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Jet noise: a perspective on recent developments and future directions

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Jet noise predictions can be made based on empirical correlations, methods combining RANS CFD and an acoustic analogy, or by numerical simulation. To make predictions for new designs, only the latter two approaches are viable. Recent predictions based on the RANS-based methods have shown some promise, though they are very limited in the range of conditions that have been considered. These approaches are described and differences between them are highlighted. How these methods fit into the concept of jet noise generation by two source mechanisms (fine scale and large scale) is addressed. They are also contrasted briefly with ideas based on wavepacket models of noise generation by large scale turbulent structures. Noise predictions based on direct simulation, sometimes coupled with a wave extrapolation method, offer a wealth of information. How this database can be used to identify noise source mechanisms as well provide guidance for noise reduction is discussed very briefly.

1 Introduction

For engineers, the ultimate goal in a study of jet noise is it's reduction. On the other hand, scientists are motivated by the desire to understand the fluid dynamic mechanisms for sound generation as well as develop an ability to make quantitative predictions. These two goals are generally complementary, and can involve experimental, analytical and computational studies. Engineers tend to lean towards to empirically-based methods that offer rapid turn-around for design purposes. However, predictions for designs outside existing geometries or operating conditions may be unreliable. Absolute predictions are not necessarily essential for design purposes: just an ability to predict changes relative to some base conditions. An understanding of noise generation mechanisms may be helpful in guiding noise reduction studies. However, since the noise is generally generated by a high Reynolds number turbulent flow, the likelihood of obtaining a definite and unambiguous understanding is remote. This has led to methods that replace the turbulent sources by *equivalent* sources. These methods are referred to as *acoustic analogies*. Alternative methods have been proposed that are more phenomenological in nature. Examples include *instability wave models* and *wavepacket models*. Many recent advances in predictions are being made with numerical simulations in which different ranges of the turbulent scales, and hence the noise sources, are simulated. This paper provides a brief overview of each of these approaches and also raises some unanswered questions with each approach. Due to space limitations the focus is primarily on acoustic analogy approaches.

2 Acoustic Analogies

Acoustic analogies, first introduced by Lighthill [1], are based on a rearrangement of the full compressible equations of motion into a propagation operator, acting on the selected dependent variable, and the remaining terms, which are designated as the equivalent sources. The choice of the dependent variable and propagation operator differs depending on the analogy. Lighthill selected the density fluctuation and the wave equation in a uniform medium at rest: as did Powell [2], Curle [3] and Ffowcs Williams and Hawkings [4]. Other choices for the dependent variable include the logarithm of the pressure $\pi = (1/\gamma)\ln(p/p_o)$, (Phillips [5], Lilley [6]), and the stagnation enthalpy (Howe [7], Möhring [8], Doak [9]). Most recently, Goldstein [10] and Goldstein and Lieb [11] used a nonlinear definition of the dependent variables: $p'_e \equiv p' + (\gamma-1)(\rho v'^2 - \bar{\rho}\widetilde{v'^2})/2$ and $u_i \equiv \rho v'_i$, where primes denote fluctuations about a Favre-averaged basic state, whose average is denoted by a tilde and, in this case, the propagator is represented by the linearized inhomogeneous Euler equations, rather than a single equation. It should be noted that though the dependent variable is different in each acoustic analogy, they all reduce to an acoustic variable away from the source region.

Each acoustic analogy was developed with certain aims and choices in mind. Analogies based on the wave equation in a uniform medium at rest have the obvious advantage of providing an analytical form of solution. However, all real propagation effects then had to be included in the equivalent source terms. The effects of a sheared flow on sound propagation was included in Lilley's [6] acoustic analogy. He also showed that for a homogeneous shear flow all the equivalent source terms are second order in the fluctuations. Thus the analogy reduces to the Pridmore-Brown [12] equation in the limit of infinitesimal disturbances. Goldstein [10] developed a generalized acoustic analogy in which the base flow could take any chosen form. Examples included a homogeneous base flow, a steady non-parallel base flows, and an unsteady compressible base flow. The different analogies also attempt to identify the physical source mechanisms - even though the sources are *equivalent*. For example Powell's [2] showed the importance of the Lamb vector or vortex force. The same separation of the source terms appears in the analogy developed by Morris and Farassat [13], Morris and Boluriaan [14] and Raizada and Morris [15]. However, it is unwise to assign too much in the way of physical mechanisms to equivalent sources.

The acoustic analogy approach is not without its critics. Fedorchenko [16] argued at length that there is no way to "justify the validity of this approach to the definition of aerodynamic sound sources." His concern, acknowledged by many users of the acoustic analogy approach, is the inability of acoustic analogies to provide a clear "separation between sound waves and non-acoustic disturbances." [1] However, this misses the intent of an acoustic analogy which, as it name suggests, is an *ansatz*. It's a method to develop a mathematical model that can then be tested by finding its a solution and then comparison with experiment. Since aeroacoustics is concerned with turbulent flow, such an approach seems eminently sensible. Tam [18] argued that the acoustic analogy can fail to identify noise sources. Several examples were given. But these are not necessarily cases where an acoustic analogy would be the sensible choice. In fact, situations where the source terms are known exactly provide some good examples of cases where an acoustic analogy has been successful. In the case of rotorcraft and propeller noise, the thickness and loading noise are well predicted based on surface geometry and aerodynamic loads using solutions to

[1] An interesting new approach to the separation of radiating and non-radiating components of a turbulent shear layer is described by Sinayoko et al. [17], who use a sharp wavenumber filter to perform the separation.

the Ffowcs Williams - Hawkings (FWH) equation [4] - see Brentner and Farassat [19]. Additionally, solutions to the FWH equation for a permeable acoustic data surface have been valuable in extrapolating near field numerical flow solutions to far field observers. Of course, the use of an acoustic analogy for wave extrapolation is not without its problems. Because of practical limitations, in the case of jet flows, it is necessary to close the acoustic data surface through the jet far downstream of the jet exit. This can generate false noise sources. Methods to minimize this problem are described by Shur et al. [20] and justifications for some of these choices, such as the use of a pressure fluctuation, rather than density fluctuation, in the FWH solution, are given by Morfey and Wright [21]. It should also be noted that the FWH solution is predicated on the sound radiation being linear and in uniform flow exterior to the acoustic data surface (assuming that the “quadrupole” volume integral is not included in the solution). Finally, it should be noted that if the unsteady, compressible flow solution is known exactly, then an acoustic analogy can be used to predict the noise. Samanta et al. [22] have demonstrated that some acoustic analogies are more robust than others in the actual implementation of this strategy. However, if such a solution is available, there would no need to invoke an acoustic analogy.

The most useful measure of the value of an acoustic analogy is its ability to make noise predictions. These should also be possible in a timely manner. To this end, the coupling of acoustic analogies with a Reynolds-averaged Navier-Stokes (RANS) solution, remains an attractive proposition. The JeNo method [23], which is an updated version of the original MGB (Mani, Gliebe, Balsa) prediction method [24] is an example of such a method. It couples solutions of Lilley’s equation (for a locally parallel model of the jet flow) with outputs from a RANS solver. Since the RANS solution contains no statistical information on the turbulence, it is necessary to provide a model for the two-point space-time statistics of the turbulence. These models are scaled according to length and time scales provided by the RANS solution. JeNo predictions are satisfactory for larger angles to the jet downstream axis but perform poorly in the peak noise directions, particularly for high Mach number heated jets. The same difficulty has been observed in other similar approaches (see Morris and Boluriaan [14] and Raizada and Morris [15]). Tam and Auriault [25], in their fine-scale mixing noise prediction, argue that this is because the mechanism for noise radiation in the peak noise direction is different to that at larger angles to the jet downstream axis. This particular issue remains a very important open question. However, two recent applications of acoustic analogies appear to have overcome this problem. The first is the method described by Goldstein and Leib [11] and the second is that of Karabasov et al. [26]. Both are based on Goldstein’s [10] generalized acoustic analogy (with minor variations). In both cases, the presented predictions agree well with experiments [2]. The interesting and revealing features of the predictions are the components of the models that are considered to be important in each case.

In both cases, the effect of the slow divergence of the jet mean flow is included and considered to be essential. Goldstein and Leib [11] use an asymptotic analysis to account for the slow divergence of the jet, whereas Karabasov et al. [26] use a numerical solution based on the adjoint approach introduced by Tam and Auriault [27]. Karabasov et al. use a complementary Large Eddy Simulation (LES) to provide the scaling of the modeled two-point space-time correlations of the equivalent sources, whereas Goldstein and Leib base their statistical model on hot-wire measurements (Bridges and Podboy [28]). The form chosen for the statistical models is quite different in the two cases. Karabasov et al. use a relatively simple Gaussian model whereas Goldstein and Leib use a much more complicated form involving exponential and series forms. Other differences involve whether the assumption of compactness in the the radial direction has any importance. Goldstein and Leib make this assumption, which is an essential simplification in their analysis, whereas it is not necessary in the numerical Green’s function calculated by Karabasov et al. The operating condition considered by Karabasov et al. was an unheated axisymmetric jet with $M_j = 0.75$. Goldstein and Leib considered unheated axisymmetric jets with M_j ranging from 0.5 to 1.4. In both cases the results are very impressive - especially given the limited success of previous acoustic analogy-based approaches. However, the subtle differences in the two implementations do raise interesting questions. In their calculations Karabasov et al. compare noise predictions using the full diverging jet Green’s function with predictions based on a locally parallel flow assumption for the mean jet flow field. The predictions show little difference at larger angles to the jet downstream axis, but in the peak noise direction, at 30^o to the jet downstream axis, the locally parallel flow case overpredicts the noise levels at low Strouhal numbers by 5-10dB. This is in contrast to previous predictions using the locally parallel approximation (for example, Raizada and Morris [15]) where the levels are typically underpredicted. However, it should be noted that Raizada and Morris used an isotropic form for the source term, whereas Karabasov et al. used the LES simulation to include the anisotropy of the sources. Very large differences in predictions are shown by Karabasov et al. if the radial variation of the Green’s function is neglected. But, as noted above, Goldstein and Leib make such an approximation with apparently no adverse effect - at least within the overall framework of their model. The range of conditions considered with the two approaches is somewhat limited. Consideration of higher Mach numbers and temperatures would be interesting for at least two reasons. The ability of Goldstein and Lieb’s asymptotic analysis, which accounts for the slow jet divergence, to include convectively supersonic conditions would be tested. In principle, this should not be an issue with the numerical evaluation of the Green’s function as used by Karabasov et al. Secondly, this would help to address the issue of whether there are two mechanisms for jet noire generation as proposed by Tam et al. [29]. As a matter of interest it should be acknowledged that the general acoustic analogy approach described by Karabasov et al. has now been used to predict the effect of chevrons of jet noise. This is described by Xia et al. [30] and does represent an attempt to extend acoustic analogy approaches to the study of jet noise reduction devices.

Before discussing the role of large scale turbulent structures in jet noise generation, some comments on the modeling of the two-point space time statistics of the noise sources is given. In the case of turbulent mixing noise, at least the fine-scale mixing noise, the noise sources formed in an acoustic analogy, for example, Goldstein [10], are *fluctuations* in the instantaneous Reynolds stress. It is an interesting point that the *sources* in several acoustic analogies do not have a

[2] Though the experiments selected were not the same

zero average. When an expression is then formed for the far field intensity the source terms appear as the two-point space time correlations of these fluctuations. For example, in the form,

$$R_{ijkl}(\mathbf{y}, \mathbf{\Delta}, \tau) = \overline{T'_{ij}(\mathbf{y}, t) T'_{kl}(\mathbf{y} + \mathbf{\Delta}, t + \tau)} \tag{1}$$

where,

$$T'_{ij}(\mathbf{y}, t) = -\left(\rho v''_i v''_j - \bar{\rho} \widetilde{v''_i v''_j}\right) \tag{2}$$

Models for the two point statistics are often constructed based on second-order correlations of the velocity fluctuations rather than fourth-order correlations of the fluctuations in the Reynolds stress. As noted earlier, Karabasov et al. [26] used LES simulations to help to model the fourth order statistics and Goldstein and Leib [11] used hot-wire measurements. Morris and Zaman [33], following Lighthill [32] showed that a general relationship exists between the second and fourth order statistics for the velocity fluctuations. It should be noted that this assumes incompressible flow. For example, for $i = j = k = l = \alpha$,

$$R_{\alpha\alpha\alpha\alpha} = (R_{\alpha\alpha})^2 \tag{3}$$

where, $R_{\alpha\alpha} = \overline{v_\alpha(\mathbf{y}, t) v_\alpha(\mathbf{y} + \mathbf{\Delta}, t + \tau)}$. The implication of Eq. (3) is that the fourth-order correlation has no negative loops. The is consistent with the hot-wire measurements by Morris and Zaman [33] and is also approximately true in the LES evaluation by Karabasov et al. [26]. The latter two-point correlations also included the density, which would affect the incompressible general result given by Eq. (3). The large negative loops that are seen in second-order two-point correlations less evident in the fourth-order correlations, because of the squaring effect. It should be noted that the model for the fourth-order correlations used by Karabasov et al. had no negative values, being exponential and Gaussian in nature. Based on the associated predictions, it appears that the details of the cross correlations may not be that important. However, the same is not the case for acoustic analogy predictions of broadband shock-associated noise (BBSAN).

An acoustic analogy for BBSAN has been developed by Morris and Miller [33]. The noise is generated by the interaction of turbulence in the jet shear layer with the shock cell structure that appears in the jet plume when the jet is operating off-design. In this case, the relevant correlation of the turbulent velocity fluctuations is second order. Morris and Miller chose to model the two-point correlations using a combination of exponential and Gaussian forms. The results were very encouraging and more recent applications to rectangular and dual stream jets are equally promising (see, Miller and Morris [34], [35]). However, it is noticeable that the length and time scales, derived from a RANS CFD solution, need to be much larger than those required to model the mixing noise. For example, Miller and Morris [35] include predictions for both BBSAN as well as fine-scale mixing noise, based on the models of Morris and Boluriaan [13] and Tam and Auriault [25]. The length and time scales used in the scaling of the two-point cross correlations are related to the turbulent kinetic energy K and the viscous dissipation rate ε (obtained from a RANS solution) by,

$$l = c_l K^{3/2}/\varepsilon \text{ and } \tau_s = c_\tau K/\varepsilon \tag{4}$$

The corresponding coefficients for BBSAN and mixing noise are given in Table 1.

Table 1: Time and length scale coefficients

Coefficient	BBSAN	mixing
c_τ	0.85	0.40
c_l	3.00	0.45

Perhaps this is not unexpected, as the mechanism for BBSAN involves the interaction of the turbulence with several shock cells. This mechanism is the basis of BBSAN models developed by Harper-Bourne and Fisher [36] and Tam [**?**]. In the former case the interaction is modeled by an array of point sources with phasing that depends on the convection velocity of the turbulence. In the latter case the shock cell structure is modeled as a standing wave and the turbulence is modeled as a superposition of instability waves. The interference between these two wave patterns results in BBSAN. So both of these models include coherence for the turbulence over several shock cells. It could be argued that the components of the turbulence that possesses this long range coherence are the large scale structures. These structures would manifest themselves in the two-point correlations as slowly decaying and oscillating functions. So, an improved model for the two point correlation function would consist of two components. The first would have relatively short length and time scales. This would represent the fine scale turbulence. The second would involve much longer length scales and both positive and negative loops in the correlation function. A suitable form for the space-time correlation with separation in the axial direction could be,

$$R(\xi, \tau) \sim \exp\left[-|\xi|/\bar{u}_c\tau_s - (\xi - \bar{u}_c\tau)^2/l^2\right] \cos\left[\kappa(\xi - \bar{u}_c\tau)\right] \tag{5}$$

where $\bar{u}_c$ is the convection velocity, which can be approximated by the local mean axial velocity. The location in the jet where the signature of the large scale structures is not overwhelmed by energetic small scale eddies is at the inner edge of the jet potential core. Figure 1 shows a comparison between the correlation function given by Eq. (5) and measurements by Morris and Zaman in a $M_j = 0.26$ unheated jet on the jet centerline at $x/D_j = 5.0$. The agreement is very good. The parameters used are $c_\tau = 2.5$, $c_l = 1.0$, $\bar{u} = 0.85$ and $\kappa = 3.0$. This general form of correlation function is being tested by the author to determine if it provides any improvements to the present BBSAN noise prediction scheme.

Since the proposed two point cross correlation function is assumed to represent the large scale turbulent structures, it could be argued that it could be used within the framework of an acoustic analogy to predict their radiated noise. In fact, assuming that the fine-scale and large-scale turbulent structures are independent, a possible form for the cross correlation would be,

$$R(\xi, \tau) = R_{fs}(\xi, \tau) + R_{ls}(\xi, \tau) \tag{6}$$

with obvious notation. However, if the mechanisms are indeed independent then the methods for noise prediction could be quite different. A possible approach is to use a wavepacket model.

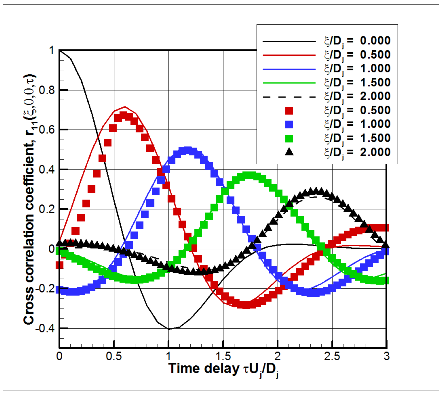

Figure 1: Second order two point cross correlation. $M_j = 0.26$, $x/D_j = 5.0$, $r/D_j = 0.0$ Symbols, measurements; lines, Eq. (6).

3 Wavepacket Models

Since the original development of instability wave models for the large scale turbulent structures a number of variants have been introduced. Of these, the wavepacket concept has become popular. A wavepacket representation includes the growth and decay of a fixed frequency traveling wave - though models that include some jittering have also been developed (Cavalieri et al [38]). On the basis of experimental observations it is possible to infer the properties of wavepackets that best represent the large scale turbulent structures and their noise radiation. A few examples include the studies by Colonius et al. [39], Reba et al. [40] and Morris [41]. The attraction of instability wave or wavepacket models is that their quasi-deterministic nature, linked to a physical phenomenon, offers opportunities for control that are not afforded by acoustic analogies. Space does not allow a complete description of the various models and experimental observations. However, at least two issues should be addressed - especially in light of the concept of the two source model of jet noise - Tam et al. [42].

Experimental evidence strongly suggests that the mechanisms of jet noise generation for both subsonic and supersonic jets are closely related. This is especially important in the peak downstream noise radiation directions. The similarity is evident in the experimental data presented, for example, by Viswanathan [43] where the radiated noise spectral shapes at small angles to the jet downstream axis are indistinguishable between supersonic and subsonic jets. However, predictions using the acoustic analogy approaches or Goldstein and Leib [11] and Karabasov et al. [26] take no explicit account of the difference between large-scale structure noise and fine-scale mixing noise. If a blending of the two approaches were to be attempted, then the possibility of *double-counting* is clear. Also, current models of large-scale structure noise based on wavepackets have been formulated primarily in the frequency domain. However, there is good experimental evidence that the events that result in noise radiation by large-scale turbulent structures are very intermittent (see Kearney-Fischer et al. [44]). This certainly complicates the modeling process.

4 Numerical Simulations

Little room remains to discuss the enormous potential and achievements of numerical simulations of jet noise. The range of operating conditions and geometries that are now amenable to numerical simulation has increased enormously in very recent years. Clearly there are computational issues that still present outstanding challenges. These include the extension of existing approaches to more complicated geometries as well as increasing the highest resolvable frequency. In the latter case it should be remembered that the dominant Strouhal numbers in terms of Overall Sound Pressure Level (OASPL) are not the most important for perceived annoyance in full scale applications. But the databases that are becoming available offer tremendous opportunities for interrogation. If confidence exists in the quality of the numerics, then any experimental technique that has been applied to the study of jet noise is *fair game* for computational studies. This includes source identification techniques using array processing (in either the jet near or far fields), flow field sampling to obtain the turbulence statistical properties, or correlations between flow properties and radiated noise. All of these opportunities are presently under investigation.

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