ACOUSTICS 2012

Numerical transmission loss calculations for perforated dissipative mufflers containing heterogeneous material and mean flow

A. G. Antebas[a], F. D. Denia[b], E. M. Sánchez-Orgaz[b] and F. J. Fuenmayor[b]

[a]CTAG (Centro Tecnológico de Automoción de Galicia), Polígono Industrial A Granxa, Calle A, parcelas 249-250, 36400 Porriño (pontevedra), Spain
[b]Centro de Investigación de Tecnología de Vehículos, Universitat Politècnica de València, Camino de Vera s/n, 46022 Valencia, Spain
anan1@upvnet.upv.es

In this work, a finite element approach has been developed to study the acoustic behavior of perforated dissipative mufflers with heterogeneous absorbent material and the presence of mean flow. The heterogeneous properties can appear, for example, during the manufacturing processes of the dissipative muffler or due to the flow of soot particles within the absorbent material. A uniform mean flow is considered within the central passage and a perforated duct separates this flow from the outer dissipative chamber. First, the finite element method is applied to the wave equation for a stationary non-homogeneous medium (outer chamber) and a moving homogeneous medium (central passage). The absorbent material is characterized as an equivalent fluid, by means of its complex density and speed of sound. To introduce the variations of these properties within the material, a coordinate-dependent function is proposed for the filling density. This yields a heterogeneous steady airflow resistivity, which produces spatial modifications of the equivalent acoustic properties. The acoustic impedance of the perforated duct has been also modified to include the heterogeneous properties of the dissipative medium. Finally, the effect of the mean flow Mach number and the filling density on the acoustic attenuation performance of the muffler is studied.

1 Introduction

Due to their broadband characteristics at mid to high frequencies, dissipative mufflers have been widely used in vehicle exhaust systems. Although plane wave models [1] are available for the prediction of the sound attenuation of mufflers at low frequency, multidimensional analytical techniques [2-4] and numerical methods [5-7] are required for higher frequencies and to consider for example, the propagation of higher order modes. While multidimensional analytical methods are desirable due to their low computational effort, they are not capable to model complex silencer geometries or non-homogeneous properties. For this reason, numerical techniques have found favor for modeling complex geometries and arbitrary boundary conditions. Among the numerical methods, the finite element method (FEM) is widely used and relevant literature can be found regarding the acoustic modeling of silencers. Young and Crocker [5] applied the finite element method to reactive concentric expansion silencers to predict their transmission loss. Kagawa et al. [8] and Craggs [9] studied the transmission loss of a lined expansion muffler assuming a locally reacting effect of the absorbent material. Finite element models for bulk reacting absorbent materials were presented by Kirby [10] to consider perforated dissipative mufflers with homogeneous properties [10,11].

In previous works, the absorbent materials considered were assumed to be homogeneous. However, in realistic cases of automotive silencers, this assumption is not always fulfilled and heterogeneous acoustic properties of the fibrous materials appear. The presence of these non-homogeneous properties may arise, for example, from uneven filling processes in dissipative mufflers [2,12] and degradation produced by the flow of soot particles within the absorbent material [13]. These two phenomena can cause significant variation in the filling density of the fibrous material, which as a consequence leads to heterogeneity of its equivalent complex density and speed of sound [14]. To the authors' knowledge, the only reference in the literature that tried to model dissipative mufflers with heterogeneous fibrous material in the presence of mean flow was the work of Peat and Rathi [6]. In this case, the heterogeneity was associated with the mean flow induced in the absorbent material.

In Ref. [6] no perforated duct was considered and the fibrous material was exposed directly to the gas in the central airway. However, in the automotive silencers, a perforated screen is usually placed to protect the fibrous material and to reduce static pressure losses over the silencer. The acoustic impedance of a perforated surface in absence of absorbent material was studied by Sullivan and Crocker [15]. Kirby and Cummings [16] obtained a semiempirical relationship for the acoustic impedance of perforated plates with a porous material backing the perforations, which depends upon various parameters including the equivalent complex density. Further investigations were carried out by Lee et al. [17] to account for the influence of the absorbent propagation medium on acoustic behavior of the perforated screen. When the bulk reacting material in contact with the perforations is considered to be homogeneous, the acoustic impedance of the perforated plate is constant from a spatial point of view [3,4,18]. However, in the case of non-homogeneous porous backing material, a modified expression of the acoustic impedance must be regarded to account for the spatial coordinate dependence of the equivalent properties.

2 Mathematical approach

2.1 Finite element formulation

Figure 1 shows the perforated dissipative muffler studied in this work, which consists of a perforated duct surrounded by a non-homogeneous absorbent material. The subdomains of the central airway and the absorbent material are denoted by Ω_a and Ω_m, respectively. Γ_a and Γ_m represent their boundary surfaces satisfying the rigid wall boundary conditions [1], except for the inlet and outlet sections Γ_i and Γ_o and the perforated surface Γ_p. A uniform mean flow of Mach number M is considered in the airway. The central passage with air is characterized by the density ρ_0 and speed of sound c_0, while its perforated surface is considered by means of the acoustic impedance $\tilde{Z}_p$. To account for the heterogeneous properties of the absorbent material, the equivalent complex density $\rho_m(\mathbf{x})$ and speed of sound $c_m(\mathbf{x})$ [19] are coordinate-dependent, which leads to spatial variation of $\tilde{Z}_p(\mathbf{x})$ as well.

For the central airway, the wave propagation is governed by the equation [1],

$$\nabla^2 P_a - M^2 \frac{\partial^2 P_a}{\partial x^2} - 2j\omega \frac{M}{c_0} \frac{\partial P_a}{\partial x} + k_0^2 P_a = 0 \qquad (1)$$

with P_a being the acoustic pressure, M the mean flow Mach number and k_0 the wavenumber in the air, defined as the ratio of the angular frequency ω and the speed of sound c_0.

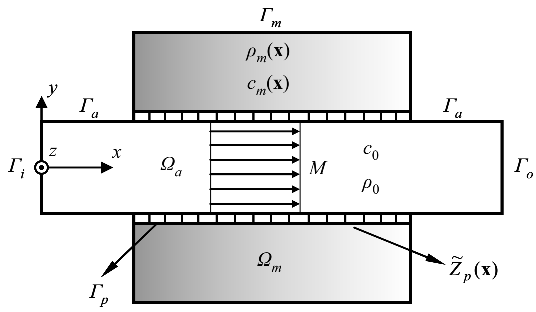

Figure 1: Perforated dissipative silencer with heterogeneous absorbent material and mean flow.

In the heterogeneous absorbent material, the wave propagation is given by the equation [20],

$$\nabla\left(\frac{1}{\rho_m}\nabla P_m\right)+\frac{1}{\rho_m}k_m^2 P_m=0 \tag{2}$$

where $k_m(\mathbf{x}) = \omega/c_m(\mathbf{x})$ is the equivalent complex wavenumber associated with the heterogeneous absorbent material.

By applying the method of weighted residuals in combination with the Galerkin approach [21], and after using Green's theorem, the weighted residual associated with the subdomain Ω_a can be written as

$$\begin{gathered}\sum_{e=1}^{N_e^a}\left(\int_{\Omega_a^e}\nabla^{\mathrm{T}}\mathbf{N}\mathbf{D}\nabla\mathbf{N}\mathrm{d}\Omega+\frac{2j\omega}{c_0^2}\int_{\Omega_a^e}\mathbf{N}^{\mathrm{T}}\mathbf{U}_0\nabla\mathbf{N}\mathrm{d}\Omega\right)\tilde{\mathbf{P}}_a^e\\ -\sum_{e=1}^{N_e^a}\left(k_0^2\int_{\Omega_a^e}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Omega\right)\tilde{\mathbf{P}}_a^e=\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e}\mathbf{N}^{\mathrm{T}}\frac{\partial P_a}{\partial n}\mathbf{n}^{\mathrm{T}}\mathbf{D}\mathbf{n}\mathrm{d}\Gamma\end{gathered} \tag{3}$$

and the weighted residual of subdomain Ω_m is

$$\begin{gathered}\sum_{e=1}^{N_e^m}\left(\int_{\Omega_m^e}\frac{1}{\rho_m}\nabla^{\mathrm{T}}\mathbf{N}\nabla\mathbf{N}\mathrm{d}\Omega-\int_{\Omega_m^e}\frac{k_m^2}{\rho_m}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Omega\right)\tilde{\mathbf{P}}_m^e\\ =\sum_{e=1}^{N_e^m}\int_{\Gamma_m^e}\frac{1}{\rho_m}\mathbf{N}^{\mathrm{T}}\frac{\partial P_m}{\partial n}\mathrm{d}\Gamma\end{gathered} \tag{4}$$

where N_e^a and N_e^m are the number of finite elements in the central perforated duct and the absorbent material subdomains, respectively, $\tilde{\mathbf{P}}_a$ and $\tilde{\mathbf{P}}_m$ are the nodal pressures, $\mathbf{N}$ are the shape functions, n is the outward unit vector and matrix $\mathbf{D}$ is given as follows,

$$\mathbf{D}=\begin{bmatrix}1-M^2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \tag{5}$$

As the rigid wall condition is satisfied at the boundary $\Gamma_a - \Gamma_p - \Gamma_i - \Gamma_o$, the right side integral of Eq. (3) is only taken over the perforated duct boundary Γ_p and the inlet and outlet surfaces, denoted by Γ_i and Γ_o, respectively. To evaluate the contribution of the perforated boundary Γ_p on the surface integral, the relation that defines the acoustic impedance of the perforated duct must be taken into account [1]. By using the Euler's equation, the normal pressure gradient at the perforated surface in the central airway can be expressed as,

$$\frac{\partial P_a}{\partial n}=-\rho_0\frac{DU_{n_a}}{Dt}=-\rho_0\left(j\omega U_{n_a}+U_0\frac{\partial U_{n_a}}{\partial x}\right) \tag{6}$$

being U_{n_a} the acoustic velocity normal to the perforations at the surface and U_0 the uniform mean flow axial velocity.

The acoustic impedance of the perforated surface is defined as the ratio of the acoustic pressure jump across the perforations to the normal acoustic velocity, and can be written as,

$$\tilde{Z}_p=\frac{P_a-P_m}{U_{n_a}} \tag{7}$$

After substituting Eq. (7) into Eq. (6), the normal pressure gradient associated with the perforated duct in the central airway is expressed as

$$\begin{gathered}\frac{\partial P_a}{\partial n}=\left(-j\omega\rho_0\frac{P_a-P_m}{\tilde{Z}_p}-\rho_0 U_0\frac{\partial}{\partial x}\left(\frac{P_a-P_m}{\tilde{Z}_p}\right)\right)=\\ \rho_0\left(-j\omega\frac{P_a-P_m}{\tilde{Z}_p}-U_0\left(\frac{1}{\tilde{Z}_p}\frac{\partial(P_a-P_m)}{\partial x}-\frac{(P_a-P_m)}{\tilde{Z}_p^2}\frac{\partial\tilde{Z}_p}{\partial x}\right)\right)\end{gathered} \tag{8}$$

The substitution of the approximated pressure fields yields

$$\begin{gathered}\frac{\partial P_a}{\partial n}=\rho_0\left(-j\omega\frac{\mathbf{N}\tilde{\mathbf{P}}_a^e-\mathbf{N}\tilde{\mathbf{P}}_m^e}{\tilde{Z}_p}-\frac{U_0}{\tilde{Z}_p}\frac{\partial\left(\mathbf{N}\tilde{\mathbf{P}}_a^e-\mathbf{N}\tilde{\mathbf{P}}_m^e\right)}{\partial x}\right.\\ \left.+\frac{U_0\left(\mathbf{N}\tilde{\mathbf{P}}_a^e-\mathbf{N}\tilde{\mathbf{P}}_m^e\right)}{\tilde{Z}_p^2}\frac{\partial\tilde{Z}_p}{\partial x}\right)\end{gathered} \tag{9}$$

A similar procedure is applied to calculate the right side integral of Eq. (4). In this case, only the contribution of the perforated boundary Γ_p is computed. By considering Euler's equation, the normal pressure gradient at the perforated surface in the heterogeneous absorbent material is given by

$$\frac{\partial P_m}{\partial n}=-\rho_m\frac{\partial U_{n_m}}{\partial t}=-\rho_m j\omega U_{n_m} \tag{10}$$

where U_{n_m} is the normal velocity at the interface. Considering the continuity of the normal particle velocity and taking into account that the outward unit vectors in the direction normal to the interface of both regions are opposite [22], yields

$$\frac{\partial P_m}{\partial n}=j\omega\rho_m\frac{\mathbf{N}\tilde{\mathbf{P}}_a^e-\mathbf{N}\tilde{\mathbf{P}}_m^e}{\tilde{Z}_p} \tag{11}$$

Replacing the normal pressure gradient in Eq. (3) by Eq. (9), yields in compact form,

$$\left(\mathbf{K}_a+\mathbf{K}_{aa}^{Z_p}+\mathbf{D}_{aa}^{Z_p}+j\omega\left(\mathbf{C}_a+\mathbf{C}_{aa}^{Z_p}\right)-\omega^2\mathbf{M}_a\right)\widetilde{\mathbf{P}}_a +\left(\mathbf{K}_{am}^{Z_p}+\mathbf{D}_{am}^{Z_p}+j\omega\mathbf{C}_{am}^{Z_p}\right)\widetilde{\mathbf{P}}_m=\mathbf{F}_a \quad (12)$$

where the following nomenclature has been introduced

$$\mathbf{K}_a=\sum_{e=1}^{N_e^a}\int_{\Omega_a^e}\nabla^{\mathrm{T}}\mathbf{N}\mathbf{D}\nabla\mathbf{N}\mathrm{d}\Omega$$

$$\mathbf{M}_a=\frac{1}{c_0^2}\sum_{e=1}^{N_e^a}\int_{\Omega_a^e}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Omega$$

$$\mathbf{C}_a=\frac{2}{c_0^2}\int_{\Omega_a^e}\mathbf{N}^{\mathrm{T}}\mathbf{U}_0\nabla\mathbf{N}\mathrm{d}\Omega$$

$$\mathbf{C}_{aa}^{Z_p}=\rho_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Gamma$$

$$\mathbf{C}_{am}^{Z_p}=-\rho_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Gamma$$

$$\mathbf{K}_{aa}^{Z_p}=\rho_0U_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\frac{\partial\mathbf{N}}{\partial x}\mathrm{d}\Gamma$$

$$\mathbf{K}_{am}^{Z_p}=-\rho_0U_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\frac{\partial\mathbf{N}}{\partial x}\mathrm{d}\Gamma$$

$$\mathbf{D}_{aa}^{Z_p}=-\rho_0U_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p^2}\frac{\partial\widetilde{Z}_p}{\partial x}\mathbf{N}^{\mathrm{T}}\,\mathbf{N}\,\mathrm{d}\Gamma$$

$$\mathbf{D}_{am}^{Z_p}=\rho_0U_0\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p^2}\frac{\partial\widetilde{Z}_p}{\partial x}\mathbf{N}^{\mathrm{T}}\,\mathbf{N}\mathrm{d}\Gamma \quad (13)$$

$$\mathbf{F}_a=\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_i}\mathbf{N}^{\mathrm{T}}\frac{\partial P_a}{\partial n}\mathbf{n}\mathbf{D}\mathbf{n}^{\mathrm{T}}\mathrm{d}\Gamma+\sum_{e=1}^{N_e^a}\int_{\Gamma_a^e\cap\Gamma_o}\mathbf{N}^{\mathrm{T}}\frac{\partial P_a}{\partial n}\mathbf{n}\mathbf{D}\mathbf{n}^{\mathrm{T}}\mathrm{d}\Gamma$$

where the integrals over Γ_i and Γ_o are associated with the usual excitation boundary conditions [1], and the integral over Γ_p represents a coupling term between Ω_a and Ω_m.

Substituting now Eq. (11) into Eq. (4) provides

$$\left(\mathbf{K}_m+j\omega\mathbf{C}_{mm}^{Z_p}-\omega^2\mathbf{M}_m\right)\widetilde{\mathbf{P}}_m+j\omega\mathbf{C}_{ma}^{Z_p}\widetilde{\mathbf{P}}_a=\mathbf{0} \quad (14)$$

with the notation

$$\mathbf{K}_m=\sum_{e=1}^{N_e^m}\int_{\Omega_m^e}\frac{1}{\rho_m}\nabla^{\mathrm{T}}\mathbf{N}\nabla\mathbf{N}\mathrm{d}\Omega$$

$$\mathbf{M}_m=\sum_{e=1}^{N_e^m}\int_{\Omega_m^e}\frac{1}{\rho_m}\frac{1}{c_m^2}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Omega$$

$$\mathbf{C}_{ma}^{Z_p}=-\sum_{e=1}^{N_e^m}\int_{\Gamma_m^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Gamma \quad (15)$$

$$\mathbf{C}_{mm}^{Z_p}=\sum_{e=1}^{N_e^m}\int_{\Gamma_m^e\cap\Gamma_p}\frac{1}{\widetilde{Z}_p}\mathbf{N}^{\mathrm{T}}\mathbf{N}\mathrm{d}\Gamma$$

Finally Eqs. (12) and (14) are written as

$$\left(\begin{bmatrix}\mathbf{K}_a+\mathbf{K}_{aa}^{Z_p}+\mathbf{D}_{aa}^{Z_p} & \mathbf{K}_{am}^{Z_p}+\mathbf{D}_{am}^{Z_p}\\ \mathbf{0} & \mathbf{K}_m\end{bmatrix}+j\omega\begin{bmatrix}\mathbf{C}_a+\mathbf{C}_{aa}^{Z_p} & \mathbf{C}_{am}^{Z_p}\\ \mathbf{C}_{ma}^{Z_p} & \mathbf{C}_{mm}^{Z_p}\end{bmatrix}-\omega^2\begin{bmatrix}\mathbf{M}_a & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_m\end{bmatrix}\right)\begin{Bmatrix}\widetilde{\mathbf{P}}_a\\ \widetilde{\mathbf{P}}_m\end{Bmatrix}=\begin{Bmatrix}\mathbf{F}_a\\ \mathbf{0}\end{Bmatrix} \quad (16)$$

2.2 Heterogeneous acoustic properties of the absorbent material

The acoustic properties of the absorbent materials considered in the literature are characterized by the equivalent characteristic impedance $Z_m = \rho_m c_m$ and wavenumber $k_m = \omega/c_m$. These complex and frequency dependent properties are usually supposed to have a constant value from a spatial point of view throughout the fibrous material domain, assuming a homogeneous steady airflow resistivity R [3,4,18]. The resistivity can be expressed in terms of the filling density ρ_c by means of [23]

$$R = A_1\rho_c^{A_2} \quad (17)$$

where the values A_1 and A_2 are independent of the filling density and can be obtained, for a given absorbent material, by a curve fitting from experimental data. For the absorbent material under consideration (Owens Corning's texturized Advantex fiber glass roving) the values A_1 = 1.083099 and A_2 = 1.827587 are taken into account [4,18] with R and ρ_c in SI units.

Although ρ_c is usually assumed to be constant in the bibliography, strong heterogeneities can be found in practical applications. During the manufacturing process of automotive dissipative mufflers, meaningful variations in the filling density can be produced causing a change in the steady airflow resistivity of the absorbent material. Also, the flow of soot particles [13] can induce spatial modifications of the material properties. In this work, a coordinate-dependent linear function $\rho_c(x) = ax+b$ is assumed to simulate the variation of the filling density along the main axial direction of the muffler and the airflow resistivity is given as follows,

$$R(x) = A_1\rho_c(x)^{A_2} \quad (18)$$

Eq. (18) leads then to a coordinate-dependent resistivity $R(x)$, and a modified version of the homogeneous models of earlier studies [3,4,18,24] is obtained. Now the characteristic impedance and wavenumber of the heterogeneous absorbent material are expressed as follows,

$$Z_m(x) = Z_0\left(\left(1+0.09534\left(f\rho_0/R(x)\right)^{-0.754}\right) + j\left(-0.08504\left(f\rho_0/R(x)\right)^{-0.732}\right)\right) \quad (19)$$

$$k_m(x) = k_0\left(\left(1+0.16\left(f\rho_0/R(x)\right)^{-0.577}\right) + j\left(-0.18897\left(f\rho_0/R(x)\right)^{-0.595}\right)\right) \quad (20)$$

$Z_0 = \rho_0 c_0$ being the characteristic impedance of the air and f the frequency. Therefore, the introduction of spatial variations in the filling density leads finally to heterogeneity associated with R, Z_m and k_m.

2.3 Acoustic impedance of perforated duct in the presence of mean flow

As it was shown in the work of Kirby and Cummings [16], the acoustic impedance of the perforated duct in the presence of homogeneous absorbent material is strongly dependent upon the acoustic properties of the fibrous material adjacent to the perforations. In this investigation, the equivalent characteristic impedance Z_m and wavenumber k_m are coordinate dependent and therefore the expression used in earlier works [22,25] for the acoustic impedance of the perforations under grazing flow conditions has been redefined here to introduce the heterogeneity of the absorbent material, giving

$$\tilde{Z}_p(x) = \frac{\rho_0 c_0\left(\zeta'_p + j0.425k_0 d_h\left(\frac{\rho_m(x)}{\rho_0} - 1\right)F(\sigma)\right)}{\sigma} \quad (21)$$

where ζ'_p denotes the nondimensionalized impedance in the absence of absorbent material [25], d_h the hole diameter, t_p the thickness, σ the porosity and $F(\sigma)$ accounts for the interaction between the perforations. Here the following expression is considered [4]

$$F(\sigma) = 1 - 1.055\sqrt{\sigma} + 0.17\left(\sqrt{\sigma}\right)^3 + 0.035\left(\sqrt{\sigma}\right)^5 \quad (22)$$

3 Results and discussion

Figure 2 shows the transmission loss (TL) predictions obtained by the implemented procedure for two cases: (1) Perforated dissipative muffler in the presence of mean flow and heterogeneous absorbent material; (2) Perforated dissipative muffler in the presence of mean flow and homogeneous absorbent material. The muffler geometry is axisymmetric, and the dimensions of the studied configuration are: central passage radius R_1 = 0.0245 m, outer chamber radius R_2 = 0.0822 m, length of the inlet and outlet ducts $L_i = L_o$ = 0.1 m and length of the absorbent material L_m = 0.3 m (see Figure 1). The perforated duct is characterized by a porosity σ = 20%, thickness t_p = 0.001 m and hole diameter d_h = 0.0035 m. For the heterogeneous absorbent material, the filling density is assumed to vary according to the function $\rho_c(x) = -653.333x+359.333$. This assumption provides a mean filling density with value 196 kg/m^3 (homogeneous absorbent material case). For the mean flow Mach number, values M = 0, M = 0.1 y M = 0.2 have been considered. The axisymmetric finite element mesh consists of eight-noded quadratic quadrilateral elements with an approximate size of 0.01 m.

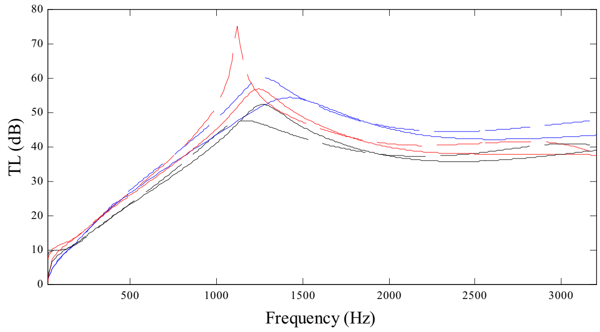

Figure 2: TL of perforated dissipative silencer: —, M = 0, heterogeneous material; – – , M = 0, homogeneous material; —, M = 0.1, heterogeneous material; – – , M = 0.1, homogeneous material; —, M = 0.2, heterogeneous material; – –, M = 0.2, homogeneous material.

As can be observed, for all the considered mean flow Mach numbers, the two cases (homogeneous and heterogeneous material) exhibit considerable discrepancies between them. Therefore, the impact of the material heterogeneity on the sound attenuation is remarkable, which justifies the approach presented in the current investigation. Also, Figure 2 shows that higher mean flow velocities lead to a decrease in the transmission loss for the two cases.

3.1 Effect of filling density

The influence of the filling density is analyzed in Figure 3, where the values ρ_c = 90 kg/m^3, ρ_c = 166.7 kg/m^3 and ρ_c = 196 kg/m^3 have been considered for the geometry analyzed in the previous figure and a Mach number M = 0.1. In the homogenous absorbent material case, these values denote the constant filling density, while in the heterogeneous absorbent material case they represent the mean filling densities. In this latter case, the axial distribution are given by $\rho_c(x) = -300x+165$, $\rho_c(x) = -555.733x+305.653$ and $\rho_c(x) = -653.333x+359.333$, respectively. As can be observed in Figure 3, the transmission loss curves associated with the heterogeneous absorbent material case are different from those obtained for the perforated dissipative silencer with homogeneous absorbent material. As the filling density increases, the two cases show higher discrepancies. For both material models (homogeneous and heterogeneous), higher filling densities tend to improve the performance of the silencer at medium to high frequencies. At very low frequencies, higher filling densities lead to lower transmission loss. These trends presented by the perforated dissipative muffler with heterogeneous properties and mean flow are similar to those observed in the absence of mean flow [14].

5 Conclusions

A finite element approach has been developed to predict the acoustic attenuation of perforated dissipative mufflers in the presence of non-homogeneous properties and mean flow. The heterogeneous properties of the absorbent material have been modeled by introducing a spatial variation of its filling density that leads to coordinate-dependent equivalent density and speed of sound. In addition, the coupling between the central passage and the dissipative outer chamber has been carried out considering a perforated screen. Since the acoustic impedance of the perforations in the presence of a backing porous medium strongly depends on the acoustic properties of the fibrous material, additional features have been implemented in comparison with the models of earlier studies. Specifically, the perforated acoustic impedance models under grazing flow conditions normally used in the bibliography have been modified to introduce the spatial influence of the heterogeneous properties of the fibrous material. To account for the simultaneous presence of the heterogeneous properties, the mean flow and the perforated screen, some modifications have been included in the standard finite element calculations.

Finally, a study has been presented to assess the influence of the heterogeneity, the mean flow and the filling density on the sound attenuation of the dissipative mufflers.

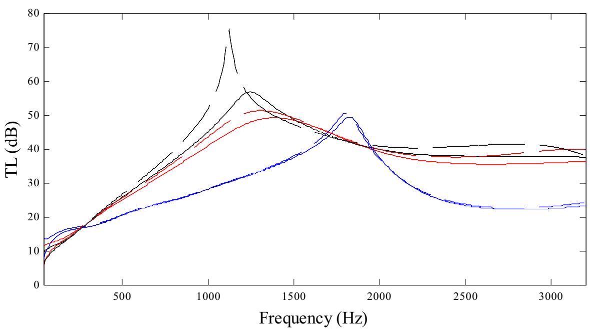

Figure 3: TL of perforated dissipative silencer, $M = 0.1$: —, $\rho_c = 90$ kg/m^3, heterogeneous material; – – , $\rho_c = 90$ kg/m^3, homogeneous material; —, $\rho_c = 166.7$ kg/m^3, heterogeneous material; – – , $\rho_c = 166.7$ kg/m^3, homogeneous material; —, $\rho_c = 196$ kg/m^3, heterogeneous material; – –, $\rho_c = 196$ kg/m^3, homogeneous material.

Acknowledgments

Dr. Antebas, now at CTAG, is grateful to Universitat Politècnica de València for providing financial support. The results presented in this paper are related to his research activities during the last period of his contract at UPV. The authors gratefully acknowledge the financial support of Ministerio de Ciencia e Innovación by means of the project DPI2010-15412.

References

[1] M.L. Munjal, *Acoustics of ducts and mufflers*, Wiley-Interscience, New York (1987)

[2] A. Selamet, M. B. Xu, I. J. Lee, N. T. Huff, "Dissipative expansion chambers with two concentric layers of fibrous materials", *Int. J. Vehicle Noise Vib.* 1(3-4), 341-357 (2005)

[3] F. D. Denia, A. G. Antebas, A. Selamet, A. M. Pedrosa, "Acoustic characteristics of circular dissipative reversing chamber mufflers", *Noise Control Eng. J.* 59(3), 234-246 (2011)

[4] F. D. Denia, A. Selamet, F. J. Fuenmayor, R. Kirby, "Acoustic attenuation performance of perforated dissipative mufflers with empty inlet/outlet extensions", *J. Sound Vib.* 302(4-5), 1000-1017 (2007)

[5] C. I. J. Young, M. J. Crocker, "Prediction of transmission loss in mufflers by the finite element method", *J. Acoust. Soc. Am.* 57(1), 144-148 (1975)

[6] K. S. Peat, K. L. Rathi, "A finite element analysis of the convected acoustic wave motion in dissipative silencers", *J. Sound Vib.* 184(3), 529-545 (1995)

[7] T. W. Wu, *Boundary Element Acoustics*, WIT Press, Southampton (2000)

[8] Y. Kagawa, T. Yamabuchi, A. Mori, "Finite element simulation of an axisymmetric acoustic transmission system with a sound absorbing wall", *J. Sound Vib.* 53(3), 357-374 (1977)

[9] A. Craggs, "A finite element method for modelling dissipative mufflers with a locally reactive lining", *J. Sound Vib.* 54(2), 285-296(1977)

[10] R. Kirby, "Transmission loss predictions for dissipative silencers of arbitrary cross section in the presence of mean flow", *J. Acoust. Soc. Am.* 114(1), 200-209(2003)

[11] R. Kirby, "A comparison between analytic and numerical methods for modelling automotive dissipative silencers with mean flow", *J. Sound Vib.* 325(3), 565-582(2009)

[12] A. Selamet, M. B. Xu, I. J. Lee, N. T. Huff, "Effects of voids on the acoustics of perforated dissipative silencers", *Int. J. Vehicle Noise Vib.* 2(4), 357-372 (2006)

[13] S. Allam, M. Åbom, "Sound propagation in an array of narrow porous channels with application to diesel particulate filters", *J. Sound Vib.* 291(3-5), 882-901(2006)

[14] A. G. Antebas, F. D. Denia, A. M. Pedrosa, F. J. Fuenmayor, "A finite element approach for the acoustic modeling of perforated dissipative mufflers with non-homogeneous properties", *Math. Comput. Model.* In Press, corrected proof available online, http://dx.doi.org/10.1016/j.mcm.2012.01.021

[15] J. W. Sullivan, M. J. Crocker, "Analysis of concentric-tube resonators having unpartitioned cavities", *J. Acoust. Soc. Am.* 64(1), 207-215(1978)

[16] R. Kirby, A. Cummings "The impedance of perforated plates subjected to grazing gas flow and backed by porous media", *J. Sound Vib.* 217(4), 619-636(1998)

[17] I. J. Lee, A. Selamet, N. T. Huff, "Acoustic impedance of perforations in contact with fibrous material", *J. Acoust. Soc. Am.* 119(5), 2785-2797(2006)

[18] A. Selamet, I. J. Lee, N. T. Huff, "Acoustic attenuation of hybrid silencers", *J. Sound Vib.* 262(3), 509-527 (2003)

[19] J. F. Allard, *Propagation of sound in porous media: modelling of sound absorbing materials*, Elsevier Sience Publishers Ltd., (1993)

[20] J. E. Murphy, S. A. Chin-Bing, "A finite element model for ocean acoustic propagation and scattering", *J. Acoust. Soc. Am.* 86(4), 1478-1483(1989)

[21] O. C. Zienkiewicz, R. L. Taylor, J.Z. Zhu, *The finite element method: Its basis and fundamentals*, Elsevier Butterworth-Heinemann, (2005)

[22] R. Kirby, F. D. Denia, "Analytic mode matching for a circular dissipative silencer containing mean flow and a perforated plate", *J. Acoust. Soc. Am.* 122(6), 3471-3482(2007)

[23] R. Kirby, A. Cummings, "Prediction of the bulk acoustic properties of fibrous materials at low frequencies", *Appl. Acoust.* 56(2), 101-125(1999)

[24] M. E. Delany, E. N. Bazley, "Acoustical properties of fibrous absorbent materials", *Appl. Acoust.* 3(2), 105-116(1970)

[25] S. H. Lee, J. G. Ih, "Empirical model of the acoustic impedance of a circular orifice in grazing mean flow", *J. Acoust. Soc. Am.* 114(1), 98-113 (2003)