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Mathematical modeling of the acoustic radiation by submerged elastic structures

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We consider a number of non-stationary fluid-structure interaction problems that represent several different scenarios of impulse acoustic loading on a fluid-contacting elastic circular cylindrical shell. Both the submerged evacuated and submerged fluid-filled shells are addressed, along with shell systems of higher structural complexity. We discuss the semi-analytical approach that was developed to model such systems, and highlight its advantages and limitations. In particular, we discuss the recent enhancements of the approach that allowed for a much more accurate modeling of the fine structure of the radiated field including the adequate reproduction of all types of the structure-induced waves seen in experiments. We also address a number of interesting and practically important acoustic effects that we observed in the systems in question. In particular, we discuss the reflection and focusing in the internal fluid volume, and analyze the propagation of the waves induced by additional structural elements incorporated into the system.

1 Introduction

The interaction between non-stationary acoustic pulses and submerged elastic structures has been extensively studies for at least six decades [1]. The classical geometries such as spherical and cylindrical have been most favored by researchers due to both their extensive occurrence in practice and the possibility of obtaining analytical solutions that they provide.

In our recent work, we focused on what probably is the most practically important geometry as far as the non-stationary acoustic loading is concerned, that is, a circular cylindrical shell. We have addressed several different systems of varying degree of complexity, and our intent is to summarize some of the more important of our findings.

We discuss three systems that allow us to sufficiently address the most interesting phenomena we have observed in our recent studies, namely a submerged evacuated shell, a submerged fluid-filled shell, and a submerged fluid-filled shell with a rigid co-axial core placed inside of it. We then consider the most recent aspect of our work, namely we discuss certain more advanced models that allow for a far more accurate simulation of the radiation by submerged shells.

2 Mathematical formulation and solution methodology

The approach we use to simulate the dynamics of all the systems considered is based on combining classical methods of mathematical physics with a finite-difference methodology. In particular, we apply separation of variables to the spatial variables of the problem and the Laplace transform to the temporal one in order to reduce the fluid dynamics equations to easily solvable ordinary differential equations. Then, we inverse the Laplace transforms obtained for the harmonics of the fluid velocity potential components in order to arrive at the series expressions for the acoustic pressure. Those are coupled to the shell equations which results in an infinite number of integro-differential systems of equations that are solved using an explicit finite-difference scheme. The approach was shown to be very robust and accurate in the context of the considered systems, and it holds a significant promise for addressing more complex systems.

3 Results and discussion

Submerged evacuated shell

A submerged evacuated shell subjected to an external shock wave is, perhaps, the best-studied fluid-interacting structure. Our effort here was focused not so much on the phenomenological analysis of the process but rather on developing a reliable semi-analytical framework that would allow one to simulate the scattered and radiated fields accurately and efficiently [2,3]. We were successful in introducing such an approach, and utilized it not only to analyze the system in question but also as a reliable and efficient tool to model the external field around shells that contain fluid (and possibly other structural elements). And although the phenomenology of the process is well known, we were able to offer some insights into certain understudied aspects of the interaction, including the detailed analysis of the late-interaction radiated field, Figure 1, and the quantification of the difference between the overall external pressure for an elastic shell versus that for a rigid cylinder.

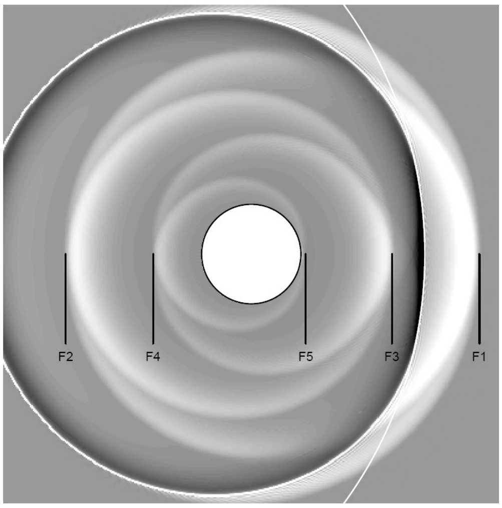

Figure 1: Late-interaction radiated field for a circular cylindrical shell subjected to an external acoustic pulse (F1-F5 denote the fronts of the radiated waves).

Submerged fluid-filled shell

A submerged fluid-filled shell is a far more interesting system to study from the phenomenological point of view in that the majority of the most important effects observed take place in the internal fluid, and the modeling of the internal fluid, until recently, had been far less developed than that of the external one. We used the same semi-analytical framework as for the evacuated shell to introduce matching solutions for both the internal and external fluid domains [1], and then have been able to analyze the dynamics of the system in response to a most common external loading.

Such an approach allowed us to study in detail a number of highly interesting effects [1,4], most notably the internal reflection and focusing. In particular, we demonstrated that the peak focusing pressure can significantly exceed the peak incident pressure, a fact of significant practical importance. A case of a shell filled with and submerged into different fluids was addressed in [5], and in that case the interaction was shown to be much more phenomenologically complex, with up to four different interaction scenarios possible, and with two completely different reflection-focusing sequences observed, Figure 2.

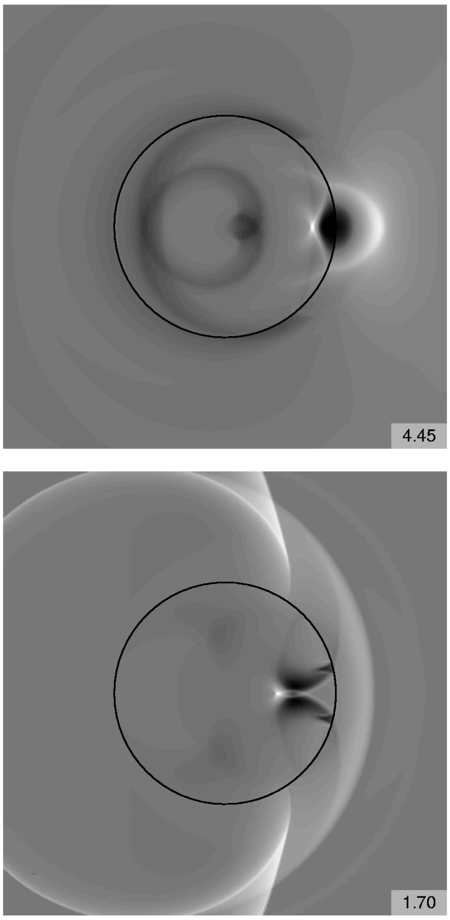

Figure 2: Different types of focusing inside a circular cylindrical shell subjected to an external pulse depending on the ratio of the sound speed in the internal fluid to that in the external one, ζ: ζ=0.50, top image; ζ=1.50, bottom image.

Submerged fluid-filled shell with a rigid co-axial core

A rather natural progression of our work on the evacuated and fluid-filled shells was to consider some degree of structural enhancement of the system, and we chose adding a rigid co-axial core as the first step in that direction. The same semi-analytical framework was employed again [6], and it resulted in another computationally efficient solution.

We observed that the core always has an effect on both the acoustic fields induced during the interaction and the stress-strain state of the structure. However, this effect was found to be not very significant for cores of small radii (in the order of 10% or less of the radius of the shell), while the influence of a core of a large radius (more than 50% of the radius of the shell) was very significant on both the fluid and structural dynamics. In particular, we have found that many of the fundamentally important wave phenomena observed in the internal fluid in the absence of the core do not take place when a large-radius core is present.

Figure 3 shows two representative snapshots of the hydrodynamic field when no core is present and when a large-radius core is placed inside the shell.

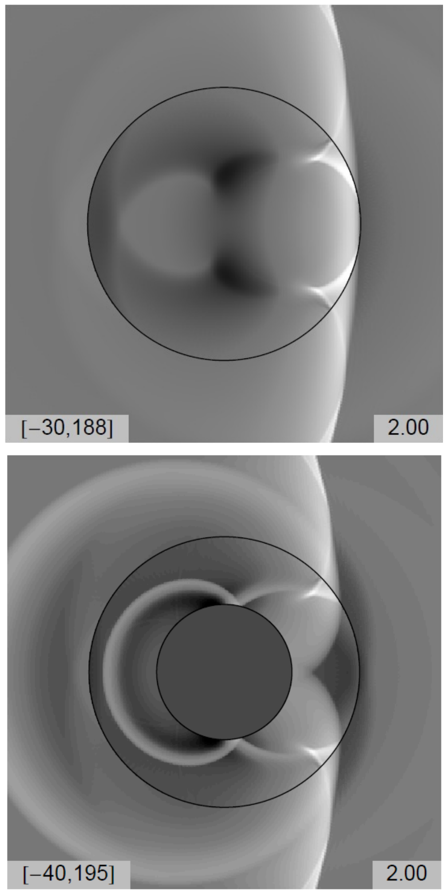

Figure 3: Acoustic field at the moment of reflection of the internal wave off the shell surface at the end of its first downstream propagation when no core is present, top image, and when a large-radius core (50% of the radius of the shell) is placed inside the shell, bottom image.

Advanced shell-fluid interaction models

Our most recent effort was focused on employing shell models that are more advanced than the traditionally used ones in order to provide fluid-structure simulations that accurately reproduce all types of waves observed in the experiments. To that end, we considered two rather different approaches, one where we relied on a more advanced shell theory, namely the Reissner-Mindlin theory [7], and the other where we used a fully elastodynamic formulation and treated the shell as a 3D body [8].

Both approaches resulted in a very significant improvement of the accuracy of simulations, as compared to the Kirchhoff-Love model. In particular, the developed models allowed for an accurate reproduction of all types of waves radiated by the shell, including the S_0 wave and A_0 wave, both of which are clearly separated in the simulated images of the radiated fields. Figure 4 shows a representative snapshot produced by the Reissner-Mindlin model in which the different types of waves are clearly visible, and Figure 5 shows a snapshot produced by a fully-elastic model.

As important as the discussed accuracy improvements are, it is worth mentioning that they do not significantly alter the overall appearance of the radiated field when the shell is very thin (the thickness-to-radius ratio of 0.01 or less).

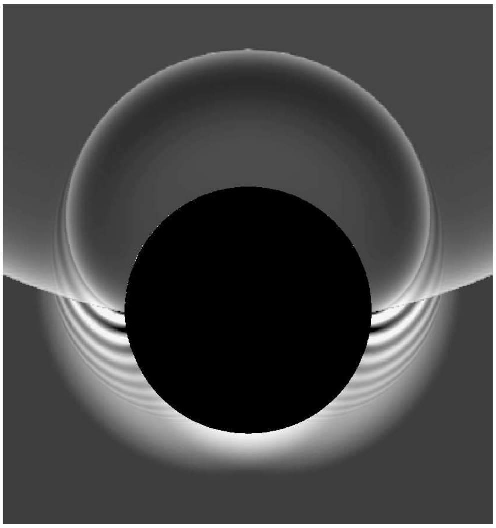

Figure 4: Radiated field simulated using the Reissner-Mindlin theory for a circular cylindrical shell with the thickness-to-radius ratio of 0.03 responding to an external acoustic pulse.

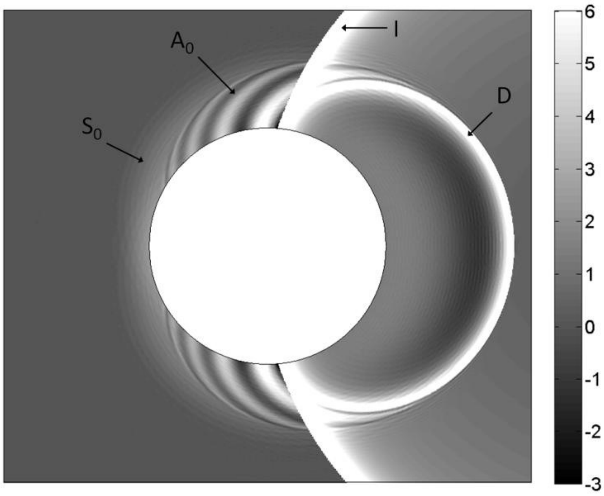

Figure 5: Radiated field simulated using the full elastodynamic model for a circular cylindrical shell with the thickness-to-radius ratio of 0.06 responding to an external acoustic pulse.

4 Conclusions

We have discussed a number of problems of non-stationary fluid-structure interaction, and approached all of them using the same semi-analytical methodology. We demonstrated that the approach developed is very suitable for this class of problems not only for single shell structures but also for shell systems with additional structural elements added to them. We also observed a number of theoretically and practically interesting effects occurring in the systems, most notably the different reflection-focusing sequences for a shell filled with and submerged into different fluids. Then, we considered two shell models that are more advanced than the most frequently used models such as the Kirchhoff-Love model, and demonstrated that the developed advanced models allow for much more accurate simulations of the radiated field, including the accurate reproduction of all types of waves seen in experiments.

Acknowledgments

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