

THE PERCEPTION OF ALL-PASS COMPONENTS IN TRANSFER FUNCTIONS

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ABSTRACT

In electro-acoustics it is common practice to process an acoustic signal by convolving it with a measured or modelled transfer function. Any transfer function can be decomposed into a linear phase component, minimum phase component and an all-pass component. An investigation was done to determine when the contribution of the all-pass component is perceptible. Through a listening experiment, thresholds of audibility were obtained, which can be used to ascertain whether discarding an all-pass component will be audible. If it is not audible discarding the all-pass component can be advantageous in many applications.

0 INTRODUCTION

A linear, time-invariant transfer function can be decomposed into a minimum phase component, a linear phase component and an all-pass component. The main aim of this work was to determine what perceptual contribution the all-pass component makes to the transfer function. Specifically, it was important to find out under which circumstances the effects of the all-pass component are audible and whether it can be neglected or discarded. The problem was addressed by recalling that the all-pass component can be decomposed into a number of first and second order all-pass sections. Although, the perception of both first and second order all-pass sections are of concern the focus of the current text will primarily be on second order sections. In a listening experiment thresholds of audibility of a single second order all-pass section were obtained. Signals were presented through headphones with the all-pass section applied to either one or both channels. The results of this experiment will be used to explain the perception of the all-pass component as a whole.

1 THEORETICAL BACKGROUND

1.1 Phase delay, group delay and pure delay

The phase of a transfer function is sometimes interpreted as introducing a certain shift in time to signals. Since this time shift can be described in different ways some terms used in this context will be repeated briefly, before looking at all-pass sections specifically. It can be shown that the time shift that a sinusoidal signal undergoes is equal to the negative phase of the transfer function at the frequency of the sinusoid divided by that frequency. By considering all frequencies this gives $\tau_p(\omega) = -\phi(\omega)/\omega$, which is called the phase delay. The group delay of the transfer function is defined as minus the derivative of the phase with respect to the frequency, i.e. $\tau_g(\omega) = -\partial\phi(\omega)/\partial\omega$. It can be interpreted as the time shift introduced to the

envelope of the signal at each frequency. Notice that both the phase delay and group delay describe the properties of the transfer function only at each frequency and do not describe a general delay introduced to a signal. For a system to perform such a delay, it must have unity gain at all frequencies and all frequencies must be shifted in time by the same amount. Therefore, the phase delay and group delay should have a constant value throughout the frequency range, i.e. $\tau_p(\omega) = \tau_g(\omega) = \tau_d$. This is called a pure delay and refers to the situation, where the output is a time-shifted copy of the input, irrespective of the signal.

1.2 Response of a second order all-pass section

The transfer function of a second order all-pass section is given by:

$$H(s)_{\text{ap2}} = \frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad (1)$$

where ω_0 is the frequency at which the phase is equal to $-\pi$. The response depends on the quality factor or Q-factor, Q , and the centre frequency, f_0 , where $f_0 = \omega_0 / 2\pi$.

1.2.1 Phase, phase delay and group delay

The transfer function of a second order all-pass section has a magnitude of unity and the phase is given by:

$$\phi(\omega)_{\text{ap2}} = 2 \tan^{-1} \left(\frac{\omega_n}{Q(1 - \omega_n)} \right) \quad (2)$$

Therefore, the phase delay is given by:

$$\tau_p(\omega)_{\text{ap2}} = \frac{2 \tan^{-1} \left(\frac{\omega_n}{Q(1 - \omega_n)} \right)}{\omega} \quad (3)$$

The group delay can be written as:

$$\tau_g(\omega)_{\text{ap2}} = \frac{2}{\omega_0 \cdot Q} \left(\frac{1 + \omega_n^2}{(1 - \omega_n^2)^2 + (\omega_n / Q)^2} \right) \quad (4)$$

where $\omega_n = \omega / \omega_0$. The phase, phase delay and group delay of a second order all-pass section is shown in Figure 1. Notice that the phase is a monotonic function of frequency and lies between 0 and -2π . The phase delay and group delay are approximately constant at low frequencies and has their highest values close to the centre frequency.

In Figure 2 examples are shown of all-pass sections with different combinations of high and low Q-factors and centre frequencies. In Figure 2a and Figure 2b it is seen that the maximum group delay is large when the Q-factor is high. In Figure 2c and Figure 2d it is seen that a relatively large group delay is introduced at low frequencies when the Q-factor is low and that this effect becomes increasingly evident as the centre frequency is reduced.

1.2.2 Impulse response

The impulse response of a second order all-pass section can be derived from Equation 1 by means of the inverse Laplace transform as:

$$h(t) = \delta(t) + Ae^{-t/\tau} \cos(\omega_1 t + \theta) \quad (5)$$

where $A = -\frac{2\omega_0}{Q} \frac{1}{\sqrt{1 - (2Q)^{-2}}}$, $\theta = \tan^{-1} \left(\frac{1}{\sqrt{(2Q)^2 - 1}} \right)$, $\omega_1 = \omega_0 \sqrt{1 - (2Q)^{-2}}$ and $\tau = \frac{2Q}{\omega_0}$.

It can be seen that the impulse response consist of a pulse followed by a decaying sinusoid, where the decay time of the sinusoidal component is controlled by the decay constant, τ .

Figure 3 shows the impulse responses corresponding to the all-pass sections shown in Figure 2. In Figure 3a and Figure 3b the ringing due to the high Q-factor is clearly seen after the initial impulse. Notice that in both Figure 3a and Figure 3b the impulse response starts with a single

high amplitude impulse. On the other hand the first values in the impulse response are not generally the highest when the Q-factor is low, as seen in Figure 3c and Figure 3d. Instead the impulse response decays quickly and the peak in the impulse response is delayed.

2 AUDIBILITY OF A SINGLE ALL-PASS SECTION

Whether the presence of a second order all-pass section is audible depends not only on its Q-factor and centre frequency, but also on whether it is present in only one ear (dichotic) or both ears (diotic), is causal or noncausal and on the stimulus signal. However, it is well known that for some signals the presence of an all-pass section can become audible if the Q-factor is high enough. The existence of such a “high Q threshold” was confirmed in an experiment described by Minnaar et al. [1]. In addition it was found that the contribution of the all-pass section can be audible if the Q-factor is low enough – referred to as the “low Q threshold”. The experiment will be described briefly as a basis for discussion of the perception of all-pass components in transfer functions.

2.1 Listening experiment

In the experiment listeners, wearing headphones, were presented with two consecutive signals with a 1 s interval in-between. An all-pass section with a given frequency was used to filter either one (dichotic) or both (diotic) channels of the second signal. The task of the listener was to adjust the Q-factor of the all-pass section until the difference between the signals was just noticeable. The all-pass section could be either causal or noncausal and different signals were used to determine the High Q and Low Q thresholds. Please see Minnaar et al. [1] for a motivation of the choice of signals and a detailed description of the procedures used. Thresholds were obtained for the six different conditions shown in the table below.

Condition	Q-factor	Centre frequency	Presentation	Causality	Signal
A	High Q	1,2,4,8,12 kHz	Diotic	Causal	Click
B	High Q	1,2,4,8,12 kHz	Diotic	Noncausal	Click
C	High Q	4,8,12 kHz	Dichotic	Causal	Click
D	High Q	4,8,12 kHz	Dichotic	Noncausal	Click
E	Low Q	1,2,4,8,12 kHz	Dichotic	Causal	Pink noise
F	Low Q	1,2,4,8,12 kHz	Dichotic	Noncausal	Pink noise

The results (mean thresholds) are shown as curves in Figure 4. Curves with positive slopes indicate high Q thresholds, whereas those with negative slopes indicate low Q thresholds. Dashed and solid lines represent diotic and dichotic presentations respectively, whereas thick and thin lines represent causal and noncausal all-pass sections respectively. The effects of an all-pass section are audible when the Q-factor is higher than the high Q threshold line or lower than the low Q threshold line.

2.2 High Q all-pass threshold

In Equation 5 it was seen that the all-pass section “rings” when the Q-factor is high and that the decay time of the ringing is controlled by the decay constant, τ . The presence of an all-pass section can be audible if the decay constant is high enough and the ringing is not sufficiently masked by the signal. Interestingly, the high Q threshold curves shown in Figure 4 can be well approximated by straight lines indicating a linear relationship between f_0 and Q , which in turn implies a constant value for τ . Traditionally the threshold of audibility has been related to the peak group delay introduced by the all-pass section, though. Since the frequency at which the peak group delay is introduced approximates the centre frequency well for high Q values the peak group delay can be found by evaluating Equation 4 at the centre frequency:

$$\tau_g(\omega_0)_{\text{ap2}} = \frac{4Q}{\omega_0} \quad (6)$$

When comparing this with the expression for τ in Equation 5 it is seen that, for high Q-factors, the peak group delay can be related mathematically to the decay constant such that $\tau_g(\omega_0)_{\text{ap2}} = 2\tau$. So, the peak group delay can be used to describe the threshold, although it should be understood that it is rather the decay of the sinusoidal component in the impulse response that determines the audibility of the all-pass section. The high Q thresholds in the experiment are found at peak group delay values of 1.5 ms and 1.2 ms for causal and

noncausal all-pass sections respectively, independent of whether the presentation was diotic or dichotic. The threshold for causal all-pass sections correspond well with previously reported values found by e.g. Preis [2] and Blauert and Laws [3].

Since the audibility of the ringing depends on masking the high Q threshold is strongly signal dependent. In pilot experiments it was found that a single click is among the signals with the lowest thresholds and that signals such as speech, music and noise have thresholds that are much higher if the presence of the all-pass is audible at all. Furthermore, in order for the effect of an all-pass section to be audible it has to be excited sufficiently at its centre frequency. The all-pass sections found in electro-acoustical transfer functions, however, are typically associated with deep dips in the magnitude at the centre frequency leading their thresholds to be even higher than for a pure all-pass. Therefore, the ringing effect of all-pass sections is not expected to be audible under normal listening conditions.

2.3 Low Q all-pass threshold

The high Q threshold describes the audibility of a second order all-pass section completely if the all-pass is applied diotically. However, when listening dichotically the presence of the all-pass can also be detected if the Q -factor is low enough due to a lateralisation of the auditory image. In contrast to the high Q threshold the low Q threshold is relatively independent of the signal as long as it contains energy at low frequencies. The lateralisation of the auditory image can be explained by inspecting the phase delay and group delay introduced by the all-pass section at low frequencies.

Figure 1 shows that the phase delay and group delay are approximately constant and have almost the same value at low frequencies. In fact it is seen that their values are the same, when evaluating the phase delay in the limit as ω approaches 0 Hz and the group delay at $\omega = 0$ Hz, as follows:

$$\tau_p(\omega)_{\text{ap2}} \underset{\lim \omega \rightarrow 0}{=} \frac{2 \cdot \left(\frac{\omega_n}{Q(1-\omega_n)} \right)}{\omega} = \frac{2}{\omega_0 Q} \quad (7)$$

$$\tau_g(0)_{\text{ap2}} = \frac{2}{\omega_0 Q} \quad (8)$$

Therefore, the value of the group delay at 0 Hz can be used to describe the phase delay and the group delay in the entire low frequency region. As with the high Q thresholds the low Q thresholds can be well approximated by straight lines. The negative slopes of the thresholds indicate that the product of f_0 and Q is constant. This in turn indicates that the group delay at 0 Hz is constant – with a value of approximately 30 μs for both causal and noncausal all-pass sections.

The results of the experiment show that the effects of any second order all-pass section with a centre frequency above 1 kHz will be audible if $\tau_g(0)_{\text{ap2}} > 30 \mu\text{s}$. A similar result should be expected for first order all-pass sections with a constant group delay at low frequencies. Since the all-pass component in a transfer function consists of several first and second order all-pass sections, its group delay can be determined by a linear superposition of the group delays of the individual sections it comprises. Furthermore, the contributions of the all-pass components at both ears should be taken into account. Therefore, the interaural group delay difference between the all-pass components should be evaluated at 0 Hz, as follows

$$IGD_{0 \text{ all-pass}} = \tau_g(0)_{\text{all-pass, left}} - \tau_g(0)_{\text{all-pass, right}} \quad (9)$$

If the absolute value of $IGD_{0 \text{ all-pass}}$ is larger than 30 μs the perceived delay introduced by the all-pass components will be audible.

Since all-pass components introduce a perceived delay it is interesting to notice that the results of the current study are in general agreement with other studies where a pure delay was applied as interaural time difference. The studies by e.g. Klump and Eady [4] and Hafter and De Maio [5] found that thresholds of audibility vary little between signals and are in the range of about 10 – 40 μs . This agreement is perhaps not surprising since there is substantial evidence in the literature indicating that the lateral displacement of an auditory event due to an interaural time difference is mainly determined by the signal components at low frequencies.

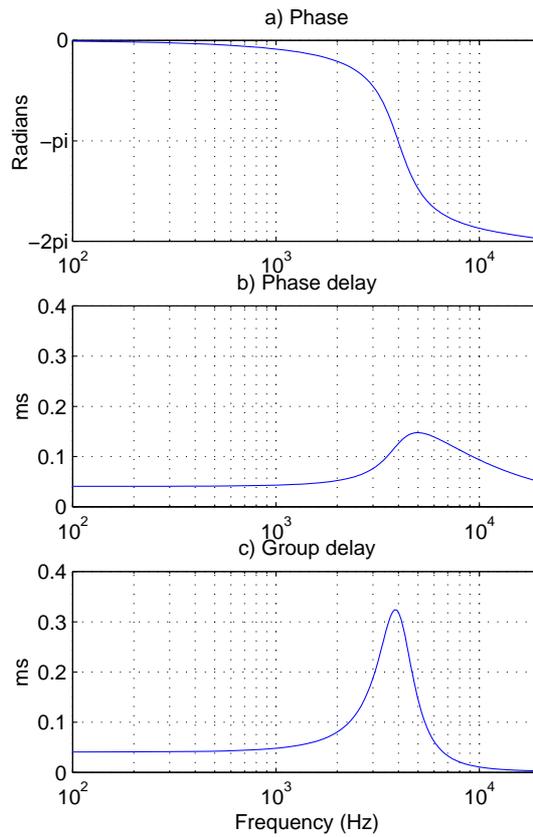


Figure 1 The a) phase, b) phase delay and c) group delay is shown for a second order all-pass section with $Q = 2$, $f_0 = 4$ kHz.

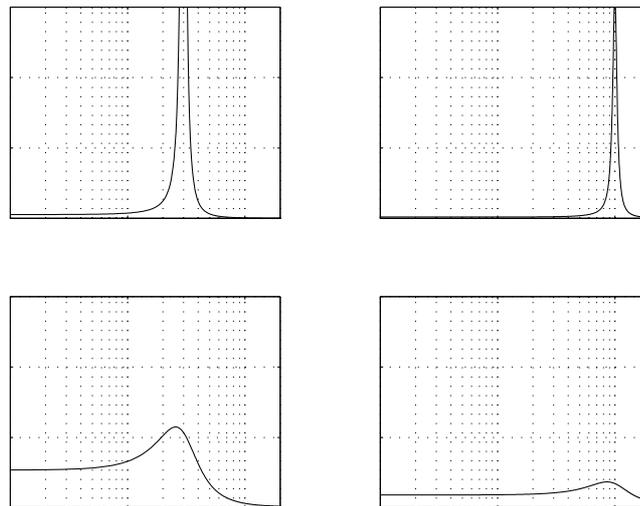


Figure 2 The group delay of a second order all-pass section is shown with the following properties: a) High Q , Low f_0 , b) High Q , High f_0 , c) Low Q , Low f_0 , and d) Low Q , High f_0 , where High $Q = 10$, Low $Q = 1$, Low $f_0 = 3$ kHz and High $f_0 = 10$ kHz.

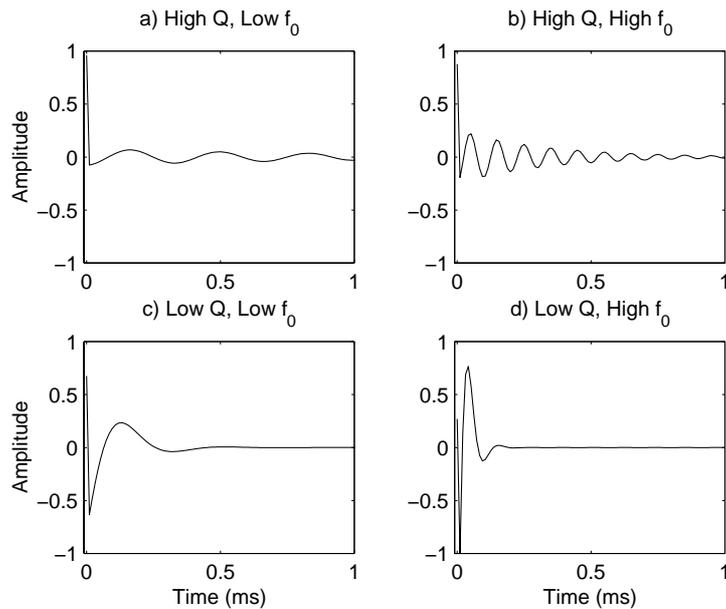


Figure 3 Impulse responses of the four second order all-pass sections shown in Figure 2.

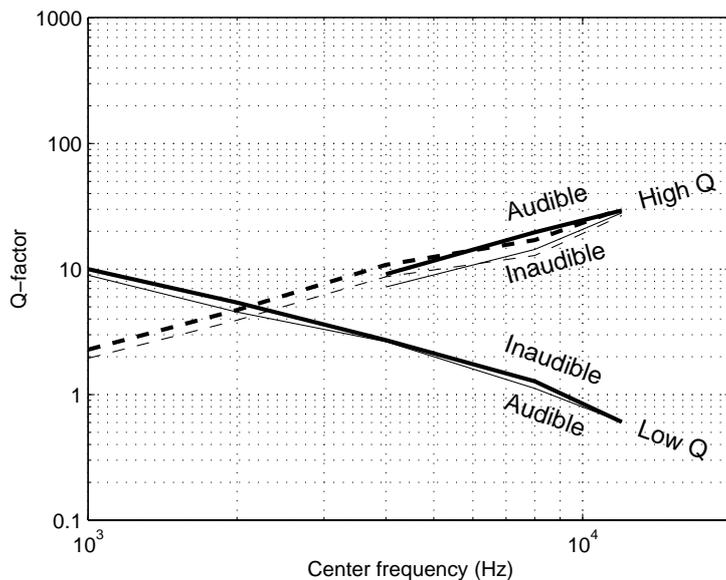


Figure 4 The thresholds are presented as curves obtained by connecting the mean thresholds by straight lines.

3 REFERENCES

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