Vibroacoustics of thin micro-perforated sound absorbers

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Lightweight Micro-Perforated Panels (MPP) backed by an air cavity constitute compact sound absorbing resonators, mostly efficient in the mid-frequency range, and that may be constructed using transparent, fibreless and recyclable materials. These soundproof devices have been intensively studied due to their important applications in building acoustics and the aeronautic, astronautic and automotive industries. However, MPPs have been often considered as rigid structures. The work presented here is a theoretical and experimental study on the influence of panel vibrations on the sound absorption and transmission properties of thin MPPs. Measurements show that the absorption performance generates extra absorption peaks or dips that cannot be understood assuming a rigid MPP. A theoretical framework is presented that exactly accounts for structural-acoustic interaction between the MPP and the cavity for general cross-sectional shapes and panel boundary conditions. This model is validated against experimental data acquired from impedance tube, laser vibrometric scans of the panels surface and transmission loss measurements. Coupled-mode analysis explains the nature of the observed spectral peaks for a wide range of perforation ratio and cavity depth.

1 Introduction

The prediction of the isolating properties of absorbing materials is a subject that has been intensively studied due to their important applications in a wide range of areas, such as building acoustics and the aeronautic, astronautic and automotive industries. Micro-Perforated Panel Absorbers (MPPAs) constitute a new type of resonance absorbers with a huge potential (mass efficient, fibreless, transparent, recyclable) in comparison with conventional porous materials. Typical MPPAs are composed of a Micro-Perforated Panel (MPP) with sub-millimetric holes backed by a cavity. The depth of the cavity is chosen to build the Helmholtz-type resonance while the thickness, the size of the perforation and the perforation ratio of the MPP determine the maximum of absorption and the performance frequency range of the MPPA [1].

In most previous analyses whether for simple or multi-layer devices, MPPs have been often considered as infinite rigid structures, eventually accounting for inertia but neglecting any vibrating effects. However, simulation and experimental studies [2,3] on thin structures have found that the absorbing performance can experience variations in the low frequency range from the results expected using the sound absorbing model initially proposed by Maa for a rigid MPP [1].

The aim of this work is to investigate the physical mechanisms that relate the acoustic and elastic properties of thin MPPAs of finite size section. The vibroacoustic properties of thin MPPAs are studied from a modal theoretical framework and from a set of experiments. Of special interest is the nature of the observed absorption peaks dominated by either Hole-Cavity (HC) or Panel-Cavity (PC) resonances. Further insight is gained from coupled mode analysis on the effect of the panel vibrations on HC resonances and on the effect of the micro-perforations on PC resonances.

2 Vibroacoustic model

An analytic model is set out to investigate the absorption and vibroacoustic properties of thin MPPAs with general cross-sectional shape and boundary conditions. Figure 1 shows a cavity-backed MPP of arbitrary cross-sectional surface \( S \) and depth \( d \). It is excited by a normal incident plane wave of amplitude \( p_0 \) and time-dependence \( e^{jωt} \). The acoustic pressure at a location \( (x,z) \) inside the cavity and the vibration velocity at a location \( x \) on the MPP panel are expressed as

\[
p(x,z;ω) = \sum_{n=0}^{N-1} a_n(z;ω) φ_n(x) = \Phi^T_a z, \quad \text{(1)}
\]

\[
v(x;ω) = \sum_{n=1}^{M} q_n(ω) ψ_n(x) = ψ^T_q, \quad \text{(2)}
\]

where \( ψ \) and \( q \) are respectively the \( M \)-length column vectors of the panel structural modes and the unknown amplitudes of the corresponding vibration velocity modes. \( \Phi_⊥ \) is the \( N \)-length column vector of the cross-sectional modes of the rigid walled cavity whose components are given by \( φ_n(x) \). Exact continuity conditions are satisfied between the normal particle velocity on the MPP surface, \( \vec{v} \), and the normal air particle velocity in the cavity close to the panel interior side, \( (j/ωρ)p\partial p/\partial z|_{z=d} \).

Figure 1: A cavity-backed MPP absorber with a flexible top surface; detail of the particle velocity distribution \( \vec{v} \) in relation to the panel velocity \( v \) and to the air particle velocity \( v_h \) averaged over each hole.

Assuming that the distance between the MPP holes is much lower than the acoustic wavelength \( λ \), \( \vec{v} \) can be considered as a distributed particle velocity, spatially averaged over each aperture "cell" adjacent to the holes, so that \( \vec{v} = (1-σ)v + σv_h \), with \( v_h \) the particle velocity averaged over the hole and \( σ \) the perforation ratio. The pressure difference, \( p_0 - p_d \), across each MPP cell results from inner and surface viscous forces generated by the airframe relative velocity, \( v_h - v \), and from inertial effects due to the air mass moving inside and beyond the hole [2]. It
reads \( p_0 - p_d = Re(Z_h)(v_h - v) + Im(Z_h)v_h \) with \( Z_h \) the impedance of a hole. Hence, the air particle velocity \( \vec{v} \) can be expressed as

\[
\vec{v} = v_\gamma + \frac{p_0 - p_d}{Z}, \tag{3}
\]

with \( Z = Z_h/\sigma \) the overall acoustic impedance of the rigid MPP, as derived by Maa [1] for circular holes and \( \gamma = 1 - \text{Im}(Z_h)/Z \). For MPPs with sub-millimetric holes, the effects due to viscous losses prevail over inertial effects so that \( \gamma \approx 1 \). Substituting Eq. (3) into the exact continuity condition and considering the equation of motion for the flexible panel, we obtain a coupled set of algebraic equations that are solved for the acoustic and structural modal amplitude vectors, \( a_j \) and \( q_1 \), to obtain \( p_d \) and \( v_\gamma \).

The MPPA absorption coefficient, \( \alpha = 1 - |R|^2 \), can be calculated in terms of the normal incidence reflection coefficient \( R = (Z_1 - Z_0)/(Z_1 + Z_0) \), \( Z_0 = \rho_0 c_0 \) is the air impedance and \( Z_1 = p_0/\vec{v} \) is the input acoustic impedance of the MPPA with \( \vec{v} \) the air particle velocity averaged over the MPP surface. Expressing \( p_d \) in terms of \( \vec{v} \), one obtains

\[
Z_1 = \frac{Z + Z_{a,0}}{1 + \gamma \frac{Z}{Z_0} v_{b,0}}, \tag{4}
\]

with \( Z_{a,0} = -jZ_0 \cot(k_0 d) \) the acoustic impedance of the cavity and \( k_0 \) the acoustic wavenumber. From Eq. (4), \( Z_1 \) can be viewed as the input acoustic impedance of the rigid MPP-cavity system, \( Z + Z_{a,0} \), modified by a factor \( \vec{v}/v_{b,0} \) which accounts for the influence of the panel elasticity. The influence factor depends on the ratio between the panel averaged velocity to the air particle velocity in the MPP holes. Assuming single cross-coupling between the first panel mode and the uniform cross-sectional cavity mode, then \( Z_1 \) can be expressed as

\[
Z_1 = Z_{a,0} + \frac{Z Z_p}{Z + Z_p}. \tag{5}
\]

\( Z_p \) is the impedance of the first panel mode. It reads

\[
Z_p = \mu_p N_1 (\omega_1^2 - \omega^2 + 2j\zeta_1 \omega_1 \omega) / j\omega \mu_p \text{ panel surface density} \text{ and } N_1 = \iint_S |\vec{v}_1|^2 d^2x . \omega_1 \text{ and } \zeta_1 \text{ are respectively the natural frequency and the damping ratio of the first panel mode.}

## 3 Experimental validation

A series of experiments was conducted on a MPPA made up of a thin micro-perforated aluminium disk of radius 50 mm and thickness 0.5 mm, backed by a rigid cylindrical cavity of depth 45 mm. The disk is uniformly perforated by circular holes with diameter 0.5 mm and perforation ratio 0.78%.

### 3.1 Impedance tube measurements

The absorption coefficient is measured in an impedance tube using the two-microphones method. A good agreement is observed in Figure 2 between the experimental results and the theoretical predictions provided that the elastic behaviour of the panel is accounted for in the model. The frequency of maximum absorption occurs at 870 Hz which is somewhat greater than the Helmholtz resonance of the rigid MPPA that occurs at \( f_{HC} = 851 \) Hz. It is associated to a HC-controlled resonance of the flexible MPPA. Although the panel vibrations tend to increase the HC resonance frequency of the absorber, this property highly depends on the perforation ratio and on the cavity depth, as discussed in Section 4.

Extra absorption peaks at 298 Hz and 1572 Hz are clearly observed and well predicted by the vibroacoustic model assuming a disk with simply-supported boundary conditions and a structural damping ratio of 0.016. They correspond to PC-controlled resonances that increase the acoustic resistance of the MPPA. Also, the panel velocity tends to slightly lower the acoustic resistance of the MPPA, thus resulting in a reduction of both the absorption bandwidth and the maximum value of the absorption coefficient (from 0.9 down to 0.87).

![Figure 2: Sound absorption coefficient of a thin MPPA: predicted assuming a rigid (grey) or an elastic (red) MPP and measured (circles); measured and predicted disk velocity due to maxima of the sound absorption coefficient.](image)
PC resonances increase the acoustic resistance of the MPPA towards the matching point. But the gain in absorption is mitigated by the low-frequency stiffness-like and high-frequency mass-like reactance of the air-cavity system. Also, out of the PC resonances, the resistance of the MPPA is systematically lowered by the panel vibrations.

![Image](image.png)

Figure 3: Phasor representation of the input impedance of a thin MPPA: predicted assuming a rigid (bold grey) or a flexible (bold black) panel; measured using the two-microphones method (circles). Circles of constant absorption coefficient (thin black).

A close agreement is observed between the experimental phasor curve and the theoretical one predicted from the vibroacoustic model. It is found that, when the locus sweeps around each modal circle, the PC resonances occur at the frequencies for which $\tau_1$ has the smallest imaginary part. Decreasing the structural damping increases the radius of each modal circle, especially the one due to the second PC resonance, which is a panel-controlled resonance due to weak coupling of the second volume displacing panel mode with the $n = 0$ cavity mode. Hence, optimal value of the structural damping may be found for which the MPPA becomes purely resistive at this resonance. Radius of the first modal circle is more influenced by the acoustic resistance $\text{Re}(Z_0)$ of the holes due to strong coupling with the uniform pressure mode.

### 3.2 Laser vibrometer measurements

Non-contact vibroacoustic measurements of the MPP vibrations have been performed using a laser vibrometer with the scanning head focused on the MPP surface through the thick transparent rear face of the MPPA. As shown in Figure 2, the absorption peaks observed at 298 Hz and 1572 Hz are clearly the $(0,0)$ and $(1,0)$ volume-displacing PC resonances of the simply-supported disk panel. Extra peaks at 881 Hz and 1319 Hz are due to non-axisymmetric disk modes excited by non-uniform shear stresses that occur along the disk circumference. We note the difference in nature between the first $(0,0)$ mode associated to a PC resonance which strongly couples with the $(0,0,0)$ uniform pressure mode and the higher-order $(0,1)$ $(0,2)$ and $(1,0)$ modes which are panel-controlled resonances with their resonance frequency close to the in-vacuo disk natural frequency.

![Image](image.png)

Figure 4: Phase difference curves between the panel averaged velocity and the air particle velocity induced by the incident wave in the MPP holes: predicted from Eq. (4) (solid grey); from Kundt's tube measurements and Eq. (4) (circles); from LSV mobility and acoustic transfer function measurements (dashed).

Of significance is the phase difference between the panel averaged velocity and the acoustic velocity induced by the incident wave in the MPP holes, e.g., the phase of the influence factor $\bar{v}/\bar{v}_{0}$, in Eq. (4) with $\bar{v}_{0} = p_{0}/Z$. Prediction results are shown in Figure 4 together with semi-experimental phase difference curves. They are obtained either from Kundt's tube measurements of the MPPA input impedance $Z_1$ and Eq. (4), or from LSV mobility measurements, $\bar{v}/p_{2}$, of the panel averaged velocity per unit pressure $p_{2}$ acquired inside the duct by a probe microphone.

When compared with Figure 2, all three curves experience a $180^\circ$ phase jump at the peaks of maximum absorption associated to the volumetric PC resonances (298 Hz and 1572 Hz). At these frequencies, one observes that the panel vibrates with a net volume velocity out of phase with the air particle velocity at the holes so that the airframe relative velocity increases and the sound absorption is enhanced. Also, it appears that the MPP absorber is sensitive to the duct acoustic axial resonances which induce maximum air particle velocity at the disk holes. They correspond to the extra phase jumps and sound absorption peaks observed at 524 Hz, 671 Hz and 1063 Hz from Kundt's tube measurements. Phase jumps of smaller amplitude are also observed in both experiments at the other PC resonances (881 Hz and 1319 Hz) which induce small absorption peaks.

The experimental study demonstrates that the theoretical model can be used with confidence to predict the influence of the panel elastic behaviour on the absorption properties of a thin MPPA. But the underlying mechanisms should be clarified on the effect of the panel vibrations on the HC resonance as well as the effect of the micro-perforations on the first PC resonance.

### 4 Coupled mode analysis

From Eq. (5), the MPPA can be viewed as a parallel connection between a mass-spring damped oscillator and a mass-resistance element coupled in series through
the air cavity stiffness and driven by the impinging sound wave of amplitude \( p_0 \). This is valid at low frequencies, i.e. when each subsystem of the MPPA is acoustically (and structurally) compact. From the equations of motion of the coupled oscillators, the resonance modes and frequencies of the undamped MPPA are obtained setting \( p_0 = 0 \) and seeking solutions of the form \( \tilde{u} = u_{0p} e^{i\omega t} \) and \( u_h = u_{0h} e^{i\omega t} \) where \( u_{0p} \) and \( u_{0h} \) are respectively the panel average displacement and the effective air mass displacement.

This yields a set of two homogeneous algebraic equations for \( u_{0p} \) and \( u_{0h} \). Nontrivial solutions are found only if its determinant vanishes, thus providing the following expressions for the two resonance frequencies of the MPPA, \( \Omega_\pm \), and the components of each resonance mode

\[
\Omega^2_\pm = \frac{\omega^2_{PC} + \omega^2_{HC}}{2} \pm \sqrt{\left( \frac{\omega^2_{PC} - \omega^2_{HC}}{2} \right)^2 + \Delta^2_{PC}\omega^2_{HC}},
\]

with \( \omega_{HC} \) the Helmholtz frequency of the rigid MPPA and \( \omega_{PC} \) the Panel-Cavity resonance of the non-perforated system.

When \( \omega_{HC} = \omega_{PC} \), the mechanical energy is equally distributed between the two resonant states. For larger values of \( \sigma \) (\( \omega_{HC} > \omega_{PC} \)), a small fraction of energy is transferred between the modes. This case is often encountered in practice. From Eq. (6), the resonances are then approximated by

\[
\Omega^2_\pm \approx \omega^2_{HC,PC} \pm \frac{m_h}{m_p} \frac{\omega^2_{HC} - \omega^2_{PC}}{\omega^2_{HC,PC}},
\]

with \( m_h \) the effective air mass moving through the holes and \( m_p \) the modal mass of the first panel mode. When \( \sigma \) increases, it can be seen from Figure 5 that the upper resonance frequency \( \Omega_+ \) stays above (and tends towards) the HC resonance whilst the lower resonance frequency \( \Omega_- \) tends towards the first uncoupled panel mode frequency \( \omega_{PC} \). Close to the HC resonance, Eq. (7) shows that the panel and air particles move in phase but with a large air-frame relative velocity due to \( \frac{u_{0p}}{u_{0h}} \ll \frac{u_{0h}}{u_{0p}} \) whereas, close to the panel resonance, the panel and the air particles move in opposite phase with an amplitude ratio given by \( \frac{u_{0p}}{u_{0h}} = 1 - \frac{\omega^2_{HC} - \omega^2_{PC}}{\omega^2_{HC}} \). This is confirmed by the experimental phase curves shown in Figure 4.

5 Conclusion

This study determines how the sound absorption properties of MPPAs are modified by the vibrating response of the perforated facing. The predicted absorption performance and panel vibrating response are validated against impedance tube as well as LSV measurements for a thin circular MPPA under normal incidence condition. It is found that the air-frame relative velocity is a key factor that alters the input acoustic impedance of thin MPPAs and generates extra absorption peaks or dips that are not observed when dealing with rigid MPPAs. The two first absorption peaks of thin MPPAs are related to either PC-, HC- or panel-controlled resonances of the panel-hole-cavity system, depending on whether the effective air mass of the perforations is greater or lower than the panel first modal mass.

References