Approach for the Characterisation of Wooden Staircases as Structure-borne Sound Sources

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At present it is not possible to predict the sound transmission into adjacent rooms from footfalls on lightweight stairs, which are connected to the separating wall. This is because the dominant transmission process is structure-borne and involves direct excitation of the separating wall, with flanking paths also contributing. A characterization of lightweight stairs as structure-borne sound sources is needed. In particular, a test method is required which will provide data, which will indicate the noisiness of the stair system when installed in a building. An approach is followed where the stair is treated as an active element, in a similar manner to that used to predict the structure-borne power from vibrating machines in buildings and other structures. Investigations on a wooden stair with one rigid contact to the wall have been carried out in the staircase test facility. To get insight into the vibration behaviour of the stair an experimental modal analysis was carried out. It is shown that the vibration behaviour of the stair is determined by beam modes of handrail and string board and plate modes of the single steps. Therefore the vibrations of the stair are strongly dependent on the point of excitation. From this it is obvious that the position of the external source (e.g. the tapping machine) is a major influence regarding the structure borne sound transmission into the wall. Hence a characterization of the stair as structure-borne sound source has to be with respect to the location of the external source. Regarding the excitation of the wall the components of excitation (forces and moments) at the connections between stair and separating wall are considered. An approach is described which allows the power through each component to be obtained in the installed condition by means of a reciprocal method. Using this method the most important components can be identified which allows a simplification of the further proceeding.

1 Introduction

At present it is not possible to predict the sound transmission into adjacent rooms from footfalls on lightweight stairs, which are connected to the separating wall. Especially at low frequencies excitation by human footfall and transmission is significant and often causes annoyance to the inhabitants. To reduce problems in the future a characterization of lightweight stairs as structure-borne sound sources is required. In particular, a test method is required which will provide data, which will indicate the noisiness of the stair system when installed in a building. Concerning this matter investigations have been carried out on a wooden staircase with string board which is a common type of stair in Germany. In this paper the investigation of the vibration behaviour of the stair and an approach for the characterisation as structure-borne sound source is outlined. Furthermore a reciprocal method is described which allows the power through each component to be obtained in the installed condition.

2 Wooden Staircases

In the case of wooden stairs, attached to walls separating dwellings, structure-borne energy enters the wall through the contact points as shown in figure 1.

![Figure 1: contact points and dominant transmission paths](image)

The stair is supported by the ceilings e.g. the floating floors. Up to now it is common practice to mount the string board directly at the wall using screws. In this case the transmission from the string board into the wall is significant. Experience shows that with the...
string board in contact with a common separating wall noise annoyance cannot be avoided. Even the normative requirements on the normalised impact sound pressure level $L'_n$ can hardly be met. The string board must be moved away from the wall as indicated in the right sketch of figure 1. However, the string board still has to be fixed at the wall, for safety reasons (stability when walking on the stair). This can be achieved through one connection [1]. Investigations show that in general transmission through this wall contact is the dominant transmission path. A detailed investigation of the structure-borne energy transmission through a single wall contact therefore is the topic of this study.

3 Investigated System

Experimental investigations on a wooden staircase with string board were carried out in a staircase test facility. The string board was moved away from the wall and resiliently supported at both ends. The contact with the wall was through one rigid screwed connection, shown in figure 2. It was confirmed experimentally that, in this set-up, the dominant structure-borne sound transmission was through the screwed contact.

![Investigated stair system](image)

Figure 2: Investigated stair system

4 Experimental modal analysis

The vibration behaviour of the stair is a major influence regarding the excitation of the wall. To get an insight into the dynamic behaviour of the stair, an experimental modal analysis was conducted, using an instrumented hammer. Accelerometers were placed on a central step (the 8th step from the floor), near the contact point and also on the edge of the 5th step. Due to reciprocity, the measured operating deflection shapes, shown in figure 3, result from

![Vibration deflection shapes](image)

Figure 3: Vibration deflection shapes

In the frequency range below 100 Hz the vibration of the stair is determined by beam modes of the handrail (47 Hz, 77 Hz) and string board (67 Hz). The vibration strength at a particular frequency is therefore strongly dependant on the position of the excited step. The excitation of steps, situated at antinodes of the handrail / string board, causes significant vibration of the whole stair assembly. In contrast, excitation of steps at nodal positions, results in reduced vibration. For example the strong vibration of the stair at 77 Hz only occurs for excitation at step 5 since step 8 is situated at a nodal point of the corresponding handrail beam mode. In the frequency range above 100 Hz the vibration of the single steps is determined by plate modes (the first plate mode occurs at 106 Hz). The handrail acts as “deliverer” of vibration energy within the stair-system. At frequencies where step plate modes and handrail beam modes coincide, the vibration of the whole stair is strong (99 Hz). At frequencies where no handrail beam modes occur, the excitation energy is mainly contained in the directly excited step (e.g. 106 Hz). This is also the case if the hand-rail has a beam mode but the excited step is situated at a node. The beam
modes of the string board determine the motion at the contact, perpendicular to the wall, and thus influence the excitation of the wall. Strong vertical motion at the wall contact follows if the wall contact and the excited step are situated at an anti-node of the string board (67 Hz). On the other hand, in the case of the excited step at a node of the string board, there can still be motion at the wall contact by energy transmission within the stair system due to handrail modes.

5 Stair as sound source

The stair assembly is a passive structure until it is excited on one or several of its steps. It then can be treated as an active source, which vibrates and transmits structure-borne power into the separating wall. The stair now can be treated in a similar manner to that used to predict the structure-borne power from vibrating machines in buildings and other structures. Accordingly the source descriptor concept [3] can be applied. It allows a characterisation of the stair as structure-borne sound source on a power basis. The source descriptor by definition is an inherent quantity of the source. The required quantities are the free velocity and mobility at the contact point formed by the rigid screw connection (figure 2). As shown the vibration behaviour of the stair strongly depends on the location of the external source. This effect can be considered e.g. by means of averaging the free velocities to be measured over all steps. From the receiver (wall) mobility the coupling function can be calculated and thus the power transmission of the stair in the installed condition can be predicted for a defined receiving wall.

\[ P_{s} = \frac{1}{2} \text{Re}\{F \cdot v^{*}\} \quad ; \quad P_{m} = \frac{1}{2} \{M \cdot w^{*}\} \]

6 Reciprocal method

For a single contact point, there are up to six degrees of freedom (3 translational; 3 rotational), which can contribute on the excitation of the wall (Figure 5). The structure-borne power imparted to the wall due to forces and moments is given by [2].

In order to obtain the structure-borne power from each component in the installed condition a reciprocal method as described in [4] will be used.

In the 1st stage a simple single component case is considered and experimentally validated. In the 2nd stage this method is extended to the multi component case which applies to the situation as found here.

6.1 Single component case

In the simplest case the receiving structure (here: the wall) is excited by a perpendicular force \( F_{ex} \) only. Under action of this force the translational response velocity at the contact point \( e \) is \( v_{ex} \). The power transmitted through the contact is (2):

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Following this the power can be calculated directly if the cross-spectrum of force and velocity at the excitation point is known. Direct measurement of the force at the mounting point(s) is difficult and often impossible since it requires the installation of a force transducer. Introducing an arbitrary remote point \( r \), equation (2) can be rearranged:

\[ P_{ex} = \frac{1}{2} \text{Re}\{F_{ex} \cdot v_{ex}^{*}\} \]

\[ = \frac{1}{2} \text{Re}\{Y_{y,ex}^{-1} \cdot v_{ex}^{*} \cdot v_{rx}\} \]

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In this arrangement \( v_{e,z} \) is the velocity at the remote point due to excitation force \( F_{e,z} \) at the contact point. The ratio \( \tilde{Y}_{v_{e,z}} \) is termed the (loaded) transfer mobility from the contact point \( e \) to the reference point \( r \). By principle of reciprocity it can be measured in the opposite direction e.g. excitation at the remote point and measure of the velocity at the contact point \( \tilde{Y}_{v_{r,z}} = \tilde{Y}_{v_{e,z}} \).

Thus this arrangement converts the problem of direct force measurement to a simpler transfer mobility measurement and cross-spectra of velocities.

The reciprocal measurement of the force and respective power was experimentally validated using a shaker attached to the receiving wall (Figure 6). A force transducer was inserted to obtain the force directly. To avoid moment excitation a piano wire was inserted.

Figure 6: shaker attached to the test wall

The contact velocity \( v_{e,z} \) was obtained by averaging the signals of the two accelerometers below and above the contact. The shaker was driven with random noise.

Figure 5 shows the force obtained directly and by the reciprocal method.

Figure 5: Force measured directly and reciprocally

The agreement in general is good. At certain frequencies discrepancies occur which result partly from the measurement of the transfer mobility which is inaccurate when the excitation or response coincides with nodal points. At approximately 800 Hz a peak of the force spectra occurs which corresponds to the first longitudinal resonance of the piano wire. Consequently the force decreases at higher frequencies as well as the signal / noise ratio and therefore the agreement between the two methods.

In Figure 6 the power obtained directly and by the reciprocal method is presented as narrow band and third octave band values. The agreement in general is promising. The power obtained reciprocally has negative values at some of the very low frequencies. This is the result of measurement uncertainty which is indicated as gaps in the curve. Although high deviations at certain frequencies occur the agreement of the third octave band values is in within ± 1 dB.

Figure 6: Power by direct and reciprocal measurement

So far it has been shown that the reciprocal method gives a reasonably good estimate of the power imparted to the wall by a perpendicular force. The total power imparted to the wall can be obtained by means of measuring the average velocity on the wall and the total loss factor [2] when the wall is treated as a reception plate:

\[
P = \omega \cdot m \cdot \eta \cdot \tilde{v}^2 \tag{4}
\]

The estimate of the input power, using this method, is shown in Figure 7. The agreement again is promising despite uncertainties in measurement of the loss factor and the average velocity.
6.2 Multi component case

The reciprocal method as described above can be expanded to the problem of multiple degrees of freedom at a single contact as given by the excitation of the wall through the screwed wall contact of the stair. It is likely that three components of excitation need to be considered: the force vertical to the wall \( F_z \) and the two moments about axes in the plane of the wall \( M_x \) and \( M_y \). The component powers are given by (5):

\[
P = \frac{1}{2} \text{Re}\left\{ F_{e,z} \cdot v_{r,1}^* + M_{e,x} \cdot w_{r,2}^* + M_{e,y} \cdot w_{r,3}^* \right\}
\] (5)

It has been assumed that the cross-mobility terms (for example, the excitation of velocity in the z-direction by a moment about the x-axis) can be neglected at a central wall location.

Three remote points \( r_1, r_2, r_3 \) are required for estimating three excitation components. The translational velocities result from a superposition of all components including cross-transfer terms:

\[
\begin{pmatrix} v_{r,1} \\ v_{r,2} \\ v_{r,3} \end{pmatrix} = \begin{pmatrix} \tilde{Y}_{v,r,1} \cdot F_{e,z} \\ \tilde{Y}_{v,r,2} \cdot M_{e,x} \\ \tilde{Y}_{v,r,3} \cdot M_{e,y} \end{pmatrix} = \begin{pmatrix} F_{e,z} \\ M_{e,x} \\ M_{e,y} \end{pmatrix} \cdot \begin{pmatrix} v_{r,1} \\ v_{r,2} \\ v_{r,3} \end{pmatrix}
\] (6)

Using reciprocity relationships which in terms of the cross mobilities is e.g. \( \tilde{Y}_{w,r,M,e} = \tilde{Y}_{w,r,F,e} \) the mobilities can be replaced by the corresponding “reciprocal” mobilities. The components are then obtained by inversion of the mobility matrix (7):

\[
\begin{pmatrix} F_{e,z} \\ M_{e,x} \\ M_{e,y} \end{pmatrix} = \begin{pmatrix} \tilde{Y}_{w,r,F,e} & \tilde{Y}_{w,r,M,e} \\ \tilde{Y}_{w,r,M,e} & \tilde{Y}_{w,r,M,e} \\ \tilde{Y}_{w,r,M,e} & \tilde{Y}_{w,r,M,e} \end{pmatrix}^{-1} \begin{pmatrix} v_{r,1} \\ v_{r,2} \\ v_{r,3} \end{pmatrix}
\] (7)

In order to obtain the required phase between the velocities, one remote point \( r_1 \) is selected as a reference value along with the velocity transfer functions between the reference point and the contact point as well as between the other remote points.

The component powers are then obtained by (8,9,10):

\[
P_{F_z} = \frac{1}{2} \text{Re}\left\{ F_{e,z} \cdot v_{r,1}^* \cdot \phi^*(v_{r,1}^*, v_{e,z}) \right\}
\] (8)

\[
P_{M_x} = \frac{1}{2} \text{Re}\left\{ M_{e,x} \cdot v_{r,2}^* \cdot \phi^*(v_{r,1}^*, w_{e,x}) \right\}
\] (9)

\[
P_{M_y} = \frac{1}{2} \text{Re}\left\{ M_{e,y} \cdot v_{r,3}^* \cdot \phi^*(v_{r,1}^*, w_{e,y}) \right\}
\] (10)

In Figure 8 are shown the results of preliminary reciprocal measurements of the powers from the three components of excitation at the contact point between stair and wall. Also shown is the total power obtained from the spatial average velocity. As external source the tapping machine was situated at step 8 (in the middle of the stair near the wall contact).

Figure 8 gives an early indication that the contribution from moments \( M_x \) and \( M_y \) is significant within the whole investigated frequency range. This early indication needs to be confirmed by further measurements.

7 Concluding remarks

A characterization of a wooden staircase with string board and a single rigid wall contact as structure-borne sound source is considered. An approach is proposed where the stair is treated in a similar manner to that used for vibrating machines. Therefore as first step the vibration behaviour of the stair was analysed by means of an experimental modal analysis. The vibration behaviour of the stair is determined by beam modes of handrail and string board and plate modes of the steps.
From this it follows that the vibration behaviour and thus the excitation of the wall is strongly dependant from the location of the external source exciting the stair. For a given external source and location on the stair the power imparted to the wall can be evaluated by means of a reciprocal method. With this method the relative contribution of the components (forces and moments) can be assessed in the installed condition and a hierarchy of the transmission paths can be established.

The capability of the reciprocal method was investigated for a single component source, a shaker, which generates a force perpendicular to the wall surface. A comparison of the directly and reciprocally obtained forces and associated powers show a reasonably good agreement. Preliminary reciprocal measurements of the power generated at the wall by the installed stair excited by the tapping machine, indicate that moments as well as forces contribute to the bending vibration field on the wall and thus to the radiated sound into the adjacent room. The relative contributions of each excitation component have yet to be correctly quantified but early indications are that no component can be neglected a priori.

References


