Active electroacoustic resonators with negative acoustic properties

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Acoustic metamaterials constitute a new class of acoustic structures, composed of periodic arrangements of engineered unit-cells, that exhibit macroscopic acoustic properties not readily available in nature. These properties can either be a negative mass density or a negative bulk modulus. However, these artificial behaviours derive from the engineered arrangement of the unit-cells, which do not present individual "meta-properties", rather than from their intrinsic nature. It is although possible to achieve intrinsic metamaterial properties out of a single unit-cell, according to active control techniques, such as direct impedance control. This paper intends to highlight the metamaterial nature of such active concepts, justifying some interesting analogies between the latter and the theory of acoustic metamaterials.

1 Introduction

The metamaterial label encompasses for structures that exhibit physical properties not readily available in nature. This concept has been extensively studied in the electromagnetic realm. Among the different drivers of the developments of such artificial structures, one of the main objective is the realization of negative physical properties, such as negative permeability or permittivity in electromagnetics (the acoustics couterpart being negative mass density and bulk modulus), or even the simultaneous combination of those properties, yielding negative refraction index[1]. Another attracting pole of such developments is the realization of cloaking capabilities, through the possibility to design heterogeneous refraction indices within a bulky 2D or 3D structures [2]. More specifically to acoustics, we are interested in structures that could present either equivalent negative mass density, or negative bulk modulus, or when combined simultaneously, negative acoustic refraction index.

In recent publications [3, 5], acoustic transmission-line metamaterials, made of periodic inclusion of series mechanical-acoustical interfaces (elastic membranes) and parallel acoustic derivation (that can either be Helmholtz resonators [4] or transverse slots) have been presented. Thanks to this specific design of transmission lines, it has been shown that consecutive negative / zero / positive refraction indices can be obtained within an audible frequency range (roughly centered around 1 kHz). The achievement of negative refraction is especially made possible by the simultaneous presence of series acoustic compliance (the membrane impedance below its resonance frequency). Such acoustic meta-properties are also underlying in some other publications, employing 1D or 2D distribution of active loudspeakers, allowing distribution of controlled acoustic impedance [6], or with distributed active transverse Helmholtz resonators [7].

The active electroacoustic absorber (or more generally electroacoustic resonator) concept reported in Ref. [8] describes a feedback-based active impedance control concept, with the capability of achieving either total absorption through controlled acoustic impedance matching, but also potentially negative absorption (through the obtention of negative acoustic impedance). This specific feature reveals the capacity of a loudspeaker to reflect more sound energy than it receives from the field (negative resistance), but also to act as a negative acoustic reactance. This property reveal some similarities with the above-mentioned concept of meta-material with negative mass density or negative bulk modulus. But rather than achieving these meta-properties from a distribution of passive acoustic impedances, this active concept present intrinsic meta-properties, allowing to present the individual active loudspeaker as a meta-material by itself. The following intends to provide preliminary hints on how such behavior can be explained by analogy with acoustic metamaterials, namely the presentation of a negative mass and stiffness of the active loudspeaker.

2 Acoustic transmission-line metamaterials with variable refractive indices

2.1 Description of an acoustic transmission-line metamaterial

In Ref. [3], a 1-dimensional acoustic transmission-line filled with air (mass density ρ and bulk modulus K) with periodic mechanical-acoustical inclusions has been proposed with a view to achieving acoustic meta-properties, labelled as composite right/left-handed transmission line (CRLH-TL). In this structure, a cylindrical host guide of internal radius a, is periodically loaded with clamped circular polyimide membranes (Young modulus E, Posson coefficient ν and mass density ρ_m) of same radius a and thickness h, alternatively with transverse radial open channels derivations ("stubs") of thickness b and length L, as described in Fig. 1. The lattice constant of such periodic arrangement is denoted d, chosen here to verify $d/\lambda = 0.1$ at $f_0 = 1kHz$, with the physical parameters values given in Table 1. This frequency has been determined as the transition frequency between negative and positive acoustic properties, and corresponds to the resonance frequency of both the chosen elastic diaphragms, and the shunt resonator constituted with each stubs and the host guide volume between two consecutive membranes.

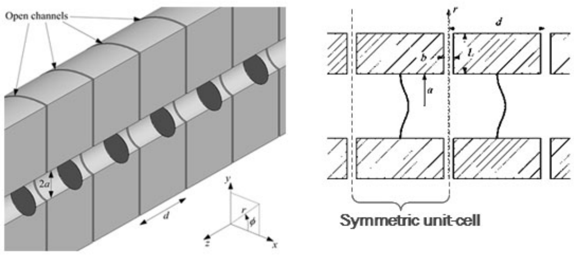

Figure 1: Acoustic transmission-line metamaterial concept, composed on periodic inclusions of elastic membranes (black disks) and radial open channels ("stubs"). Left: isometric view of the structure; right: cut-view of 2 unit-cells of the structure.

2.2 Acoustic characterization of the metamaterial

If we consider a unit-cell of the proposed transmission-line metamaterial with a symmetric convention (Π-type unit-cell), that is to say, comprised between two consecutive half-stubs (equivalent acoustic mass m_{at}), surrounding a single

Table 1: Physical parameters of the unit-cell components

Quantity	Notation	Value	Unit
Young modulus	E	2.758	GPa
Poisson coefficient	ν	0.34	(-)
density	ρ_m	1420	kg.m^{-3}
Membrane radius	a	9.06	mm
Membrane thickness	h	125	μm
Stubs length	L	42.67	mm
Stubs thickness	b	1	mm
Lattice constant	d	34	mm
air density	ρ	1.18	kg.m^{-3}
air bulk modulus	K	137.4	kPa

elastic membrane (of equivalent acoustic mass m_{am} and compliance C_{am}, limiting the discussion to lossless acoustical systems). The lumped element model for such unit-cell can be illustrated on Fig. 2. Here, each half-stub at the extremities of the unit-cell can be modeled as an equivalent shunt acoustic impedance, constituted of an acoustic mass m_{at}, and a shunt compliance C_{at}. The connecting duct section between the membrane and each half stub can be modeled as a conventional transmission-line elements, that is to say, a series acoustic mass m_{aTL} and a shunt acoustic compliance C_{aTL}. Considering the proposed values for the design of membranes and stubs described in the preceding section, the corresponding acoustic masses and compliances can be derived, according to Ref. [9]. The resulting expressions of all these components are summarized in Eq. 1:

$$\begin{array}{ll} m_{am} = 1.8830\dfrac{\rho_m h}{\pi a^2} & C_{am} = \dfrac{12\pi a^6(1-\nu^2)}{196.51 E h^3} \\ m_{at} = \dfrac{\rho}{2\pi b}\ln\left(1+\dfrac{L}{a}\right) & \\ m_{aTL} = \dfrac{\rho}{S}(d-h) & C_{aTL} = \dfrac{S}{K}(d-h) \end{array} \qquad (1)$$

The lumped-element model of one unit-cell of the CRLH TL is illustrated on Fig. 2, which "global" components m_{as}, C_{as}, m_{ap} and C_{ap} are expressed in Eq. 2:

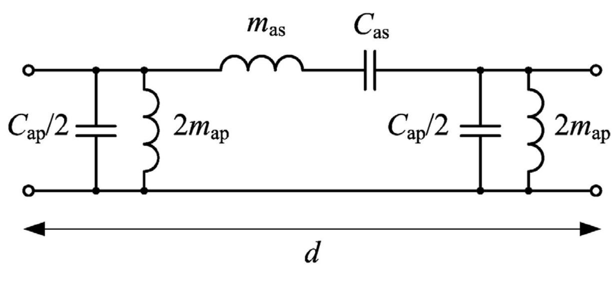

Figure 2: Lumped element model of the Π-type unit-cell of the transmission-line acoustic metamaterial

$$\begin{cases} m_{as} = m_{am} + m_{aTL} \\ C_{ap} \approx C_{aTL} \end{cases} \text{and} \begin{cases} m_{ap} = m_{at} \\ C_{as} = C_{am} \end{cases} \qquad (2)$$

The computed values of acoustical components for the unit-cell described in Ref. [3] are given in Table. 4

Table 2: Values of the lumped elements associated with the transmission line described in Ref [3].

	m$_a$ (kg.m^{-4})	C$_a$ (10^{-12} m^3.Pa^{-1})
Membrane	m$_{am}$=1296	C$_{am}$=17.42
Radial stub	m$_{at}$=348.3	
Host TL sections	m$_{aTL}$=156.1	C$_{aTL}$=63.60
Total series elements	m$_{as}$=1452	C$_{as}$=17.42
Total parallel elements	m$_{ap}$=348.3	C$_{ap}$=63.40

2.3 Meta-properties of the CRLH TL

The metamaterial can be described with the Bloch theory [3]. Among the different properties of the transmission line that can be derived from formalism are the equivalent medium parameters (equivalent mass density ρ_{eq} and bulk modulus K_{eq}), illustrated on Fig. 3 with the chosen values of parameters of Table 1, and defined as:

$$\begin{cases} \rho_{eq} = \dfrac{Z_{as}}{j\omega}.\dfrac{S}{d} \\ K_{eq}^{-1} = \dfrac{Y_{ap}}{j\omega}.\dfrac{1}{S d} \end{cases} \text{with} \begin{cases} Z_{as} = j\omega m_{as} + \dfrac{1}{j\omega C_{as}} \\ Y_{ap} = j\omega C_{ap} + \dfrac{1}{j\omega m_{ap}} \end{cases} \qquad (3)$$

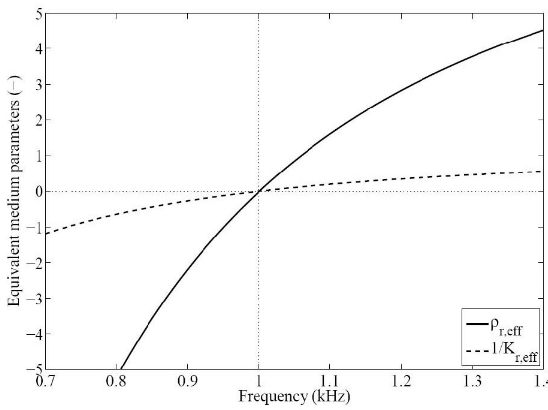

Figure 3: Equivalent mass density and bulk modulus (relative values) for the considered CRLH TL around the transition frequency

It can be seen that negative acoustic properties can be obtained on the proposed CRLH transmission line concept, below the transition frequency defined by the resonances of the series mechanical resonator and the shunt stub structure. The effective mass density presents negative values below 1 kHz, with a dependance in $1/\omega^2$. The same applies for the equivalent bulk modulus which is also negative below 1 kHz, and the dependance of which is in ω^2. These achieved negative properties are only due to the specific arrangement of passive acoustical-mechanical systems within a 1 dimensional air duct, resulting in a frequency-dependant effective medium properties:

- below f_0, simultaneous increasing negative mass density and decreasing negative bulk modulus,
- above f_0, simultaneous increasing positive mass density and decreasing positive bulk modulus

Despite the real advantage of achieving simultaneous negative properties below the transition frequency f_0, the frequency dependancy might be seen as a significant drawback of the proposed structure. The following intends to highlight an innovative manner to achieve such negative acoustic properties without frequency dependency.

3 The electroacoustic resonator with negative acoustic impedance

3.1 Description of the electroacoustic resonator concept

Let us consider a moving-coil loudspeaker, being inserted within an infinite cylindrical waveguide with same cross-section as the loudspeaker diaphragm, the front and rear faces seing the same acoustic radiation impedance Z_c=ρc (ρ = 1.18 kg.m^{-3} is the density of air, and c = 340 m.s^{-1} is the celerity of sound in the air (for a temperature of 15°C and a static pressure of 980 hPa, see Ref. [10], p.16). The Thiele-Small parameters of the loudspeaker are defined in FIG. 4 and numerical values are given in Table 3. In the following, we denote $p = p^- - p^+$ the difference between the acoustic pressure at the rear and the front faces of the loudspeaker, v the velocity of the diaphragm, e the voltage applied to the electric input of the loudspeaker, i the current circulating through the coil.

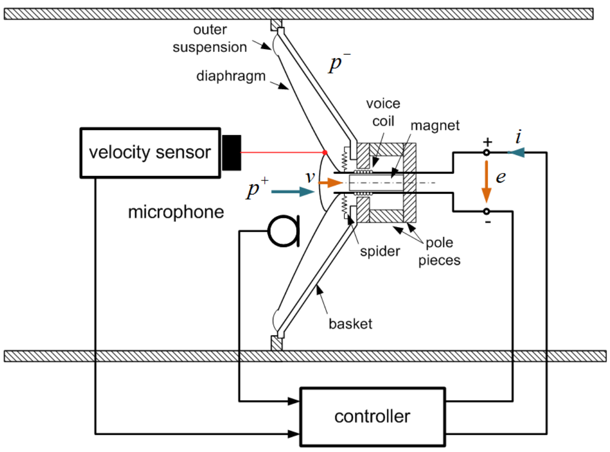

Figure 4: Sketch of an electrodynamic loudspeaker with direct impedance control.

The concept of passive or active electroacoustic resonator refers to the capability of modifying the acoustic impedance at the diaphram of the loudspeaker by either shunting its electric terminals to a dedicated complex (and even negative) electric load, or by feeding back a voltage proportional to a combination of acoustic quantities, such as described in [8] (which is summarized in the following section).

3.2 Acoustic characterization of the electroacoustic resonator

An electrodynamic loudspeaker is a linear time-invariant system that, under certain hypotheses, can be described with differential equations [10]. From Newton's law of motion on the acoustic side, and from the mesh equation on the electric side, expressed in terms of Fourier transforms, one can obtain the following equation system:

$$\begin{cases} S\,P(j\omega) = & \left(j\omega M_{ms} + R_{ms} + \dfrac{1}{j\omega C_{ms}}\right) V(j\omega) - Bl I(j\omega) \\ E(j\omega) = & (j\omega L_e + R_e) I(j\omega) + Bl V(j\omega) \end{cases}, \tag{4}$$

where P, V, E and I are the Fourier transforms of sound pressure p, air velocity at the diaphragm (opposed to diaphragm velocity) v, electric voltage e and current i, and the small signal parameters of the loudspeakers (M_{ms},) being defined in Table 3.

It is always possible to derive the system of Eq. 4 in order to express the normalized acoustic admittance of the loudspeaker face as a function of the sound pressure $P(j\omega)$ and volume flow velocity $Q(j\omega) = S\,V(j\omega)$, whatever the load or feedback at its electrical terminals:

$$Z_{as}(j\omega) = \frac{P(j\omega)}{S\,V(j\omega)}. \tag{5}$$

Now, let's consider the loudspeaker is fed back by a combination of an electrical voltage proportional to the diaphragm velocity (sensed with a laser velocimeter), and another proportional to the sound pressure at the diaphragm (through microphone sensing), as described on Fig. 4. This configuration is generally denoted "direct impedance control" in the literature [11]. The feedback voltage can then be written as:

$$E(j\omega) = \Gamma_p P(j\omega) - \Gamma_v V(j\omega) \tag{6}$$

where Γ_p and Γ_v are the gains applied on the pressure and velocity feedback voltage. With this type of active impedance control, it is shown that variable acoustic properties can readily be obtained at the loudspeaker diaphragm, the acoustic impedance of the loudspeaker taking then the form:

$$Z_{as}(j\omega) = \frac{P(j\omega)}{S\,V(j\omega)} = \frac{1}{S^2}\frac{Z_{ms}(j\omega)Z_e(j\omega) + (Bl)(Bl + \Gamma_v)}{Z_e(j\omega) + Bl\Gamma_p}. \tag{7}$$

w where $Z_{ms}(j\omega) = R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{ms}}$ and $Z_e(j\omega) = R_e + j\omega L_e$ are the mechanical and electrical impedance of the loudspeaker.

It is especially possible to identify equivalent acoustical components (m_{as},R_{as},C_{as}) out of this expression of Eq. 7. In this case, these effective electroacoustic resonator parameters can be written as:

$$Z_{as}(j\omega) \approx j\omega m_{as} + R_{as} + \frac{1}{j\omega C_{as}} \tag{8}$$

where

$$\begin{cases} m_{as} = \dfrac{M_{ms}}{S^2}\left(1 + \Gamma_p \dfrac{Bl}{S R_e}\right)^{-1} \\ R_{as} \approx \dfrac{R_{ms}}{S^2}\left(1 + \dfrac{Bl(Bl + \Gamma_v)}{R_e R_{ms}}\right)\left(1 + \Gamma_p \dfrac{Bl}{S R_e}\right)^{-1} \\ C_{as} = S^2 C_{ms}\left(1 + \dfrac{\Gamma_p Bl}{S R_e}\right) \end{cases} \tag{9}$$

This active impedance control concept can then be described as an active 1 degree of freedom acoustic resonator, the parameters of which can easily be derived from the feedback gains and the loudspeaker small signal parameters. The following will show how this formalism can highlight some interesting meta-properties, such as negative acoustic mass or negative acoustic compliance.

Note on stability: In general, assuming this simple model is valid over the frequency range of interest, it can be observed that the stability of this kind of active impedance control is ensured if velocity gain Γ_v is strictly positive. This stability criterion can be further discussed, especially when feedback gains (and consequent nonlinearities) increase, but it is generally experimentally validated, with the usual range of loudspeaker, up to feedback gains of the order of 100 (as a rule of the thumb).

3.3 Meta-properties of electroacoustic resonators

In the following, the numerical simulations will be performed on a Visaton AL-170 low-midrange loudspeaker, with enclosure volume of 10 l. The small signal parameters of the loudspeakers are given in Table 3:

Table 3: Visaton AL 170 small signal parameters.

Parameter	Notation	Value	Unit
DC resistance	R_e	5.6	Ω
Voice coil inductance	L_e	0.9	mH
Force factor	Bl	6.9	$N.A^{-1}$
Moving mass	M_{ms}	13.0	g
Mechanical resistance	R_{ms}	0.92	$N.m^{-1}.s$
Mechanical compliance	C_{ms}	1.2	$mm.N^{-1}$
Effective area	S	133	cm^2
Box volume	V_b	10	l

In Ref. [8], only positive values of acoustic impedances have been reported. This implies that the feedback gains are both positive, which induce the impedance of Eq. 7 to present equivalent positive acoustic mass, resistance and compliance. But, providing the stability criterion is respected, negative values of acoustic acoustic impedance of Eq. 9 can be achieved with such electroacoustic resonator concept, the only gain on pressure Γ_p allowing the modification of both m_{as} and C_{as}. These negative values can especially be achieved if gain Γ_p is set such that:

$$\Gamma_p < -S R_e / Bl \tag{10}$$

In such a case, the acoustic mass and compliance can be made strictly negative, the acoustic resistance decreasing as gain Γ_p increases. It is noticeable that, for gains Γ_p very close to the threshold of Eq. 10, the absolute value of mass and compliance can reach significant values. An example of a theoretically achievable (meaning stable) acoustic reactance is given on Fig. 5:

This simple example shows that an active electroacoustic resonator can be seen as a meta-material by itself, presenting

Table 4: Values of active acoustic mass and compliance for different settings of Γ_p.

Γ_p ($V.Pa^{-1}$)	m_{as} ($kg.m^{-4}$)	C_{as} (10^{-8} $m^3.Pa^{-1}$)
-0.01	998.85	1.76
-0.02	-86.17	-20.36
-0.03	-41.30	-42.47
-0.04	-27.16	-64.58
-0.05	-20.23	-86.69

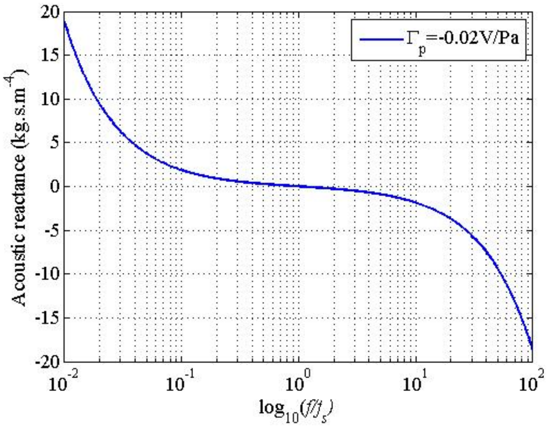

Figure 5: Example of achievable active reactance.

negative values of acoustic mass and compliance. One potential application of such result is to substitute active electroacoustic resonators for the membranes of Ref. [3], with a view to achieve constant negative mass or compliance within the frequency band of interest, or to electronically drive the series membranes for obtaining further meta-properties.

4 Conclusions

This preliminary investigation on active electroacoustic resonators intrinsic meta-materials gives some ideas for novel ways of envisaging acoustic transmission lines with negative / zero / positive refractive index, such as the one described in Ref. [3], but with the capability to either vary those properties by active means, or providing non frequency-dependant values of negative effective parameters as the ones illustrated on Fig. 3.

Acknowledgments

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