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Searching for a theoretical relation between reverberation and the scattering coefficients of surfaces in a room

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The reverberation time of a room is related to its global absorption properties through the Sabine or Eyring formula. Surfaces' scattering also influences the reverberation. However, even if the physical principles governing this influence are already known, a clear relation is missing between the "quantity" of scattering, the surfaces' scattering coefficients and reverberation. This would help to specify the scattering properties of surfaces in room acoustics projects. This paper does not give a solution to this problem, but it is rather a reflection based on existing research. It also proposes an approach based on the acoustic radiative transfer (or transport) equation. This equation is first of all expressed for room acoustics problems, especially concerning the boundary conditions which explicitly include the surfaces' scattering coefficients. It is then applied to a "diffuse" sound field and to some other simple configurations, to illustrate the possibilities of the method.

1 Introduction

It is well known that surfaces' scattering influences the reverberation in a room [1,2,3]. However, a clear relation is missing between the "quantity" of scattering, the scattering coefficients of surfaces and reverberation, something like the Sabine or Eyring formula which link absorption and reverberation. This would help to specify the scattering properties of surfaces in room acoustics projects.

A survey of the scientific literature reveals that only few studies have been dedicated to this subject. Kuttruff [4] studied the reverberation of rooms fitted with lambertian surfaces (scattering coefficient $s=1$) but this author did not consider other values of the scattering coefficient. Several authors [4,5] have analysed the relative distribution of specular and diffuse reflected energies, as a function of the (uniformly distributed) scattering coefficient. But the influence on the reverberation time was not investigated.

A tentative approach has been made by Hanyu [5] to define the "average" scattering coefficient in a room. Like the average absorption coefficient, he proposes a simple surface-weighted average. This author also defines the "diffusion time" as the "time when the ratio of specularly to total reflected energy becomes -60dB". However, these definitions and relationships rely on the assumption of exponential energy decays and, therefore, they cannot address all kinds of reverberation. In particular, there's no influence of the "average" scattering coefficient on the reverberation time in Hanyu's study.

Besides theoretical developments, some authors have also collected experimental results through computer simulations. Onaga and Rindel [6] have analysed the effect of the scattering caused by building facades on the sound levels and reverberation times in a street canyon. They show that in low-facade streets "the reverberation time is determined by the sum of absorption coefficient and scattering coefficient". In high-facade streets, the reverberation time essentially depends on absorption, except for very low scattering coefficients. In another paper by Sumarac-Pavlovic and Mijic [7], the reverberation time T30 has been computed in 52 rooms, all of them with uniform scattering coefficient and uniform absorption coefficient ($\alpha=0.1$). These authors showed that T30 values depend on the room's shape. In particular, non exponential decays were present for some rooms, for example rooms with large parallel surfaces and low scattering ($s=0$). More interesting, they defined four groups of rooms which present different average relationships between T30 and the scattering coefficient. One group in particular is characterised by significant variations showing a minimum T30 around $s=0.3$ and a maximum at $s=0$. The computed value of T30 corresponds with the Eyring's value in the completely diffuse case ($s=1$), except for one group of rooms for which T30's value can exceed Eyring's value by (at most) 20 percents. Clearly, these results can be considered as test cases, if verified. They again illustrate for some rooms the influence of scattering which is able to significantly decrease the reverberation time, even with $s<0.4$. The configurations in which this might occur are however not clearly identified, nor is the way to combine different scattering coefficients in the same room.

In this paper, we propose to apply the acoustic radiative transfer (or transport) equation to this problem. A first tentative is made to derive a theoretical relationship between the reverberation time and scattering. In section 2, the radiative transfer equation is presented and a formulation of its boundary conditions is proposed, including the scattering coefficient. In section 3, the assumption of a perfectly diffuse field is analysed in relation with this equation. In section 4, the equation is solved for rooms with lambertian surfaces. In section 5, we consider the case of mixed reflections, but the solution is only developed under specific assumptions made on the cloud of image sources.

2 The acoustic radiative transfer equation

In geometrical acoustics, the sound energy can be described by sound particles and their energy distribution function $N(\vec{r},\hat{s},t)$ in (J.m^{-3}.sr^{-1}), depending on the position $\vec{r}$ in the room, the direction of propagation $\hat{s}$ (unit vector) and the time t [8,9]. This function satisfies a differential equation called the radiative transfer (or transport) equation. In this paper we use the formulation of this equation described by Navarro et al [8].

If the distribution function is integrated for all possible directions of propagations around the position $\vec{r}$, then we obtain the sound energy density $w(\vec{r},t)$ in (J.m^{-3}). This function obeys to the following equation [8]:

$$\frac{\partial w(\vec{r},t)}{\partial t} + \nabla.\vec{J}(\vec{r},t) + mc\, w(\vec{r},t) = q_0(\vec{r},t) \quad (1)$$

in which m is the air absorption attenuation in (m^{-1}), c is the sound celerity (m.s^{-1}) and q_0 is the sound power density (W.m^{-3}) generated by the sources at the position $\vec{r}$. The sound energy flow vector $\vec{J}$ is defined by [8] as:

$$\vec{J}(\vec{r},t) = \iint_{4\pi} \hat{s}\, c\, N(\vec{r},\hat{s},t)\, d\Omega \quad (2)$$

Now, if we define $\overline{w(t)}$ as the volume-averaged energy density, integrating (1) on the whole volume of the room V gives:

$$V\frac{d\overline{w}(t)}{dt} + \int_S \vec{J}(\vec{r}_b,t)\,.\,\hat{n}_{ext}\,dS \; + mcV\,\overline{w}(t) = W(t) \tag{3}$$

In this last expression, $W(t)$ is the total power generated in the room at time t , S is the surface enclosing the volume V of the room and $\hat{n}_{ext}$ is the unit vector normal to this surface at the position $\vec{r}_b \in dS$ and directed towards the exterior of V.

Boundary conditions are necessary to solve these equations and introduce the absorption and scattering properties of surfaces. The more general form of these boundary conditions is expressed by Eq. (17) of [8]:

$$\begin{aligned} &for\ \hat{s}\in\Omega_{in}:\\ &N(\vec{r}_b,\hat{s},t) = \iint_{\Omega_{ext}} R_F(\vec{r}_b;\hat{s}',\hat{s})\,N(\vec{r}_b,\hat{s}',t)\left(\hat{s}'.\hat{n}_{ext}\right)d\Omega' \end{aligned} \tag{4}$$

The total solid angle around position $\vec{r}_b$ has been divided into Ω_{in} and Ω_{ext} containing the unit vectors $\hat{s}$ directed towards the interior and the exterior of the volume of the room, respectively. Equation (4) therefore represents the energy transported by the sound particles which are reflected by the surface at position $\vec{r}_b$, in the direction $\hat{s}$. R_F is the surface reflecting function with units of sr^{-1}. On the other hand, equation (2) gives :

$$\begin{aligned} \vec{J}(\vec{r}_b,t).\hat{n}_{ext} \; &= \iint_{\Omega_{ext}} c\left(\hat{s}'.\hat{n}_{ext}\right)N(\vec{r}_b,\hat{s}',t)\,d\Omega' \\ &\quad + \iint_{\Omega_{in}} c\left(\hat{s}.\hat{n}_{ext}\right)N(\vec{r}_b,\hat{s},t)\,d\Omega \end{aligned} \tag{5}$$

The first term on the right represents the total flux ($W.m^{-2}$) incident on the surface at position $\vec{r}_b$, while the second term is the total reflected flux multiplied by (-1). We can therefore rewrite (5) as:

$$\vec{J}(\vec{r}_b,t).\hat{n}_{ext} = \phi_{inc} - \phi_{refl} = \phi_{abs}(\vec{r}_b,t) \tag{6}$$

We can further define two groups of incident sound particles: those which have undergone at least one diffuse reflection (N_d) and those which have not (N_s). Then, from (5) and (6), the incident flux ϕ_{inc} is itself decomposed into a specular incident flux $\phi_{inc,s}$ and a diffuse incident flux $\phi_{inc,d}$. The specular reflected flux is created by the specular incident flux only, while the diffuse reflected flux is composed of the non-absorbed part of $\phi_{inc,d}$ and the diffusely reflected part of $\phi_{inc,s}$:

$$\begin{aligned} &\phi_{refl,s}(\vec{r}_b,t) = (1-s(\vec{r}_b))\,(1-\alpha(\vec{r}_b))\,\phi_{inc,s}(\vec{r}_b,t)\\ &\phi_{refl,d}(\vec{r}_b,t) =\\ &\qquad (1-\alpha(\vec{r}_b))\left[\phi_{inc,d}(\vec{r}_b,t) + s(\vec{r}_b)\,\phi_{inc,s}(\vec{r}_b,t)\right] \end{aligned} \tag{7}$$

With this model of reflection, we assume like Kuttruff [4] that “the conversion of diffuse energy into specular energy never occurs”. s and α represent the scattering and absorption coefficients respectively. Finally, introducing (7) into (6) for mixed reflections leads to $\left(\vec{J} \;=\vec{J}_s + \vec{J}_d\right)$:

$$\begin{aligned} \vec{J}_s(\vec{r}_b,t).\hat{n}_{ext} &= \alpha_s(\vec{r}_b)\,\phi_{inc,s}(\vec{r}_b,t)\\ \vec{J}_d(\vec{r}_b,t).\hat{n}_{ext} &= \alpha(\vec{r}_b)\,\phi_{inc}(\vec{r}_b,t) - \alpha_s(\vec{r}_b)\,\phi_{inc,s}(\vec{r}_b,t)\\ &= \alpha(\vec{r}_b)\,\phi_{inc,d}(\vec{r}_b,t) - s(\vec{r}_b)(1-\alpha(\vec{r}_b))\,\phi_{inc,s}(\vec{r}_b,t) \end{aligned} \tag{8}$$

with the specular absorption coefficient defined as:

$$\alpha_s(\vec{r}_b) = \alpha(\vec{r}_b) + s(\vec{r}_b)\left(1-\alpha(\vec{r}_b)\right) \tag{9}$$

3 The diffuse sound field

In a perfectly diffuse sound field, the distribution function $N(\vec{r},\hat{s},t)$ would not depend on the direction $\hat{s}$ and the sound energy density $w(\vec{r},t)$ would be the same at all positions in the room. This would imply that $\vec{J} = 0$ in (2) and, therefore, $w(\vec{r},t)$ in (1) would not depend on the surfaces’ absorption properties, but only on air absorption.

This simple observation proves that the “diffuse sound field” model is not directly compatible with the radiative transfer equation, except if all surfaces in the room are perfectly reflecting ($\alpha=0$).

It will be shown in the following section that the diffuse sound field can be considered as an asymptotic result of the diffuse reflections’ assumption.

4 Diffuse reflections

Rooms with diffusely reflecting walls do not always create diffuse sound fields [4, 5]. However, several studies have shown that in a reverberation experiment, when the sound source is cut off, the resulting time-energy decay in these rooms is close to an exponential function [4, 7, 9, 10]. Moreover, the slope is nearly identical at different positions in the room, though sometimes significantly different from the one predicted by Sabine or Eyring formula. The absolute value of the energy decay can also differ from one position to another. In a reverberation experiment, if the source is cut off at $t=0$, we therefore assume that:

$$\begin{aligned} &w(\vec{r},t) = W(\vec{r})\,e^{-\gamma t}\\ &\overline{w}(t) = \frac{e^{-\gamma t}}{V}\int_V W(\vec{r})\,dV = \overline{W}e^{-\gamma t} \qquad t > t_a \end{aligned} \tag{10}$$

The exponential decay is generally established after an initial transient period t_a.

Adequate boundary conditions can be deduced from the development of the particle distribution function $N(\vec{r},\hat{s},t)$ as a first order spherical harmonics expansion at all positions in the room, including on its surface [8]. A first order expansion is justified in a room with diffusely reflecting boundaries since the sound field is "not very far from" a diffuse field. This development gives [8]:

$$\vec{J}(\vec{r}_b,t).\hat{n}_{ext} = \frac{c}{2}\frac{\alpha(\vec{r}_b)}{2-\alpha(\vec{r}_b)}\,w(\vec{r}_b,t) = \frac{c}{4}\,\chi(\vec{r}_b)w(\vec{r}_b,t) \tag{11}$$

Note that $\chi(\vec{r}_b)$ is a function of the surface's absorption coefficient at position $\vec{r}_b$.

Applying the approximations (10) and (11) in (3) and considering that $W(t)=0$ for $t>0$ in a reverberation experiment leads to:

$$\gamma = mc + \frac{c}{4V}\int_S \chi(\vec{r}_b)\frac{W(\vec{r}_b)}{\overline{W}}\,dS \tag{12}$$

This expression is more general than the perfectly diffuse field model since it allows different absolute time-decays at different positions $\vec{r}_b$. The diffuse sound field model is recovered if the sound energy density tends to uniformity $\left(W(\vec{r}_b) \to \overline{W}\right)$ and the surfaces' absorption is low $\left(\chi(\vec{r}_b) \to \alpha(\vec{r}_b)\right)$.

In the next section, we will need to precise the relation between $\overline{W}$ and W_0, the power of the source in a reverberation experiment, if the equations (10-12) are extended up to $t=0$ $(t_a \to 0)$. In this situation, $\left(V\,\overline{W}\,\gamma\right)$ represents the total absorbed power in the room just before cutoff in $t=0^-$, which must equal the total emitted power in a stationary field:

$$W_0 = V\,\overline{W}\,\gamma \qquad (if\ t_a \to 0) \tag{13}$$

Expression (10) is the solution of the transfer equation (3) with $\vec{J}(\vec{r}_b,t).\hat{n}_{ext}$ given by (11) and the source contribution $W(t) = W_0$ for $t \le 0$ and $W(t) = 0$ for $t > 0$. Therefore, the solution of the same transfer equation for a Dirac pulse excitation $W(t) = \delta(t)$ is given by:

$$\begin{aligned} w_\delta(\vec{r},t) &= \frac{W(\vec{r})}{W_0}\,\gamma\,e^{-\gamma t} \qquad t > t_a \\ w_\delta(\vec{r},t) &= 0 \qquad\qquad\qquad t < 0 \end{aligned} \tag{14}$$

5 A model for mixed specular and diffuse reflections

At each position in the room and at every instant, we define two groups of sound particles: those which have already undergone at least one diffuse reflection (called the "diffuse" particles) and those which have not (called the "specular" ones):

$$\begin{aligned} w(\vec{r},t) &= w_s(\vec{r},t) + w_d(\vec{r},t) \\ \overline{w}(t) &= \overline{w_s}(t) + \overline{w_d}(t) \end{aligned} \tag{15}$$

We also assume that, during each reflection of a group of sound particles at time t, a given percentage $s(1-\alpha)$ of the incident flux of specular particles is transformed into a diffuse reflected flux, while the diffuse incident flux never gives rise to a reflected specular flux (see section 2).

It can be shown that both components of the sound energy density satisfy equations (1) and (3) separately. Note that once emitted by the source, the sound particles are initially "specular", since they have not undergone any diffuse reflection yet. Therefore, the source's contributions q_0 in (1) and W in (3) are included in the "specular" equation.

The specular component $w_s(\vec{r},t)$ could be expressed by the sum of the contributions of all image sources, if their position and power were calculated independently. In this section, we restrict our model to those rooms for which the cloud of image sources is approximately isotropic, at least from a certain distance from the real source. This implies that the specular particle distribution function N_s can be developed as a first order spherical harmonics expansion and that (11) holds for the specular flow vector $\vec{J}_s$. Furthermore, we consider only the rooms for which the cloud of image sources is approximately the same at all receptor's positions in the room, meaning that:

$$w_s(\vec{r}_b,t) \cong \overline{w_s}(t) \tag{16}$$

With these two assumptions on the cloud of image sources, the expression of the reverberation decay can be obtained from (3), (11) and (16):

$$\begin{aligned} &\frac{d\,\overline{w_s}(t)}{dt} + \gamma_s\,\overline{w_s}(t) = 0 \qquad t > t_b \\ &\gamma_s = mc + \frac{c}{4V}\int_S \chi_s(\vec{r}_b)\,dS\ , \quad \chi_s = \frac{\alpha_s}{1-\alpha_s/2} \end{aligned} \tag{17}$$

As can be seen in (17), the validity of the two assumptions is restricted to those image sources which are distant from the real source by at least ct_b. The solution of (17) is given by:

$$\overline{w_s}(t) = \overline{w_s}(t_b)\,e^{-\gamma_s(t-t_b)} \qquad t > t_b \tag{18}$$

The diffuse component $w_d(\vec{r},t)$ can be obtained by the following analysis. First, we still consider that the diffuse particle distribution function N_d can be developed as a first order spherical harmonics expansion. Since N_d and N_s verify this approximation, so is their sum. Therefore, as explained in section 4, $\vec{J}.\hat{n}_{ext}$ is approximated by (11). Secondly, considering that $\vec{J}_d = \vec{J} - \vec{J}_s$ leads to:

$$\begin{aligned} \vec{J}_d.\hat{n}_{ext} &= \frac{c}{4}\left(\chi(\vec{r}_b)\ w(\vec{r}_b,t) - \chi_s(\vec{r}_b)\,w_s(\vec{r}_b,t)\right) \\ &= \frac{c}{4}\left(\chi(\vec{r}_b)\ w_d(\vec{r}_b,t) - \left(\chi_s(\vec{r}_b) - \chi(\vec{r}_b)\right)w_s(\vec{r}_b,t)\right) \end{aligned} \tag{19}$$

The diffuse energy density $\overline{w_d}(t)$ therefore satisfies the following equation:

$$V\frac{d\,\overline{w_d}(t)}{dt}+\frac{c}{4}\int_S \chi_{\vec{r}_b}\,w_d(\vec{r}_b,t)\,dS+mcV\overline{w_d}(t) = \frac{c}{4}\int_S (\chi_s-\chi)_{\vec{r}_b}\,w_s(\vec{r}_b,t)\,dS \;\cong K_s\,\overline{w_s}(t) \tag{20}$$

The solution of (20) is that of a diffuse sound field with a source contribution of the type $W(t)=K_s\,\overline{w_s}(t)$. Applying the "impulse" solution (14) gives:

$$w_d(\vec{r},t)=K_s\int_{-\infty}^{\infty}\overline{w_s}(\tau)\,w_\delta(\vec{r},t-\tau)\,d\tau \tag{21}$$

In the following, the solution of this integral is developed in the case of a rapidly established exponential decay in the diffuse field, i.e. $t_a \rightarrow 0$. For $t>t_b>0$, applying (13) and (14) leads to:

$$\overline{w_d}(t)=\overline{w_d}(0)\,e^{-\gamma t}+\frac{K_s}{V}\int_0^{t_b}\overline{w_s}(\tau)\,e^{-\gamma(t-\tau)}\,d\tau + \frac{K_s}{V}\int_{t_b}^{t}\overline{w_s}(t_b)\,e^{-\gamma_s(\tau-t_b)}\,e^{-\gamma(t-\tau)}\,d\tau \tag{22}$$

The first integral can be neglected if $t>>t_b$, which finally leads to:

$$\overline{w_d}(t)=\overline{w_d}(0)\,e^{-\gamma t} + \frac{K_s}{V}\,\frac{\overline{w_s}(t_b)}{\gamma-\gamma_s}\left(e^{-\gamma_s(t-t_b)}-e^{-\gamma(t-t_b)}\right) \tag{23}$$

Adding (18) and (23) gives the average sound energy density $\overline{w}(t)$ as the sum of two exponential decays:

$$\overline{w}(t)=\overline{w}(t_b)\,\left(\mu\,e^{-\gamma_s(t-t_b)}+(1-\mu)\,e^{-\gamma(t-t_b)}\right) \qquad t>t_b$$
$$\mu=\frac{c\,\overline{w_s}(t_b)}{4V(\gamma-\gamma_s)\overline{w}(t_b)}\int_S \chi(\vec{r}_b)\left(\frac{W(\vec{r}_b)}{\overline{W}}-1\right)dS \tag{24}$$

The influence of the scattering coefficient on the sound energy decay and the reverberation is contained in the parameter μ, through γ_s and the ratio $\overline{w_s}(t_b)/\overline{w}(t_b)$. If all surfaces in the room are diffuse reflectors $(s\rightarrow 1)$, then this last ratio and μ tend towards zero and the model expressed by (10) and (12) is retrieved. If however all surfaces are specular reflectors, $(\mu\rightarrow 1)$ and the model expressed by (18) is retrieved.

The parameter μ in (24) must always be comprised between 0 and 1, otherwise some unexpected behaviours could appear in the graph of $\overline{w}(t)$ (negative values or increasing segments).

The influence of scattering also depends on the uniformity of the sound field when the room is limited by diffusely reflecting boundaries: if $\left(W(\vec{r}_b)\rightarrow\overline{W}\right)$, then $(\mu\rightarrow 0)$ and both exponential decays have the same slope. In that case, we retrieve the diffuse field results.

Therefore, according to (24), the scattering coefficient can have an influence on the slope of the decay and the reverberation time only in rooms where the sound energy density along the boundaries is different from the average energy density, if their surfaces are diffuse reflectors. Remember also that (24) holds for rooms which have an isotropic and uniform cloud of image sources.

To verify these first conclusions, some computations have been performed with a ray-tracing program [11] in a cubic room, which is known to approximately fulfil this last condition about the image sources' distribution. Figure 1 illustrates the results obtained in a cubic room with *L=10m*, *α=0.1* for the six walls, a point source is placed at the centre of the cube and 124 receptors are uniformly distributed in the volume of the room (in fact, the whole volume has been divided into 125 identical smaller cubes, with a receptor at the centre of each of them, except at the source's position). 10^7 rays have been emitted by the source such that the reverberation times T30 are significant up to two decimal places (estimated random error = 0.01s). Figure 1 shows the mean values of T30 obtained by averaging for all receptors. In this cubic room, the reverberation decays are approximately linear (in dB) and the T30 values are nearly identical at each receptor's position (differences less than 0.02s).

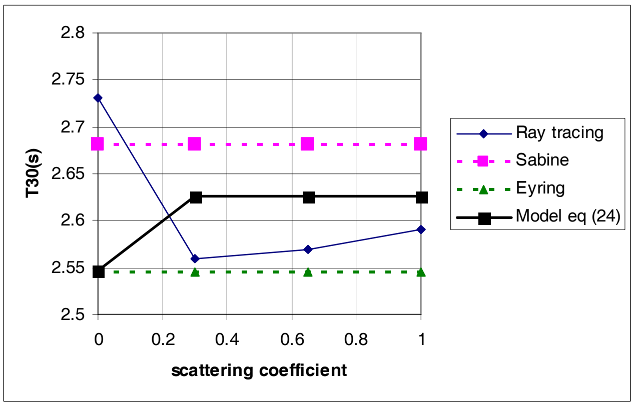

Figure 1: Reverberation time T30 computed by four models in a cubic room with *L=10m* and *α=0.1*. The scattering coefficient is the same for all surfaces and its value is one of the following {0, 0.3, 0.65, 1}.

The "ray tracing" curve in figure 1 shows a continuous significant decrease of T30 from *s=1* to *s=0.3* (the scattering coefficient is the same for all surfaces). Then, the reverberation time suddenly rises for *s=0*. This behaviour is similar (though in a less extent) to the results presented by Sumarac-Pavlovic and Mijic [7]. An explanation for this sudden increase for low scattering coefficients is proposed in the following.

To compute T30 values with the model (24), we need some more approximations. For example, the ratio $W(\vec{r}_b)/\overline{W}$ which appears in the expression of γ in (12) and μ in (24) will be approximated in the cubic room by identifying in the ray tracing results the SPLs computed at those receptors which are the closest from the cube's envelope. Another example is the ratio $\overline{w_s}(t_b)/\overline{w}(t_b)$ which must be estimated as a function of the scattering coefficient (space is missing to develop and describe this estimation in this paper) .

The model (24) predicts a continuous decrease of T30 from *s=1* to *s=0* in the cubic room with α*=0.1*. The *s=1* value is determined by the exponential constant γ in (12). The average value of the ratio $W(\vec{r}_b)/\overline{W}$ on the surface equals 0.97, which leads to a reverberation time T30=2.62s in figure 1. This prediction is a bit closer to the ray tracing value than Sabine and Eyring's values. The *s=0* value is determined by the exponential constant γ_s in (17). It would also be a good prediction of the ray tracing results if the T30 values were still continuously decreasing between *s=0.3* and *s=0*. However, the predictions of the model (24) are not suitable between *s=0* and *s=1*, suggesting that the approximation that we have used for the ratio $\overline{w_s}(t_b)/\overline{w}(t_b)$ is not adequate.

Figure 2 shows the same results obtained in the same cubic room, but with α*=0.3*. The conclusions are similar.

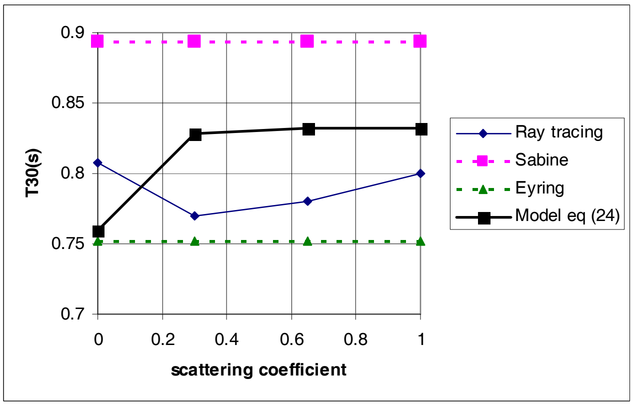

Figure 2: Same as figure 1 with α*=0.3*.

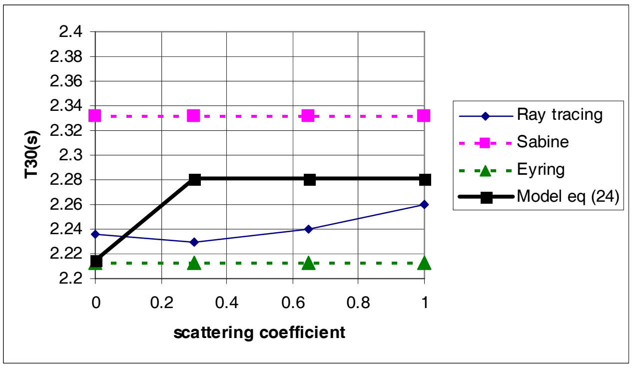

Figure 3: Reverberation time T30 computed by four models in an irregular room (V=733m³, S=506m²) and α*=0.1*. The scattering coefficient is the same for all surfaces.

An explanation of the differences observed between the ray tracing model and (24) at low scattering values could be that large parallel surfaces create image sources in well defined directions, which obviously contradicts the assumptions that lead us to the specular contribution (18). To verify this, we computed T30's values in an irregular room with approximately the same volume and surface than our cubic room. The results illustrated in figure 3 clearly show that the increase of T30 for *s=0* has almost disappeared (compare with figure 1).

6 Conclusion

The acoustic radiative transfer equation is a possible approach to model the influence of the scattering coefficient on the reverberation time. However, much research work must still be done to find an adequate formulation of the model, particularly for:

- the parameter μ in (24),
- the contribution of large parallel surfaces which should be isolated,
- the contribution of surfaces with different scattering properties.

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