Matryoshka locally resonant sonic crystal

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The results of numerical modelling of sonic crystals with resonant array elements are reported. The investigated resonant elements include plain slotted cylinders as well as various their combinations, in particular, Russian doll or Matryoshka configurations. The existence of a separate attenuation mechanism associated with the resonant elements, which increases performance in the lower frequency regime has been identified. The results show a formation of broad band gaps positioned significantly below the first Bragg frequency. For low frequency broadband attenuation a most optimal configuration is the Matryoshka sonic crystal, where each scattering unit is composed of multiple concentric slotted cylinders. This system forms numerous gaps in the lower frequency regime, below Bragg bands, whilst maintaining a reduced crystal size viable for noise barrier technology.

1 Introduction

Recent years have seen a growing interest in sonic crystals [1, 2] and their potential for use as noise barriers, with reported sound attenuation up to 20 dB [3] and 25 dB [4]. An advantage of sonic crystal noise barriers is that, by varying the distance between the scatterers, it is possible to attain peaks of attenuation in a selected frequency range. The relationship between the lattice parameter and operating frequency suggests extremely large barriers will be required to attenuate lower frequency noise such as traffic. We investigate the effects of elastic wave propagation through a new class of locally resonant sonic crystal (LRSC) with multiple acoustic resonances, capable of broadening the range of attenuation. The proposed LRSC forms broad attenuation bands in the lower frequency regime and comprises concentric slotted cylinders. The results of this work are presented in full in Ref [5]. Wave propagation in a sonic crystal with a Helmholtz resonator defect was studied by Wu *et al.* [6], where the Helmholtz resonator is placed as a point defect of the sonic crystal and exhibits local resonance phenomena. Movchan *et al.* investigated the asymptotic analysis of an Eigenvalue problem for the Helmholtz operator in a periodic structure involving split-ring resonators and associated multistructures where the position of stop bands were deduced [7]. It has been found that the behaviour of our resonators with a large slot is best described by the standard formula for Helmholtz resonators [8] even when arranged concentrically.

In the present paper an array of the resonant elements that have broad resonances below the Bragg band gaps have been studied. In particular the elements having a shape of slotted cylinders and concentric configurations have been considered. The interaction between their resonances produces band gaps and gives rise to phenomena that can lead to acoustic attenuation. The continuum band of the surrounding effective medium interacts with resonance states by hybridization (mixtures of different waves states) giving rise to hybridization gaps such as those found in three dimensional solid phononic crystals [9] and experimentally in colloidal films [10].

2 C-shaped LRSC

2.1 Eigenvalue Analysis

First we consider infinite arrays of locally resonant scatterers. Their band structures are obtained using the FEM which was developed in the framework of Comsol Multiphysics [11]. An advantage of using Comsol Multiphysics to compute the acoustic band structure is the capability of modelling more complex scatterer geometries. The structure is assumed to be infinite and periodic in the direction x with the period a_1 and in the direction y with the period a_2 and described by two basis vectors: $(a_1, 0)$ and $(0, a_2)$. According to the Floquet-Bloch theorem, the relation for the pressure distribution p for nodes lying on the boundary of the unit cell can be expressed as:

$$p(\vec{x} + \vec{a}_1 + \vec{a}_2) = p(x)\exp[i(k_x a_1 + k_y a_2)], \qquad (1)$$

where x is the position vector in the unit cell and $\vec{k} = (k_x, k_y)$ is the Bloch wavevector. Considering the periodic boundary conditions above allows the reduction of the model to a single unit cell see Figure 1. Periodic boundary conditions are applied to truncate the two-dimensional simulation plane in the x and y directions, by reducing the system to one unit cell. For a LRSC in a two-dimensional square array, the unit cell (seen in Figure 1) is used as a basis for the calculations. Analysis of the first ten Eigenfrequencies and the corresponding Eigenvectors is computed. The Eigenvectors are related to the pressure distribution of the mode. Figure 2 gives the computed band structure of a two-dimensional locally resonant sonic crystal, comprising slotted tubes with inner radius 5 mm, external radius 6.5 mm and slot width 4 mm arranged in a square lattice in air. The period is 22 mm. We call each resonating inclusion, a C-shaped resonator.

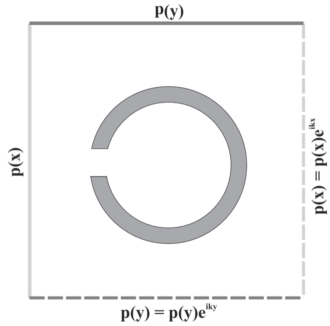

Figure 1: Unit cell for a C-shaped locally resonant sonic crystal with Floquet-Bloch boundary conditions described.

We note the appearance of a flat band in the band structure (Figure 2). Modes associated with a flat band should have a group velocity equal to zero and exhibit strong spatial localisation. In practice, such localised modes are often created by inserting a defect in a periodic structure, i.e. creating a cavity [6]. It is clear that the acoustic resonance owing to the C-shaped inclusions leads to the appearance of this flat band, forming a complete acoustic band gap that is induced by the local acoustic resonance of each individual scatterer.

The introduction of the extra, flat resonance band could lead to the construction of viable acoustic barriers in the low frequency regime, which offer sound attenuation in all crystal lattice planes. The flat band (originating from the localised acoustic resonance seen in the band structure) is a large anticrossing gap; this is generally referred to as a hybridization gap in the context of sonic crystals [9]. Hybridisation band

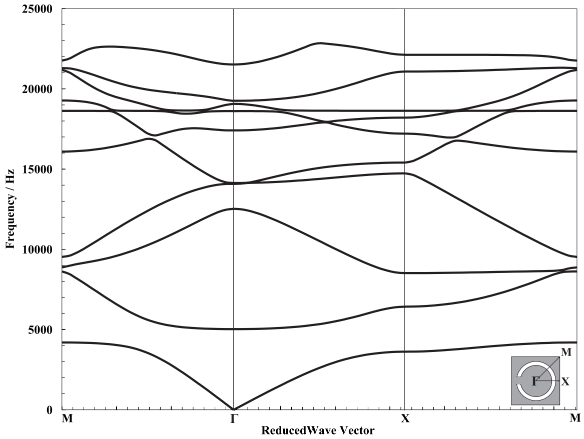

Figure 2: Finite Element computed band structure for a C-shaped locally resonant sonic crystal.

gaps can form if the scattering objects constituting a sonic crystal have strong resonance states in the frequency range of interest. Such sonic crystal systems are best thought of as an analogy of a tight-binding system of single resonators [12]. The identical resonators all have the same resonance frequencies ω_{res} if they are independent of each other. If interactions are allowed we can expect a degenerate state to form symmetrically around ω_{res}. In the dispersion relation this interaction occurs at the intersection of the flat resonance band with the linear continuum band at frequencies close to ω_{res}. This frequency is independent of the structure of the sonic crystal. The resonant frequencies are mainly affected by the size and internal wave velocity of the scatterers. In general, the two scattering regimes can overlap in a sonic crystal. Resonance scattering occurring in the same frequency range as Bragg scattering favours the formation of broad (and therefore more likely omnidirectional) band gaps.

2.2 Transmission Analysis

The finite element method has been utilised to calculate the pressure field behind a sonic crystal and to generate a pressure map of the system at fixed frequencies. The Comsol Multiphysics software is adopted to solve the acoustic wave propagation in the sonic crystals. The equation used to analyse the acoustic wave problems is expressed as

$$\frac{1}{\rho_0 c^2}\frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left(-\frac{1}{\rho_0}\nabla p\right) = 0. \quad (2)$$

This reduces to a Helmholtz equation for a time harmonic pressure wave excitation, $p = p_0 e^{i\omega t}$

$$\nabla \cdot \left(-\frac{1}{\rho_0}\nabla p_0\right) - \frac{\omega^2 p_0}{\rho_0 c^2} = 0, \quad (3)$$

where $\omega = 2\pi f$ is the angular frequency. By solving equation (3), the pressure field can be obtained.

A two-dimensional locally resonant sonic crystal system in a 10×10 square lattice is described in Comsol Multiphysics, with lattice parameter a = 22 mm, cylinder radius r_o = 6.5 mm r_i = 5.0 mm and slot width s = 4.0mm. Material parameters for this system are ρ_s = 7800 kgm^{-3}, c_s = 6100 ms^{-1}, ρ_a = 1.25 kgm^{-3}, c_a = 343 ms^{-1} where subscripts denote the internal and external radius and air domain and steel scatterers. In the case of the rigid scatterers in the LRSC system, sound-hard boundary conditions have been applied; i.e. the normal component of the velocity of the air particles is zero in the walls of the cylinders. The radiation boundary conditions at the exterior edges of the rectangular domain are considered to be perfectly absorbing. In the simulations, a rising tone noise source at the left edge of the domain, from 0 – 30000 Hz, is modelled as a radiation condition with pressure source set to 1 Pa, which is equivalent to a 90 dB source. For the numerical simulation, we use a triangular mesh of approximately 10^6 elements, with at least ten elements per wavelength to solve the wave equation across the domain.

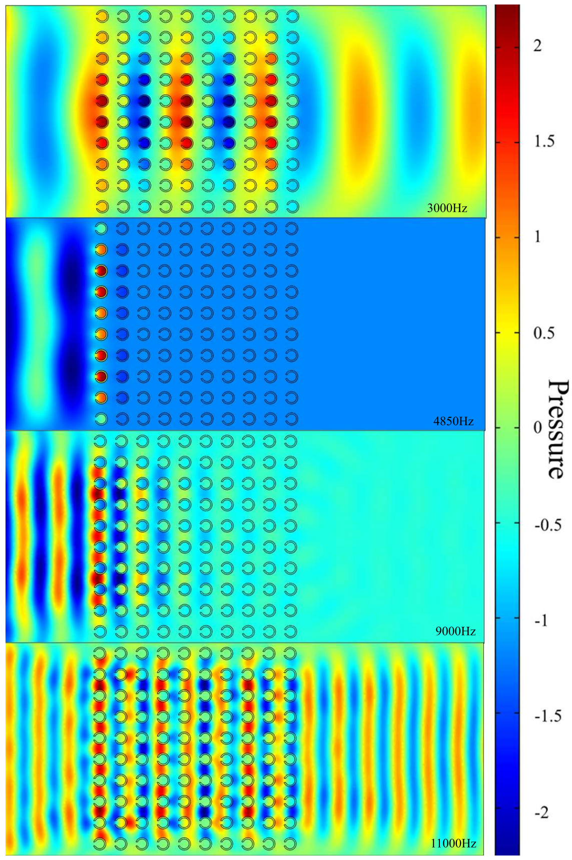

Figure 3: (Colour online) Finite Element computed pressure level maps for a C-shaped locally resonant sonic crystal at 3000, 4850, 9000 and 11000 Hz.

Computed pressure maps, taken at four frequencies of interest, demonstrate the propagation of an acoustic plane wave through the C-shaped LRSC. At frequencies below the active frequency (3000 Hz), the incoming wave propagates as if the system was a homogeneous medium. At 4850 Hz the computed pressure map shows that the C-shaped LRSC attenuates the wave in this region. The pressure map, see Figure 3, indicates that regions of maximum pressure are localised to the inclusions, at the resonance frequency. Above the resonant frequency around 6500 Hz, the acoustic wave is free to propagate through the system. As we approach the Bragg band gap frequency of 9000 Hz, again we see the appearance of band gap attenuation. Looking at the pressure maps it becomes apparent that the two regions of attenuation are controlled by two different mechanisms; resonance and Bragg scattering.

In the transmission spectrum, see Figure 4, additional attenuation peaks can be observed (~20000 Hz). If we compare the location of these peaks with the computed band structure, the peaks can be attributed to anticrossing regions present in the band structure. In general, such gaps originate from

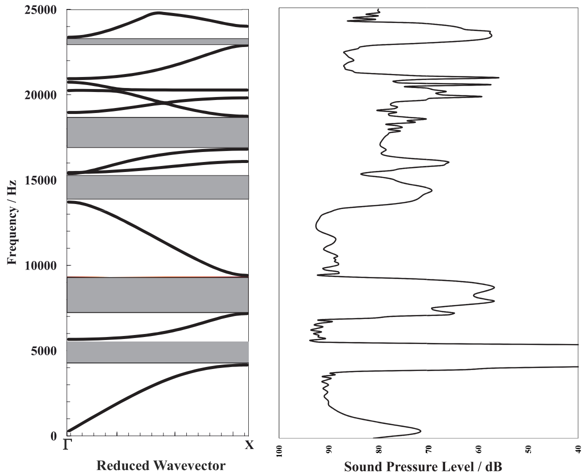

Figure 4: A comparison between Finite Element computed band structure and Finite Element transmission simulation for a C-shaped locally resonant sonic crystal.

level repulsion, when two bands of the same symmetry avoid crossing each other. The appearance of these anticrossing regions are beyond the scope of this investigation, but should be investigated further to enhance the performance of the C-shaped LRSC. The reader is directed toward a seminal paper by Wu *et al.* [13] detailing this phenomenological effect. The physical origin of these anticrossing gaps, is different when compared with those induced by the acoustic resonance.

Figure 5, presents a comparison between the finite element computed band structure, transmission spectrum and that obtained using the experimental methods [15]. The overall agreement in general is good; the two distinct regions of attenuation are presented by both spectra and corresponds well to the computed band structure. The small increase in Bragg band gap width are most likely to be attributed to the finite crystal size effects in the transmission results, we can use an analogy to line broadening in x-ray diffraction in finite crystal samples [14], to explain the increased band gap widths obtained from that in the transmission measurements.

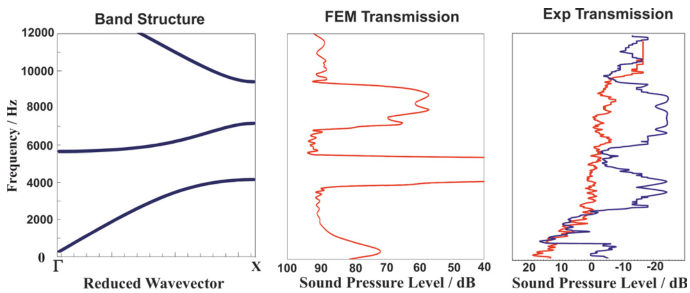

Figure 5: A comparison of Comsol computed band structure and transmission measurements with experimental transmission experiments for an locally resonant sonic crystal system.

3 Matryoshka LRSC

The C-shaped tubes act as acoustic resonators which give rise to a single flat band that extends across all high symmetry directions and is located below the Bragg gap. Its position is dependent upon the cavity dimensions and is independent of the sonic crystal periodicity. For practical applications of sonic crystals as noise barriers it is desirable to be able to broaden the width of this resonance gap. One method to achieve this is to include multiple resonator sizes and overlap the individual resonance peaks. We have investigated mixed arrays that display this ability [15] however, in order to save space and reduce the overall barrier thickness we now propose a design of sonic crystal with resonators placed concentrically inside one another, extending the multistructure describe by Movchan *et al.* [7]. We coin this the Matryoshka (Russian doll) configuration. Specifically we investigate a Matryoshka LRSC whose unit cell is defined with six concentric C-shaped resonators, all tuned to frequencies that lie within 200 Hz of each other in the low frequency regime. This is achieved by increasing the dimensions of the resonating inclusions and lattice parameter (see Figure 6).

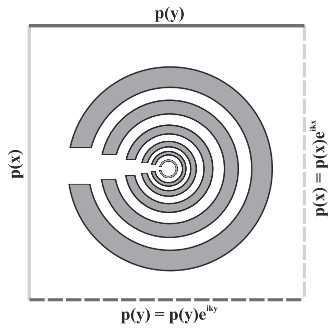

Figure 6: Schematic of the unit cell used in band structure calculations for the six concentric Matryoshka system.

Applying periodic boundary conditions, to replicate an infinite array of these concentric inclusions in a square array with lattice parameter a = 15.5 mm, the acoustic band structure can be computed. The dimensions of each C-shaped resonator are designed so that they can be placed concentrically inside each other. The largest C-shaped resonator has an, external diameter = 132 mm, and an internal diameter = 109 mm with a slot width 31 mm. Subsequent concentric resonators have a scale factor of $\frac{1}{1.3}$, giving the smallest of the nested resonators an external diameter = 22.5 mm, and an internal diameter = 14.1 mm with a slot width 11.3 mm. Figure 7 presents the Finite Element computed band structure in all high symmetry directions.

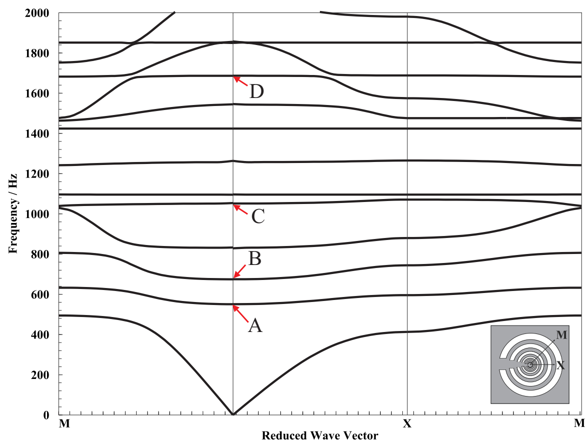

Figure 7: Finite Element computed band structure for the six concentric Matryoshka system. A–D correspond to the Eigenmode pressure distribution in Figure 8.

The band structure has been computed in the low frequency regime < 2000 Hz, corresponding to the first thirteen Eigenvalues, by varying the wave vector in the first Brillouin zone. It can be seen that a Matryoshka system, with many individual resonating units, induces the formation of multiple band gaps. Due to the periodic nature of these inclusions, this sonic crystal system possesses the characteristic Bragg band gaps, although it is hard to identify which bands are attributed to the separate band gap formation mechanisms from the band structure alone. A conventional sonic crystal system with a lattice parameter a = 15.5 mm should possess a Bragg band gap around 1120 – 1360 Hz, therefore the other band gaps present in the band structure must be caused by the acoustic resonance of each C-shaped inclusion. It can be seen that the induced resonance band gaps are complete acoustic band gaps, inhibiting wave propagation across all lattice planes, without the need for a large packing fraction as found with the characteristic Bragg band gap. For com-

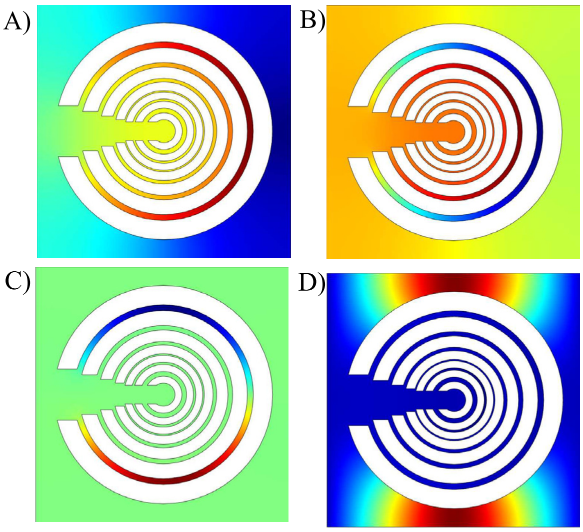

Figure 8: (Colour online) Finite Element computed Eigenmode pressure distribution: *A*) indicating individual resonance of largest resonator *B*) second largest resonator, *C*) multiple harmonic resonance of largest resonator, and *D*) propagating mode, indicating a Bragg gap edge.

pleteness, Finite Element Methods have been employed to obtain a transmission spectrum for this array, see Figure 9. A 10×10 array of the Matryoshka inclusions (each containing six concentric C-shaped resonators) is described in Comsol. The spectrum extends to 2000 Hz, and demonstrates the appearance of multiple regions of attenuation, owing to the individual resonances of the six C-shaped resonators as well as a Bragg band gap.

The fact that we have large slot sizes which are aligned concentrically with the same orientation, means that the asymptotic model in Ref [7] does not hold for our alternative structure. We can use a modified Helmholtz resonator equation [8] to predict the location of each resonance gap, attributed to the different sized inclusion. A comparison of the band gap locations from Finite Element computation to the modified Helmholtz resonator equation can be seen in ref [5] showing good agreement.

Figure 8 presents the corresponding Eigenmode pressure diagrams computed for the first two bands A) and B) respectively. It can be seen that each individual resonator experiences an increase in pressure inside the cavity, caused by the acoustic resonance of each C-shaped inclusion. It should be noted that multiple harmonic resonance gaps are formed, shown in C) and induce the formation of extra gaps in the band structure. We can confirm the existence of a Bragg gap, by again studying the Eigenmode pressure distribution D). It is clear to see that this band is attributed to a propagated mode. This allows us to confirm the band gap formation mechanism, either resonance or Bragg that is responsible for each region of attenuation present in the frequency spectrum.

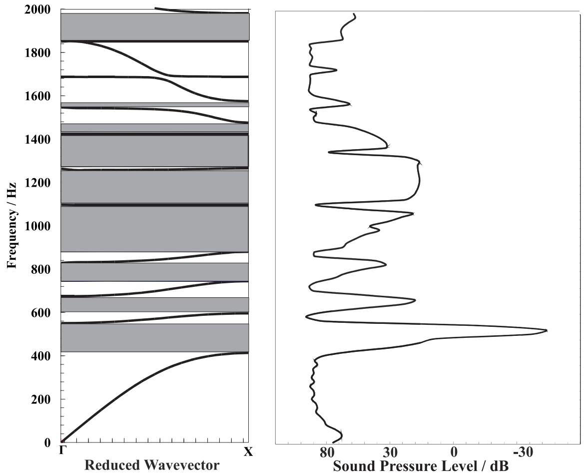

Figure 9: A comparison of the Finite Element computed band structure with the Finite Element computed frequency spectrum for a six concentric Matryoshka system.

For comparison, Figure 9 shows both the Finite Element computed band structure, limited to the ΓX direction, and the computed frequency spectrum. The frequencies at which the band gaps occur in the band structure are in good agreement with the regions of attenuation present in the transmission spectrum. A small attenuation band is present in the transmission spectrum at around 1700 Hz. At the corresponding frequency in the band structure, an anticrossing region appears, induced by the level repulsion effect. Since the resonances are very close in frequency to the frequency that satisfies the Bragg condition, the two band gap regimes appear to overlap in this Matryoshka sonic crystal. Resonance scattering occurring in the same frequency range as Bragg scattering favours the formation of broad band gaps.

4 Conclusion

The proposed Matryoshka LRSC offers a viable solution to overcome the inherent dependence on spacing experienced with conventional sonic crystal designs. It has been discovered that such systems can form multiple resonance band gaps in the lower frequency region, below that of Bragg formation. These resonance bands can be combined to form broad regions of attenuation, either by selecting close acoustic resonances or further by tuning the structure to combine the characteristic Bragg band gap with the resonance band gaps. This configuration is particularly suited for noise barrier applications.

Acknowledgments

be found at http://link.aip.org/link/?JAS/130/2746/

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