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Shear wave speed and attenuation in water-saturated glass beads and sand

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The frequency dependence of shear wave attenuation in water-saturated glass beads and sand contains distinguishable frequency bands in which different power laws apply. The bands are separated by relaxation frequencies. This is unlike the case of the dry granular medium, which is known to be proportional to the first power of frequency. The Biot-Stoll model of wave propagation in porous media captures the lower relaxation frequency, which is driven by a relaxation process in the relative motion between the pore fluid and the granular frame. A modification that includes squirt flow at the grain contacts is necessary to capture the higher relaxation frequency, which is a consequence of a relaxation process in the fluid motion within the grain contact region. There is evidence to suggest that the viscosity of the thin fluid film in the grain contacts may deviate from that of the bulk fluid due to ionic forces. Comparisons are made to published experimental measurements.

1 Introduction

One of the most influential references on shear wave propagation in marine sediments is Hamilton 1976 [1], in which the shear wave speed is assumed to be approximately constant and shear wave attenuation is "proportional to the first power of frequency". It is based on the published measurements of shear waves that were available at the time [2]. The model belongs under the "constant-Q" category.

A brief review of the different definitions of attenuation is useful [1]. Given a shear wave of the form,

$$u = Ae^{i(-\omega t + k_{so}x) - \alpha_s x} \qquad (1)$$

where u is the transversal particle displacement, A the arbitrary amplitude, k_{so} the real part of the wavenumber, α_s the attenuation (Nep/m), x the distance in the wave direction, ω the frequency (radians/s), and the subscript "s" is used to denote shear wave. The wave speed c_s is given by

$$c_s = \omega / k_{so} \qquad (2)$$

The quality factor Q, is defined as the loss tangent, and for small attenuation values, it is approximately related to attenuation divided by frequency. Thus, if attenuation is proportional to the first power of frequency, then the value of Q would be independent of frequency.

$$1/Q_s = \tan(\phi) \approx 2\alpha_s c_s / \omega \qquad (3)$$

Measurements by Stoll [3] using a resonant column apparatus, in the frequency band from 200 Hz to 2 kHz, indicated that the constant-Q model may be applicable to dry sand, but not water-saturated sand, as illustrated in Figure 1.

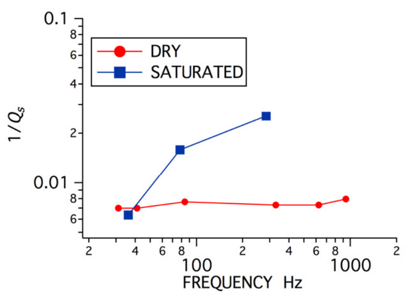

Figure 1: Measurements of attenuation in dry and water-saturated coarse Ottawa sand from [3]; 1 mm mean grain diameter.

This result was obtained using a coarse sand with a grain size of 1 mm. In Figure 2, measurements with smaller grain sizes, 0.3 and 0.03 mm, suggest, that there is some grain size dependence. At a grain size of 0.03 mm, there is no significant change in 1/Q.

For the coarser grains, the value of 1/Q should not be expected to increase indefinitely, therefore it may reach a maximum or saturation value, and it may even decrease at higher frequencies. This can be observed in measurements, in the band 2 kHz to 20 kHz in an open sand box, by Bell [4], as illustrated in Figure 3. The water-saturated sand clearly shows 1/Q decreasing with frequency, while the dry sand shows an approximately constant Q.

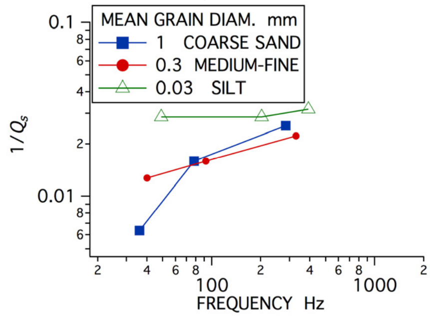

Figure 2: Measurements of attenuation in water-saturated sand from [3] using different grain sizes.

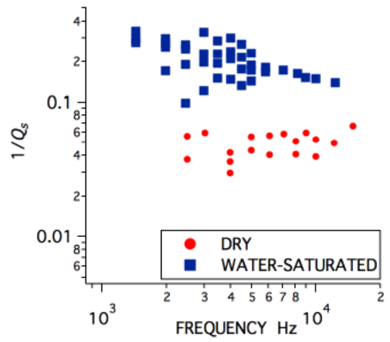

Figure 3: Measurements of attenuation in dry and water-saturated Panama City sand from [4]; 0.3 mm mean grain diameter.

This paper addresses, in particular, the case of the water-saturated coarse sand, in which there is significant change in the value of 1/Q as a function of frequency.

2 Frequency dependence

There is a set of measurements by Brunson [5] that straddles the peak frequency. It shows both the increase in 1/Q at the lower end of the frequency band, and the decrease at the higher end. Both glass beads and sands were used. The measurements on a sample of glass beads are as shown in Figure 4. The average value of the measured 1/Q, with plus and minus one standard deviation bars, are shown as a function of frequency. A significant part of Brunson's dissertation was to make the measurements and compare them with Biot's theory. The parameters of the Biot model [6], [7], as reformulated by Stoll [8] are shown in Table I. The grain and fluid densities and bulk moduli are taken from standard tables for the glass beads and water, respectively. The viscosity is that of water at room temperature and pressure. The permeability and the porosity were measured by Brunson [5]. The pore size and tortuosity were estimated from porosity and grain size [8] [9]. The shear modulus (real part) was adjusted to match the measured shear wave speed of 225 m/s. The frame bulk modulus has no effect on the shear speed but it is listed in the table for completeness. In the original Biot model, the last two parameters in the table, the log decrements, were set to zero. A comparison with the measurements gave a very poor fit, as shown by the dashed line in Figure 4. Following Stoll [3], Brunson adjusted the value of the shear log decrement to get the best fit, as shown by the solid line in the figure. The bulk log decrement has no effect but it is listed for completeness.

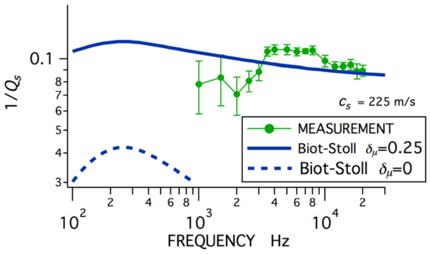

Figure 4: Measurements of attenuation in water-saturated glass beads, mean values and plus, minus one standard deviation bars, from [5] compared with the Biot-Stoll model, with and without a shear log decrement (δ_μ).

Table 1: Biot-Stoll parameters

Parameter	Value	units
ρ_s Grain density	2420	Kg/m^3
□$_r$ Grain bulk modulus	36	GPa
ρ_f Fluid density	1000	Kg/m^3
k_f Fluid bulk modulus	2	GPa
β Porosity	0.355	
η Viscosity	0.001	
κ Permeability	110	μm^2
a_p Pore size	70	μm
c Tortuosity	1.65	
μ_o Frame shear modulus	0.087	GPa
K_{bo} Frame bulk modulus	-	GPa
δ_μ Shear log decrement	0.25	
δ Bulk log decrement	-	

3 Modified BICSQS and BIMGS

In sand and sandstones, it was realized that the fluid film at the grain-grain contact may have an important effect on the frame moduli. As the frame stress increases, the fluid film in the grain-grain contact may be "squirted" out. This approach lead to the Biot and squirt flow (BISQ) model by Dvorkin and Nur [10] for sandstones. A similar model was developed for unconsolidated sand, based on the Biot model, with contact squirt flow and shear drag (BICSQS) [11]. It was followed by a more rigorous derivation, called Biot model with gap stiffness (BIMGS), by Kimura in 2006 [12]. In the modified BICSQS, some errors in the original model are corrected, and the more rigorous form from BIMGS are used for the fluid gap stiffness.

3.1 Grain contact model

The contact between grains of sand or glass beads may be considered as the contact between two rough surfaces, as illustrated in Figure 5. For the purpose of modeling, the contact region is idealized as a solid contact surrounded by a circular, thin fluid film.

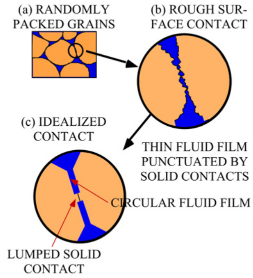

Figure 5: Idealized contact between grains: a solid contact surrounded by a thin fluid film

The tangential and normal contact stiffnesses, S_t and S_n, from [11] and [12] may be expressed in terms of the sum of the solid and fluid components.

$$S_t = S_{to} + S_{tg} = S_{to} - iS_{to}\frac{\omega}{\omega_\mu} \tag{4}$$

$$S_n = S_{no} + S_{ng} = S_{no} + S_g\left(1-\left(\frac{\kappa a J_o(\kappa a)}{2J_1(\kappa a)}\right)^{-1}\right) \tag{5}$$

The subscripts "*o*" and "*g*" refer to the contributions from the solid contact and the fluid film in the gap, respectively. In the case of the tangential stiffness, the fluid film causes a viscous drag, represented by an imaginary term that increases with frequency. In the normal stiffness, the fluid film contribution is more complicated. It is modeled as a complex reactive force caused by squirt flow. The stiffness term S_g is the asymptotic stiffness of the fluid film as frequency tends to infinity. At which point, the fluid film, unable to respond to the stress variations, is effectively immobilized and behaves as a spring. From [12], it is given by,

$$S_g = \pi k_f a^2 / h \tag{6}$$

where a is the radius of the fluid film, h the fluid film thickness, and k_f the fluid bulk modulus. The argument κa may be expressed in terms of a bulk relaxation frequency, ω_k, where [12]

$$\kappa a = \sqrt{-i\omega / \omega_k} \tag{7}$$

$$\omega_k = 2\pi\frac{k_f}{12\eta}\left(\frac{h}{a}\right)^2 = 2\pi f_k \tag{8}$$

The relaxation frequency ω_k is a function of the aspect ratio (h/a) of the fluid film, fluid bulk modulus k_f and viscosity η. Assuming that the solid component behaves as a Hertzian contact, there is a relationship between the solid components of contact stiffness through the Poisson's ratio of the grain material.

$$S_{no} = S_{to}\frac{2-\nu}{2(1-\nu)} \tag{9}$$

From the above expressions, the complex, frequency-dependent macroscopic frame moduli, K_b and μ, may be formulated using standard expressions for a granular medium of spherical grains of constant size [13],

$$K_b = \frac{n(1-\beta)}{12\pi R_g} S_n = K_{bo} + K_b \left(1 - \left(\frac{\kappa a J_o(\kappa a)}{2 J_1(\kappa a)} \right)^{-1} \right) \qquad (10)$$

$$\mu = \mu_o + \mu_g = \frac{n(1-\beta)}{20\pi R_g} (S_n + 1.5 S_t) \qquad (11)$$

where n is the average number of grains per unit volume, R_g the grain radius, and β the porosity. As before, the subscript "o" refers to the contributions from the solid contact which is static, and "g" refers to contributions from the fluid film in the gap, which is frequency dependent. The frame moduli are used in the standard Biot-Stoll equations to solve for the shear wave attenuation and speed. The value of the grain Poisson's ratio is from tables. The values of μ_o, K_g, and f_k were adjusted to obtain the best fit to the measured attenuation curve and wave speed. The best fit attenuation curve is shown in Figure 6, and the parameter values are shown in Table 2. The value of the shear relaxation frequency ω_μ was so high that it could be ignored.

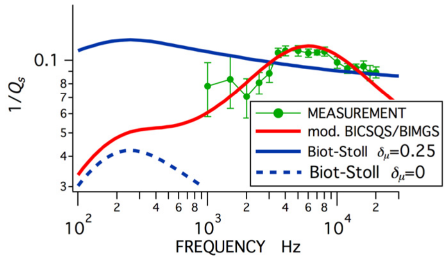

Figure 6: Measurements of attenuation in water-saturated glass beads: mean values, and plus, minus one standard deviation bars, from [5] compared with the Biot-Stoll model, with and without a shear log decrement (δ_μ), and the modified BICSQS/BIMGS model.

Table 2: Modified BICSQS and BIMGS parameters.

Parameter	Value	Units
ρ_s Grain density	2420	Kg/m^3
$\square_r$ Grain bulk modulus	36	GPa
ρ_f Fluid density	1000	Kg/m^3
k_f Fluid bulk modulus	2	GPa
β Porosity	0.355	
η Viscosity	0.001	
κ Permeability	110	μm^2
a_p Pore size	70	μm
c Tortuosity	1.65	
μ_o Static frame shear modulus	0.087	GPa
ν Grain Poisson's ratio	0.08	
K_g Asymptotic fluid contribution	0.086	GPa
f_k Bulk relaxation frequency	1800	Hz

3.2 Fluid viscosity and film thickness

Equations (6) and (8) may be inverted to give expressions for the dimensions of the fluid film.

$$h = 144 \frac{\eta K_g f_k R_g}{k_f^2 n(1-\beta)} \qquad (12)$$

$$a = \frac{K_g R_g}{n(1-\beta)} \sqrt{1728 \frac{\eta f_k}{k_f^3}} \qquad (13)$$

If the values from Table 2 were used in these equations to invert for the fluid film dimensions, h and a, the result is approximately 0.0002 and 2 nm (nanometers), respectively. It is observed that the value of the fluid film thickness, h, is problematic because the width of a water molecule is about 0.3 nm. Since the fluid is water, it is unphysical for the water film to be several orders of magnitude smaller than a water molecule.

The solution was found in the field of microfluidics. Recent papers by Goertz, Houston and Zhu [14] and Riedo [15] show that the viscosity of a thin film of water, confined by hydrophilic surfaces such as silica, is increased by several orders of magnitude as the film width is decreased below a few nanometers. Using the data in Goertz, Houston and Zhu, the viscosity of water as a function of film thickness was constructed as shown in Figure 7.

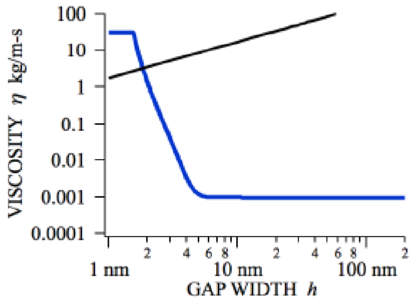

Figure 7: Viscosity of a thin film of water as a function of the film thickness (gap width) based on measurements by Goertz, Houston and Zhu [14] in blue, and the straight line from Eq. (14) in black.

Equations (12) and (13) may be rearranged to give viscosity as a function of film thickness,

$$\eta = h \frac{k_f^2 n(1-\beta)}{144 K_g f_k R_g} \qquad (14)$$

This straight line intersects the curve of viscosity as a function of film thickness to give the effective viscosity in the fluid film and the physical film thickness. The results are tabulated in Table 3.

Table 3: Modified BICSQS and BIMGS parameters.

Parameter	Value	Units
η_e Effective film viscosity	9.1	N s/m^2
h fluid film thickness	1.7	nm
a fluid film radius	170	nm

4 Conclusion

It is shown from published measurements that the shear attenuation in dry sands may be approximated as a constant Q process. In water saturated sands and glass beads, however, this approximation is inadequate. The value of 1/Q is observed to change with frequency – increasing with frequency at low frequencies, and decreasing with frequency at high frequencies. Measurements by Brunson [5] were chosen for comparison with theory because they cover the increase with frequency, the peak and the decrease at higher frequencies. Using measurements by Brunson, it is shown that basic the Biot theory does not provide a good match. The Stoll approach, in which loss mechanisms are injected by making the frame moduli complex, can be made to fit the measurements, but only in an average sense. The modified BICSQS/BIMGS model, which is an extension of the Biot model in which the frame moduli is governed by the squirt flow of the fluid film trapped within the grain-grain contact, is able to give a better fit.

This model predicts a direct connection between the macroscopic frame moduli and the dimensions of the fluid film at the grain-grain contact. In the process of fitting the model to the measurements, it became apparent that the microfluidic effects, that significantly increase the viscosity in thin films of water, needed to be taken into account. Without the microfluidic effects, the estimated water film thickness was too small to be physically possible. With the microfluidic effects, the estimated dimensions of the water film were reasonable. Therefore, it is deduced that microfluidic effects play a significant role in the acoustics of water saturated glass beads and sand.

Acknowledgments

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