Using the 2.25 MHz pair of transducers, the two signals transmitted through water and through a Plexiglas plate of thickness $E = 8$ mm are shown in Figure 3(a). The associated FFT magnitude of the two waveforms are plotted in Figure 3(b).

4 Inverse procedure

As shown previously, the transmission coefficient with diffraction correction is defined by the set of parameters (p_1, p_2, p_3, p_4, p_5, p_6, q_1, q_2), which can be determined from experimental data by inversion. For this purpose, we employ the least squares method for the minimization of the sum of squared deviations between the calculated T_{cal} and the experimental T_{exp} transmission complex coefficients considering parameters (13) and (20) as variables in a multidimensional space

$$\min \sum_{n=1}^{N} \Re e^2\left(T_{cal}^{(n)} - T_{exp}^{(n)}\right) + \Im m^2\left(T_{cal}^{(n)} - T_{exp}^{(n)}\right) \quad (24)$$

Here, N is the number of data points at different frequencies, and $\Re e$ and $\Im m$ designate the real and imaginary parts.

4.1 Sensitivity analysis

Sensitivity analysis is a good indicator of inversion stability because it allows one to determine whether a parameter is clearly identifiable. Here we consider only the local sensitivity which consists in changing only one parameter at a time. In this case, we define a sensitivity coefficient denoted S_i which describes the variation in the complex transmission with respect to small change in one parameter:

$$S_i = \sqrt{\frac{\sum_{n=1}^{N} \Re e^2\left(T_{cal}^{(1)} - T_{cal}^{(2)}\right) + \Im m^2\left(T_{cal}^{(1)} - T_{cal}^{(2)}\right)}{\sum_{n=1}^{N} \Re e^2\left(T_{cal}^{(1)}\right) + \Im m^2\left(T_{cal}^{(1)}\right)}} \quad (25)$$

where $T_{cal}^{(1)}$ is the calculated transmission, computed with standard parameters and $T_{cal}^{(2)}$ is the calculated transmission updated when the value of a parameter is modified.

Table 1 provides the sensitivity coefficients S_i calculated for a 1‰ increase in each parameter from its reference value, with the other parameters set at their reference values. Calculations are done using properties typical for Plexiglas plate immersed in water, and results are provided for three different frequency ranges: 0.6-1.4, 1.5-3 and 3-6 MHz. Several conclusions can be made. First, the transmission coefficient is most sensitive to p_1 and p_2 which are related to the thickness and wave velocity. Second the sensitivities to p_3, p_4, p_5 and q_1 are significantly smaller. Third, the transmission coefficient is found to be extremely insensitive to changes in p_6 and q_2. Therefore, in order to simplify the inversion and reduce convergence difficulties that could arise due to lack of sensitivity of the transmission coefficient to p_6 and q_2, these two parameters will not be considered further in the inverse process. Finally, one can see except for parameter p_4, that the higher the frequency range of interest, the higher the sensitivity coefficient, so that all plate properties with the exception of density, can be determined with better precision from the data measured at 5 MHz than at 1 MHz. It must be noted here that the sensitivity coefficient, although being a good indicator of the inversion stability, does not fully reflect the accuracy of the reconstructed parameters. Other factors such as signal-to-noise ratio and bandwidth used for reconstruction can significantly affect the result.

Table 1: The sensitivity coefficients S_i calculated for a 1‰ increase in each parameter.

Parameters (+1‰)	Sensitivity coefficients S_i (‰) 0.6-1.4 MHz	1.5-3 MHz	3-6 MHz
$p_1 = \frac{E}{V_w}$	85	189	370
$p_2 = \frac{E}{V_{0p}}$	48	106	205
$p_3 = \alpha_{0p} V_{0p}$	0.22	0.38	0.72
$p_4 = \frac{\rho_p V_{0p}}{\rho_w V_w}$	0.41	0.39	0.38
$p_5 = n - 1$	0.46	0.83	1.7
$p_6 = \alpha_{0w} V_w$	0.00067	0.0032	0.012
$q_1 = \frac{a^2}{(L-E)V_w}$	0.11	0.14	0.17
$q_2 = \frac{E}{L-E}$	0.022	0.016	0.016

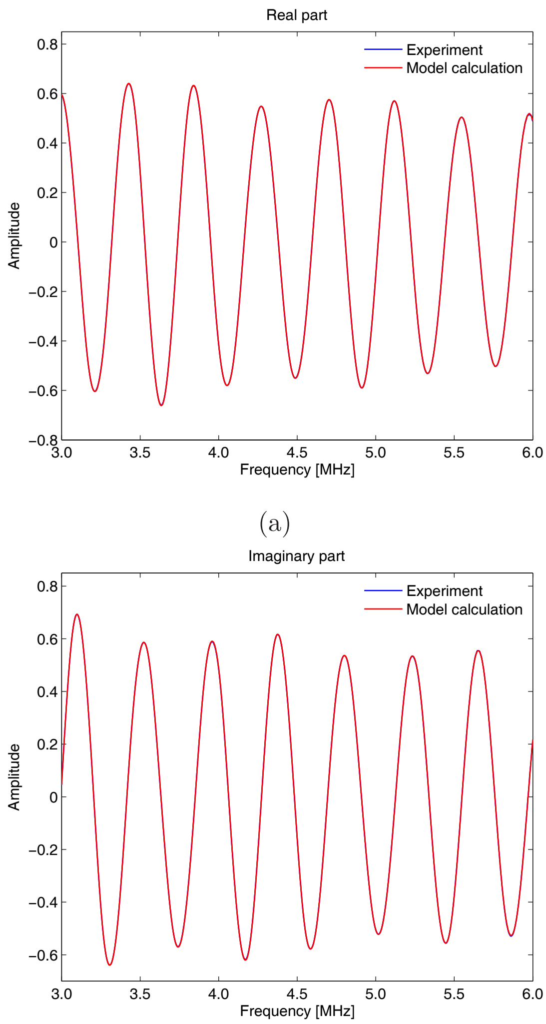
4.2 Parameters identification

Our goal is to determine the five plate properties (V_{0p}, α_{0p}, n, E, ρ_p) without any prior knowledge, as is often required in practice. To realize this, first we determine the set of parameters (p_1, p_2, p_3, p_4, p_5, q_1) from experimental data by inversion. Next, considering the properties of water (ρ_w, V_w, α_{0w}) as known, the plate properties are calculated as follows:

$$E = p_1 V_w, \; V_{0p} = \frac{p_1}{p_2} V_w, \; \alpha_{0p} = \frac{p_3 p_2}{p_1 V_w}, \quad \rho_p = \frac{p_4 p_2 \rho_w}{p_1}, \; n = p_5 + 1 \quad (26)$$

where the properties of water are taken to be $V_w = 3.2246\,\theta + 1420.02\; m/s$, $\rho_w = -0.20783\,\theta + 1002.31\; kg/m^3$ and $\alpha_{0p} = 3 \times 10^{-14}\; N_p/m/Hz^2$

The six parameters are determined simultaneously using the Matlab® routine *lsqnonlin*, where the Gauss-



(b)

Figure 4: Complex transmission of a 8 mm Plexiglas plate calculated and measured in the 3-6 MHz frequency range: (a) real and (b) imaginary parts.

Table 2: Properties determined for the 8 mm Plexiglas plate using data measured at 1, 2.25 and 5 MHz.

Plexiglas (8 mm) \ Frequency range	$0,6-1,4\ MHz$	$1,5-3,0\ MHz$	$3,0-6,0\ MHz$
$E\ (\sigma_E)\ [mm]$	7.558 (0.009)	7.555 (0.005)	7.556 (0.005)
$\rho_p\ (\sigma_{\rho_p})\ [kg/m^3]$	1188.2 (20.4)	1188.5 (15)	1192.4 (21.1)
$V_{0p}{}^*(\sigma_{V_{0p}})\ [m/s]$	2741.6 (6.6)	2747.4 (3)	2745.6 (2.6)
$\alpha_{0p}\ (\sigma_{\alpha_{0p}})\ [Np/m/Hz^n]$	$2.47\ (6.32) \times 10^{-4}$	$1.07\ (1.13) \times 10^{-4}$	$9.2\ (6.1) \times 10^{-5}$
$n\ (\sigma_n)$	0.8 (0.18)	0.856 (0.072)	0.867 (0.043)

$^*\ f_0 = 2.5\ MHz$

Newton or Levenberg-Marquardt algorithm is used. Lower and upper bounds are specified for all the variables. Initial guesses are found from a forward procedure considering especially the periodicity and decreasing amplitude with frequency of the measured spectrum data. Statistical confidence intervals and standard deviations σ for the predicted parameters are then computed using a combination of *nlinfit* and *nlparci* functions.

Figure 4 shows the experimental transmission coefficient measured for the plate of Plexiglas in the 3-6 MHz frequency range. The red lines represent real and imaginary parts calculated from the parameters determined. The comparison is found to be excellent. The discrepancy between the two curves is very small: 0.5%. Similar results are obtained for 0.6-1.4 MHz and 1.5-3 MHz frequency ranges.

Numerical values are compared in Table 2 for the three frequency ranges. The value for the phase velocity is determined at $f_0 = 2.5\ MHz$. Table 2 also presents statistics (mean and standard deviation) of the parameters reconstructed by the inverse algorithm. One can see that error for ρ_p, α_{0p} and n is larger than that for E and V_{0p} (properties responsible for minima positions) which correlates with the sensitivity analysis presented in Table 1. The most pronounced error is observed for α_{0p} of which values estimated for the three frequency ranges are not close to each other. The error in α_{0p} and n determination is however smaller when data is measured at 5 MHz, again in accordance with corresponding sensitivities. In addition, the calculated resultant three frequency phase velocity and attenuation using Eqs. (3) and (4) indicate that although the measurements are made under different measurement conditions such as different transducers, temperature and sample surface investigation, the results of dispersion and attenuation measurements are quite stable and should be reliable.

5 Summary and conclusions

This paper describes an ultrasonic method that allows simultaneous determination of some properties (thickness, density, longitudinal phase velocity and attenuation) of a viscoelastic plate immersed in water, using measured transmission coefficient at normal incidence. By introducing intermediate parameters, the number of parameters describing the transmission coefficient with diffraction correction is reduced. The sensitivity of the proposed method to the individual parameters is studied and the inversion is performed accordingly. The properties of the plate are calculated from the intermediate parameters determined assuming the properties of water as known.

It is shown that the largest standard deviation/mean ratio for the thickness determined is about 0.11%, for phase velocity 0.24%, for density 1%, and for attenuation exponent and coefficient 23% and 260%. We conclude that even though the theory and the experiment curves agree extremely well (see Figure 4), this data cannot be inverted to deduce attenuation due to a lack of sensitivity of transmission to α_{0p} and n. However, as the results suggest, estimation of n is comparatively better than that for α_{0p}. Also it can be noticed that the estimation becomes more precise, in particular for α_{0p}, when high frequency data is used.

Identification results from other specimens demonstrate the accuracy of our method. The attenuation power law and the dispersion predicted by the Szabo's model are found to be suitable for modeling the viscoelastic nature of these samples.

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