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Simplified model of a harmonic point source moving above an impedance ground

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Transportation noise is an important source of annoyance in urban and suburban areas. Modelling sound propagation from these moving sources needs an efficient prediction tool. The classical theory due to Morse and Ingard is limited to free space. A reference model has been proposed for modelling the pressure field generated by a harmonic point source moving above a flat impedance ground which takes into account the spherical wave reflection coefficient. The need to reduce the complexity of the reference model leaded to a heuristic model intuitively derived from a model for a motionless harmonic sound source, by including Doppler corrections. However the predictions of this heuristic model are significantly different from those of the reference model. In this paper, we propose a simplification of the reference model, by approximating the variables by constants or by linear expressions on small time intervals. Compared with the reference model, this simplified model is computationally faster. In addition, it makes the physical link between the reference and the heuristic models. Numerical simulations confirm the accuracy of the simplified model.

1 Introduction

A great difficulty of designing a prediction tool from the physics of a phenomenon stands on a compromise between its accuracy and its computational burden. If the physical parameters of the model describing the phenomenon are numerous and if calculations require a lot of computational time, the complexity of the model may present some limitations of use. On the other side, if the model makes assumptions with few parameters that speed up the calculations by neglecting some aspects of the phenomenon, the model may still prove useless because of poor results. This issue can be applied to moving sources, and the main idea of this article is to propose a simplified model that is time-saving and whose results are in good agreement with the reference one.

Time-domain formulation of sound pressure produced by a subsonic moving noise source on a motionless receiver is well-known, especially when considering harmonic source and homogeneous atmosphere. For free field, the problem may be regarded as solved [1, 2, 3, 4]. The studies of the effects of the ground on the propagation of moving source are more recent [5, 6, 7, 8]. All these authors use the plane wave reflection coefficient in place of the spherical wave reflection coefficient to compute the boundary wave term [1]. This was corrected in [9, 10, 11], defining a Time-domain Model (actually a harmonic model whose parameters are time-dependant) referred as TM hereinafter.

In free space, source motion presents two shifts compared to a motionless source: the amplitude shift and the Doppler effect [3, 12]. Motion effect with a ground is more complicated. This is why a heuristic approach has been proposed [1, 10], involving amplitude shift and Doppler effect in the time domain. This Heuristic Model is noted HM thereafter. However, HM tends to overestimate the effects of source motion [1].

In this paper, a Simplified Time-domain Model (STM) is derived from TM, in order to produce a fast computational model. It provides also a physical understanding of moving sources. The simplification is based on the approximation of TM's variables by linear expressions or constants on short time intervals.

The paper is organized as follows. The problem is stated and notations are explained in section 2. Then the Time-domain Model (TM) is presented in section 3. Section 4 is devoted to the definition of the Simplified Time-domain Model (STM). Finally, numerical simulations with TM as reference show that STM outperforms the existing Heuristic Model (HM) and point out the accuracy of STM.

2 Problem and notations

2.1 Statement of the problem

This problem is stated in [11]. Consider a harmonic monopole source S with angular frequency ω and complex source strengh $A\, e^{-\jmath\psi}$. (x_S, y_S, z_S) are the rectangular coordinates of space and t is time. S is moving at constant speed v along the x-axis above a homogeneous ground surface. Without loss of generality, its instantaneous position along the x-axis expresses as

$$x_S = v\,t, \tag{1}$$

y_S and z_S held constants. A motionless receiver is located at $M(x_M, y_M, z_M)$.

The source is assumed to move at subsonic speed v. Hence the associated Mach number $M_a = v/c$ (with c the sound speed in the air) verifies $M_a \leq 0.3$ (or equivalently, $v \leq 367$ km.h^{-1}). This condition ensures linearity of the governing equations according to [1]. The atmosphere is supposed to be homogeneous. Air absorption and atmospheric turbulence are not considered in this paper. Furthermore, turbulence induced by the movement and the roughness of the ground are not considered.

The aim is here to determine an analytical expression for the sound field due to the moving source S. The governing equation is the space-time wave equation given by

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = A\, e^{-\jmath(\omega t+\psi)}\delta(x-x_S)\delta(y-y_S)\delta(z-z_S) \tag{2}$$

and the boundary condition, at $z = 0$, is determined by

$$\frac{1}{c}\frac{\partial\phi}{\partial t} - \frac{1}{\beta}\frac{\partial\phi}{\partial z} = 0 \tag{3}$$

Here, ϕ is the velocity potential relating to the acoustic pressure p by

$$p = -\rho\,\partial\phi/\partial t \tag{4}$$

where ρ is the density of the air and β the specific normalized ground admittance. The one-parameter Delany-Bazley-Miki's model [13] is choosen. This model consists in revised expressions for the complex wave number and characteristic impedance of Delany-Bazley mo-del [14]. Indeed, in the case of multiple layers, Miki noticed that the real part of the surface impedance when computed with the original Delany-Bazley model sometimes becomes negative at low frequencies denoting a non-physical result.// That is why Miki's corrections are used in this paper.

In the Delany-Bazley-Miki model, $\beta(f)$ is expressed as

$$\beta(f) = \left(1 + 5.5\left(\frac{f}{\sigma}\right)^{-0.632} + \mathrm{j}\,8.43\left(\frac{f}{\sigma}\right)^{-0.632}\right)^{-1} \quad (5)$$

where σ is the specific flow resistivity expressed in kN.s.m^{-4}. Note that the imaginary part of β is negative (in other words, the imaginary part of the denominator is positive) due to the convention chosen for time (a dependence in $e^{-\mathrm{j}\omega t}$ of $p(t)$ is assumed).

A locally reacting ground is supposed, which appears sufficient for most outdoor grounds (reflective road, grass fields). The extended reaction could be used in the case of porous roads.

2.2 Notations

Figure 1 illustrates the emission time geometry of the problem. All variables concerning the direct wave (super-

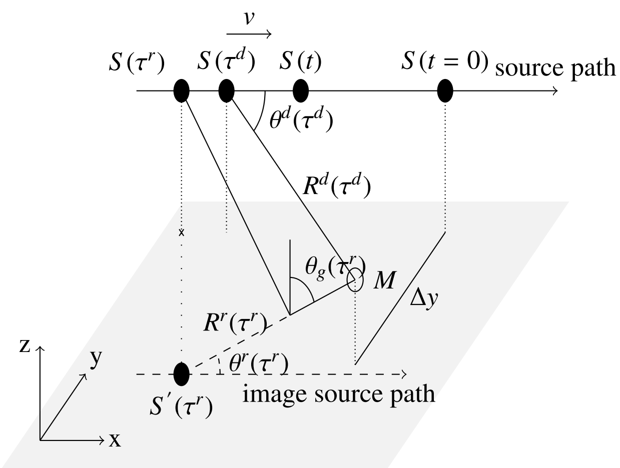

Figure 1: Moving sound source S with its image source S' and a motionless receiver M.

script d) depend on $\tau^d(t)$, which is the emission time of the direct wave arriving at M at instant t. All variables concerning the perfectly reflected wave (superscript r) depend on $\tau^r(t)$, which is the emission time of the reflected wave arriving at M at instant t. For instance, θ_g, the incidence angle (defined from the outward normal to the ground) of the reflected wave, depends on $\tau^r(t)$. The direct path length R^d (respectively the reflected path length R^r) is the distance between the receiver M and the source S (respectively the image source S'). R^d and R^r are expressed as

$$\begin{cases} R^d(\tau^d) = \sqrt{(x_M - x_S(\tau^d))^2 + \Delta^2_{yz-}} \\ \\ R^r(\tau^r) = \sqrt{(x_M - x_S(\tau^r))^2 + \Delta^2_{yz+}} \end{cases} \quad (6)$$

and

$$\begin{cases} \Delta^2_{yz-} = (y_M - y_S)^2 + (z_M - z_S)^2 \\ \\ \Delta^2_{yz+} = (y_M - y_S)^2 + (z_M + z_S)^2 \end{cases} \quad (7)$$

where Δy corresponds to the closest distance between the source and the receiver (along the y-axis). θ^d (respectively θ^r) is the angle between the source-receiver line (respectively the image source-receiver line) and the x-axis such that

$$\begin{cases} \cos\theta^d(\tau^d) = \dfrac{x_M - x_S(\tau^d)}{R^d(\tau^d)} \\ \\ \cos\theta^r(\tau^r) = \dfrac{x_M - x_S(\tau^r)}{R^r(\tau^r)} \end{cases} \quad (8)$$

where $\cos\theta_g(\tau^r)$ is computed with

$$\cos\theta_g(\tau^r) = \frac{z_M + z_S}{R^r(\tau^r)} \quad (9)$$

Lastly, source movement is regularly sampled at different times $[t_1, \cdots, t_l, \cdots]$, leading to time segments I_l defined as

$$I_l = \left[t_l^b, t_l^e\right] \quad (10)$$

with

$$\begin{cases} t_l^b = t_l - \dfrac{d_I}{2} \\ \\ t_l^e = t_l + \dfrac{d_I}{2} \end{cases} \quad (11)$$

d_I being the duration of I_l. In the following, d_I equals 20 ms. This corresponds to a 50 Hz resolution in frequency domain.

In the following, the subscript l indicates that variables are measured at $t = t_l$. For example, R^d_l means $R^d(\tau^d(t_l))$.

3 Time-domain Model

The sound pressure generated by a moving source and received at a motionless receiver has already been addressed in emission time geometry [9, 10, 11]

$$p(t) = \frac{-\mathrm{j}\omega A}{4\pi} \times \left(C^d \frac{e^{\mathrm{j}\kappa R^d}}{R^d} + C^r\, Q(w^r)\, \frac{e^{\mathrm{j}\kappa R^r}}{R^r}\right) \quad (12)$$

A time dependence $e^{-\mathrm{j}(\omega t + \psi)}$ is also assumed in Eq. (12). All variables with superscript d (resp. r) depend on time τ^d (resp. τ^r). Compared to the original formulation, azimuthal and elevation angles for the direct (resp. reflected) path is replaced by θ^d (resp. θ^r) for simplicity. Moreover, the strength of the source A is taken into account and the model for acoustic impedance of ground is simpler (a one-parameter model is considered rather than a two-parameter one as originally). In Eq. (12), f is the frequency of S ($\omega = 2\pi f$), κ the wave number corresponding to f ($\kappa = 2\pi f / c$).

Q is the spherical wave reflection coefficient written as [15]

$$Q(w) = \left[R_p + (1 - R_p)F(w)\right] \quad (13)$$

where R_p is the plane wave reflection coefficient for a locally reacting ground, defined by

$$R_p(f) = \frac{\cos\theta_g(\tau^r) - \beta(f)}{\cos\theta_g + \beta(f)} \quad (14)$$

In Eq. (13), the boundary loss factor $F(w)$ is given by [15]

$$F(w) = 1 + 2\mathrm{j}\,\sqrt{w}e^{-w}\mathrm{erfc}(-\mathrm{j}\,\sqrt{w}) \quad (15)$$

where

$$\mathrm{erfc}(x) = \int_x^\infty e^{-u^2}\, du \tag{16}$$

and where w is the so-called numerical distance. For a locally reacting ground [16], we have

$$w = \frac{\mathrm{j}\kappa R^r}{2}\left(\frac{1}{\beta} + \cos\theta_g\right)^2 \tag{17}$$

An efficient computation of $e^{-w}\mathrm{erfc}(-\mathrm{j}\sqrt{w})$ is available in [17]. C^d and C^r are two coefficients defined by

$$\left\{\begin{array}{lcl} C^d & = & \dfrac{1 + \dfrac{1}{\mathrm{j}\kappa R^d}\left(\dfrac{M_a^2 - M^d}{1 - M^d}\right)}{(1 - M^d)^2} \\ C^r & = & \dfrac{1 + \dfrac{1}{\mathrm{j}\kappa R^r}\left(\dfrac{M_a^2 - M^r}{1 - M^r}\right)}{(1 - M^r)^2} \end{array}\right. \tag{18}$$

M^d and M^r being defined by

$$\left\{\begin{array}{l} M^d = M_a \cos\theta^d \\ M^r = M_a \cos\theta^r \end{array}\right. \tag{19}$$

Lastly, in Eq. (13), Q is a function of the dopplerized numerical distance w^r

$$w^r = \frac{w}{1 - M^r} \tag{20}$$

In Eq. (12), most variables $(C^d, C^r, R^d, R^r, R_p, M^d, M^r)$ are expressed in the emission time geometry, as mentionned in 2.2.

In order to express τ^d and τ^r as a function of t, we start from the following definitions

$$\left\{\begin{array}{l} c = \dfrac{R^d(\tau^d)}{t - \tau^d} \\ c = \dfrac{R^r(\tau^r)}{t - \tau^r} \end{array}\right. \tag{21}$$

Substituting R^d (resp. R^r) from Eq. (6) into Eq. (21) yields a second order polynomial of τ^d (resp. τ^r). Two solutions are possible. Considering only the physical one corresponding to $\tau \in \Re$ and $\tau \le t$ leads to

$$\left\{\begin{array}{l} \tau^d(t) = \dfrac{c\,t - M_a x_M - \sqrt{\Delta^d(t)}}{c(1 - M_a^2)} \\ \tau^r(t) = \dfrac{c\,t - M_a x_M - \sqrt{\Delta^r(t)}}{c(1 - M_a^2)} \end{array}\right. \tag{22}$$

where

$$\left\{\begin{array}{l} \Delta^d(t) = (x_M - x_S(t))^2 + (1 - M_a^2)\Delta_{yz-} \\ \Delta^r(t) = (x_M - x_S(t))^2 + (1 - M_a^2)\Delta_{yz+} \end{array}\right. \tag{23}$$

One can notice that if the speed v vanishes in Eq. (12), we get the classical "Weyl-Van der Pol formula", also called Rudnick's model in acoustics. Indeed, if $v = 0$ then $C^d = C^r = 1$ and $M^r = 0$. Therefore Eq. (12) is referred to as the "Doppler Weyl-Van der Pol formula" [11].

4 The Simplified Time-domain Model

Unfortunately, TM is quite complex and has a high computational cost. On the other side, the heuristic approach HM, simpler, is not accurate enough [11]. In consequence, a Simplied Time-domain Model (STM) is proposed. It is derived from TM, assuming additional hypotheses for simplification.

4.1 Hypotheses

Eq. (12) can be simplified, assuming four hypotheses, for each time interval I_l

. (H1): for $t \in I_l$, $C^d(\tau^d(t))$ and $C^r(\tau^r(t))$ can be approximated by constants $\tilde{C}_l^d$ and $\tilde{C}_l^r$.

. R^d and R^r can be approximated

- (H2): by constants R_l^d and R_l^r for terms outside the exponents of Eq. (12).
- (H3): by linear approximation for terms within the exponents of Eq. (12).

. (H4): Q can be approximated by a constant Q_l.

In Eq. (12), (H2) is not applied for terms in exponentials, because rapid variations in the phase of direct or reflected wave can produce high errors. For more details, all these hypotheses are discussed in [18].

4.2 Simplified Time-domain Model

When applying (H1), (H2) and (H4) to Eq. (12), the sound pressure $p(t)$ can be approximated, for each I_l, by

$$p_l(t) \approx \frac{-\mathrm{j}\,\omega\,A}{4\pi} \times \left\{\tilde{C}_l^d \frac{e^{\mathrm{j}\kappa R^d}}{R_l^d} + \tilde{C}_l^r\, Q_l\, \frac{e^{\mathrm{j}\kappa R^r}}{R_l^r}\right\} \tag{24}$$

From (H3), for each I_l, R^d is linearly approximated on time segment $[t_l - d_l/6,\, t_l + d_l/6]$ as

$$R^d(\tau^d(t)) \approx a_l^d\, t + b_l^d \tag{25}$$

with

$$a_l^d = \frac{R^d\left(\tau^d\,(t_l + d_l/6)\right) - R^d\left(\tau^d\,(t_l - d_l/6)\right)}{\dfrac{d_l}{3}} \tag{26}$$

and

$$b_l^d = \frac{R^d\left(\tau^d\,(t_l)\right)}{2} + \frac{R^d\left(\tau^d\,(t_l - d_l/6)\right) + R^d\left(\tau^d\,(t_l + d_l/6)\right)}{4} - a_l^d\, t_l \tag{27}$$

Note that the linear approximation of R^d is done on $[t_l - d_l/6,\, t_l + d_l/6]$ and not all time segment $I_l = [t_l - d_l/2,\, t_l + d_l/2]$. Indeed, when considering smooth windows, the linear approximation of $R^d(t)$ needs to be more accurate near the middle t_l of the time segment. A Hanning window is considered hereinafter.

In the same way, R^r is linearly approximated by replacing superscript d by superscript r in Eqs. (25-27).

Substitution of R^d and R^r from Eq. (25) with superscript d or r into Eq. (24) yields the following simplified pressure $\tilde{p}_l(t)$, keeping only the real part of $p_l(t)$

$$\tilde{p}_l(t) \approx A\left(N_l^d \cos\left(\mu_l^d\, t + \nu_l^d\right) + N_l^r \cos\left(\mu_l^r\, t + \nu_l^r\right)\right) \tag{28}$$

with

$$\begin{cases} N_l^d &= \dfrac{\omega\, \tilde{C}_l^d}{4\pi} \\ N_l^r &= \dfrac{\omega\, \tilde{C}_l^r}{4\pi} \times Q_l \end{cases} \tag{29}$$

and

$$\begin{cases} \mu_l^d = \omega\left(\dfrac{a_l^d}{c} - 1\right) \\ \mu_l^r = \omega\left(\dfrac{a_l^r}{c} - 1\right) \end{cases} \tag{30}$$

and

$$\begin{cases} \nu_l^d = \kappa\, b_l^d - \dfrac{\pi}{2} \\ \nu_l^r = \kappa\, b_l^r - \dfrac{\pi}{2} + \arg Q_l \end{cases} \tag{31}$$

STM makes a link between the Morse and Ingard's theory for moving sources in free space and the Time-domain Model considering an impedance ground. The sound received at a microphone is the sum of the contribution of two sources corresponding to the direct path and the reflected path, each one with its own Doppler effect and amplitude shift. Moreover, the spherical wave reflection coefficient Q is calculated at the dopplerized frequency for the reflected path.

5 First results

Numerical simulations are carried out, mainly to compare TM and STM. One result, among the worst ones (it combines high speed, high frequency, a source path near the receiver, and a reflecting ground), is illustrated here in order to show the potential of STM. Figure 2 compares the mean-square sound pressure level Lp_l for TM and STM, as a function of $x_S(t)$.

At first glance, STM and TM are superimposed. For this simulation setup, the discrepancy reaches 0.2 dB maximum (see Figure 3). It decreases when $|x_S(t)|$ increases. Furthermore, the improvement of STM on HM is clear: maximum discrepancies are respectively of 0.1 and 10 dB.

6 Conclusion

A Simplified Time-domain Model (STM) of the pressure field generated at a motionless receiver by a harmonic moving point source above a flat impedance ground is presented in this paper. STM reduces the computational burden, while being accurate. Moreover, the empirical approach taken for HM is justified.

Numerical simulations emphasize the accuracy of the simplified model STM in comparison to TM, considering the

(a)

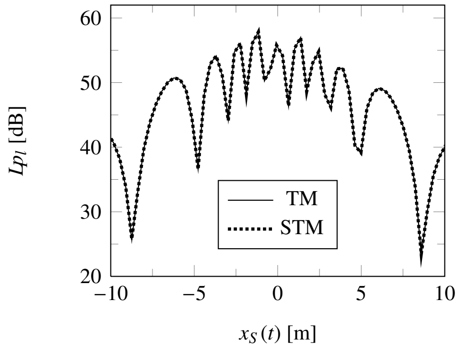

Figure 2: Comparison of TM and STM. Configuration setup: $S(x_S, 2, 0.75)$, $M(0,0,1)$, $v = 130$ km.h^{-1}, $f = 3010$ Hz, $\sigma = 30000$ kN.s.m^{-4}. Lp_l^{STM} and Lp_l^{TM} as a function of $x_S(t)$.

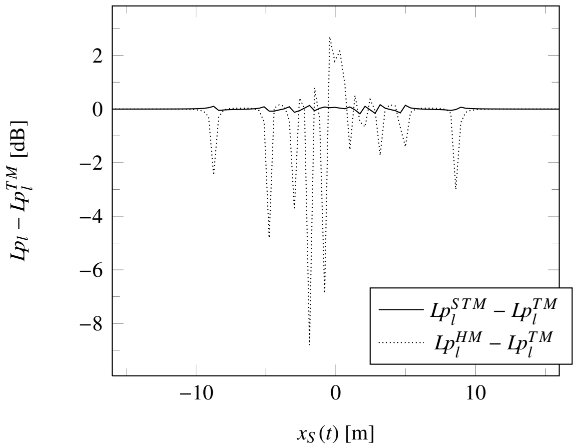

Figure 3: Mean-square sound pressure differences between STM or HM and TM, as a function of S-abscissa. Configuration setup: $S(x_S, 2, 0.75)$, $M(0,0,1)$, $\sigma = 30000$ kN.s.m^{-4}, $v = 130$ km.h^{-1}, $f = 3010$ Hz.

mean-square sound pressure level. Even by considering a 2 m distance between the source and the microphone, and a high speed (130 km.h^{-1}), STM is very accurate (0.2 dB at worst in most of cases). Discrepancies are maximum when the source is near the closest point of approach (CPA), and decrease when the source is approaching or receding from the receiver.

In conclusion, STM appears to be an accurate and effective prediction tool. The main perspective of the present work is to validate this model with experimental data, first by direct measurements to evaluate the accuracy of the model associated with outdoors sound propagation. Then, if both fit well, this would allow to use the STM model in source identification methods. This point opens a large set of possible applications for moving sources in outdoor environments.

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