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Comparison of the sound of grand and upright pianos using wavelets

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Wavelet analysis is useful for extracting patterns and thus analyzing signals. Although the Fourier analysis can reveal different features of a signal, it is less appropriate for describing transient phenomena and sudden sound changes. Wavelet analysis is capable of highlighting different attributes of a signal. Different types of wavelets are thus used in the present study in order to compare the sound produced by grand pianos with that produced by upright pianos. It was found that with the use of the discrete wavelet transform it is possible to distinguish between the sound of grand and upright pianos.

1 Introduction

Wavelet analysis is useful for extracting patterns from a big volume of data. In contrast to the Fourier transform [7] where the signal is separated into an infinite sum of sinusoidal waves of different frequencies, wavelet analysis splits the signal into parts that are not sine or cosine waves.

Fourier analysis has been used in music synthesizers in order to realistically reproduce the sound of musical instruments [6]. It has been also extended to signals that are not periodic. Despite the fact that the Fourier transform can highlight different characteristics of a signal, it is less useful for describing transient phenomena and sudden sound changes.

At the beginning of the twentieth century the Haar wavelets have been introduced that are composed of a positive pulse succeeded by a negative pulse. Then the Gabor transform [9] was created which has similarities with the Fourier transform. In 1984 Jean Morlet was capable of decomposing a signal into wavelet elements and then recomposing them into the original signal. In 1987 Ingrid Daubechies [2] created another family of wavelets that can be easily applied in other sciences. Nowadays, wavelets are mainly used for compression of data as well as for de-noising.

The measured acoustical properties that can be used in order to distinguish between a grand and an upright piano include among others the directivity of the produced sound, the inharmonicity [11] of the strings, the sound spectrum, the decay time of the sound, the efficiency of the dampers, the eigenmode density of the soundboard, and the dynamic range of the produced sound.

Grand pianos produce better balanced sound because of the higher density of eigenmodes. Moreover, grand pianos are less probable to produce after-sound fluctuations because of multiple stringing. Due to the bigger mass of the grand piano dampers, the damping effect is efficient also for lower frequencies. Grand pianos generate sound with less inharmonicity [1] as they are bigger with longer bass strings whose sound has wider spectrum. As far as the way pianos are placed inside a room is concerned, grand pianos are usually free from any direct obstacle though upright pianos are usually placed against a wall that consequently reflects the produced sound.

2 Wavelet transform

A transform consists a special category of function. A function describes the relationship between the domain and the codomain. Considering the Fourier transform $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$, its domain contains time functions whereas its codomain contains frequency functions. As a result, a transform is a map whose domain and codomain contain functions.

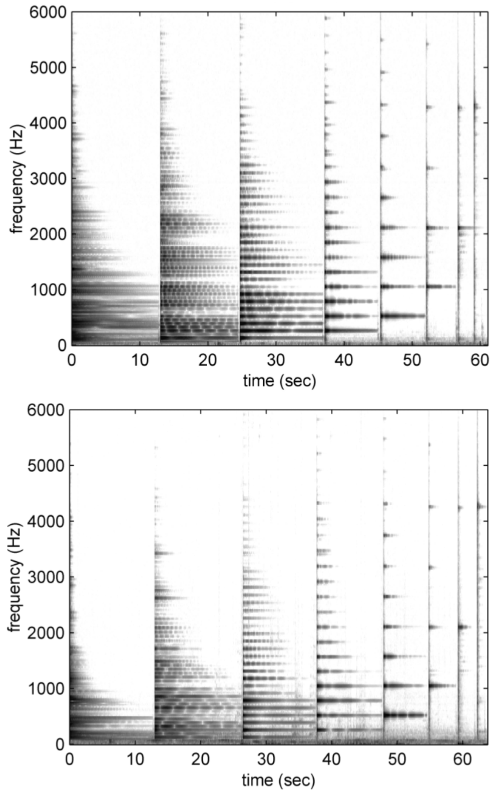

Fig.1 Short-Time Fourier Transform (STFT) with a Hanning window of length $N = 2048$ of all C from bass to treble played by a grand and an upright piano

The Fourier analysis [7] consists of the continuous-time Fourier transform, the continuous-time Fourier series, the discrete-time Fourier transform $F[k]=\sum_{n=0}^{N-1} f_n e^{-\frac{j2\pi nk}{N}}$, and the discrete-time Fourier series that correspond to continuous-time energy, continuous-time power, discrete-time energy, and discrete-time power signals, respectively. An energy signal is a signal whose energy $E=\int_{-\infty}^{\infty} s^2(t)dt$ is finite and a power signal is a signal whose power $P=\lim_{\alpha\to\infty}\frac{1}{2\alpha}\int_{-\alpha}^{\alpha} s^2(t)dt$ is finite. For discrete-time signals the energy and power are equal to $E=\sum_{n=-\infty}^{\infty} s_n^2$ and

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} s_n^2$, respectively. The fast Fourier transform is a form of discrete-time Fourier transform.

The Haar transform is a type of wavelet transform and can be calculated by

$$\begin{aligned} c_{00} &= \int_0^1 s(t)\varphi_{00}(t)dt \\ d_{jk} &= \int_0^1 s(t)\psi_{jk}(t)dt \end{aligned} \tag{1}$$

where $\varphi_{00}(t)$ is the scaling function and $\psi_{jk}(t)$ are the wavelets. The wavelet at level 0 is called the mother wavelet. Level 0 has the scaling function $\varphi_{00}(t)$ and the wavelet $\psi_{00}(t)$, level 1 has $\psi_{10}(t)$ and $\psi_{11}(t)$, level 2 has $\psi_{20}(t)$, $\psi_{21}(t)$, $\psi_{22}(t)$, as well as $\psi_{23}(t)$, and consequently each higher level has twice as many wavelets. Haar wavelet is one of the many wavelets that exist, and each set of wavelet basis functions has one wavelet transform.

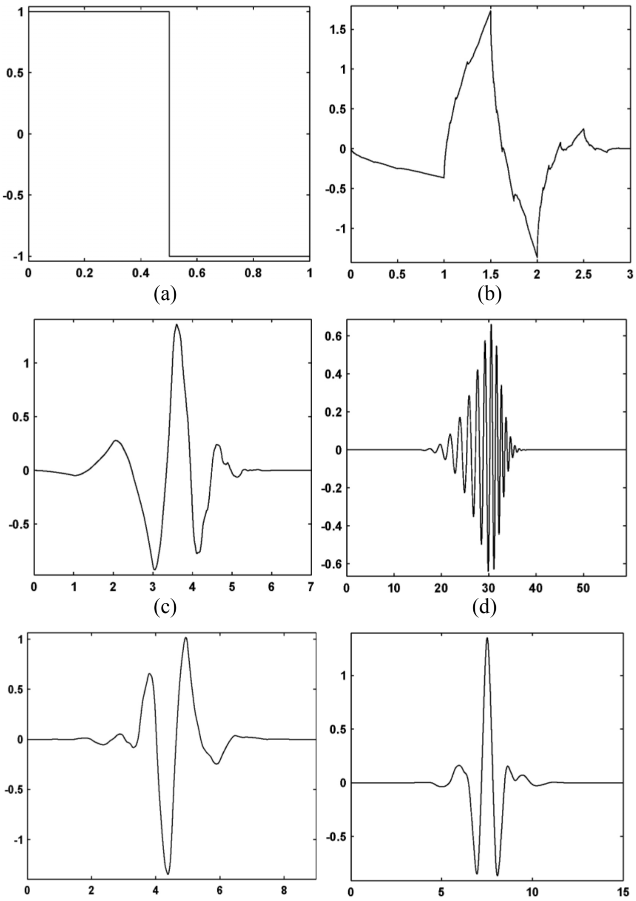

Fig.2 The (a) Haar wavelet, the Daubechies wavelets of order (b) 2, (c) 4, and (d) 30 as well as the Symlet wavelets of order (e) 5 and (f) 8

Scaling functions and wavelets [7] are defined by

$$\begin{aligned} \varphi_{jk}(t) &= 2^{\frac{j}{2}} \varphi_{00}(2^j t - k) \\ \psi_{jk}(t) &= 2^{\frac{j}{2}} \psi_{00}(2^j t - k) \end{aligned} \tag{2}$$

where $0 \le k \le 2^j - 1$. As far as the scaling function coefficients are concerned, j starts at 0 and reaches an upper limit that is defined by the sample rate whereas wavelet coefficients are calculated by starting at the other end.

By considering that scaling functions form a basis for V_j then wavelets form a basis for $V_j^\perp$ which is defined as the orthogonal complement of V_j. The combination of wavelets and scaling functions $V_j \oplus V_j^\perp$ consists a basis for the next higher space V_{j+1}. Different other combinations are possible as well such as all wavelets $\psi_{jk}(t)$ together with $\varphi_{00}(t)$, which is the combination used by the wavelet transform. Considering the case of V_2 which has the basis $b_2 = \{\varphi_{00}, \psi_{00}, \psi_{10}, \psi_{11}\}$ then V_3 has the basis $b_3 = \{\varphi_{00}, \psi_{00}, \psi_{10}, \psi_{11}, \psi_{20}, \psi_{21}, \psi_{22}, \psi_{23}\}$ that consists of b_2 and the basis of $V_2^\perp$. This is how the bases for higher levels are constructed.

Different types of wavelet transforms exist such as the Discrete Wavelet Transform (DWT) where continuous-time functions are mapped to numbers as well as other forms where continuous-time signals correspond to continuous functions and discrete-time signals are matched up with numbers. The discrete wavelet transform [7] is given by

$$\begin{aligned} c_{jk} &= \left\langle s(t) \middle| \varphi_{jk}(t) \right\rangle \\ d_{jk} &= \left\langle s(t) \middle| \psi_{jk}(t) \right\rangle \end{aligned} \tag{3}$$

whereas the inverse discrete wavelet transform is equal to

$$s(t) = \sum_{k=-\infty}^{\infty} c_{Jk}\varphi_{Jk}(t) + \sum_{j=J}^{\infty} \sum_{k=-\infty}^{\infty} d_{jk}\psi_{jk}(t) \tag{4}$$

where J is the starting index. The forward equations describe the decomposition of a signal $s(t)$ whereas the inverse equations express its synthesis. The wavelet transform can be calculated by either starting at level 0 and proceeding to level n or the other way around. As far as the n-level wavelet transform is concerned, there are $2^{n+1} - 1$ wavelet coefficients as well as one scaling function coefficient.

The wavelet transform can be also calculated with the use of a low-pass and a high-pass digital filter in parallel [7]. A discrete-time signal that passes through those filters is then downsampled by 2, which means that every other sample is discarded. Each output produced after the low-pass filter and the downsample operation undergoes the same process again. This is repeated continuously until only one sample is produced as output. The wavelet transform contains this last sample as well as all the samples produced after the high-pass filter and the downsample operation. The number of samples of the original signal is thus the same with the number of samples of its wavelet transform.

The two-scale relation [7] for scaling functions and wavelets is equal to

$$\begin{aligned} \varphi(t) &= \sum_k h_0[k]\sqrt{2} \cdot \varphi(2t-k) \\ \psi(t) &= \sum_k h_1[k]\sqrt{2} \cdot \varphi(2t-k) \end{aligned} \tag{5}$$

where h_0 and h_1 are used to make the association to the scaling functions at the next lower level. $s(t)$ can be either described with scaling functions at level $j+1$ or with scaling functions and wavelets at level j as

$$s(t) = \sum_k c_{j+1,k} 2^{\frac{j+1}{2}} \varphi(2^{j+1}t-k) = \sum_k c_{jk} 2^{\frac{j}{2}} \varphi(2^j t-k) + \sum_k d_{jk} 2^{\frac{j}{2}} \psi(2^j t-k) \tag{6}$$

where $s(t) \in V_{j+1}$. For the scaling function and wavelet coefficients holds

$$\begin{aligned} c_{jk} &= \sum_m h_0[m-2k] c_{j+1,m} \\ d_{jk} &= \sum_m h_1[m-2k] c_{j+1,m} \end{aligned} \tag{7}$$

where h_0 and h_1 represent a low-pass and a high-pass filter, respectively. All the coefficients below level $j+1$ can be thus calculated.

Wavelets can be used in pattern recognition to reduce the dimensionality by extracting features, which are further compared with each other or with already known patterns. As a result they reduce the complexity while keeping important information about the under investigation signal. Wavelets can be also used to identify the place of a discontinuity that is not clearly detectable in the signal. They are thus sometimes capable of revealing information that is hidden in a signal.

3 Recording setup

The aim when recording musical instruments is to capture the produced sound with as much realism as possible. As a result, it is important to place the microphones in an optimal place. The sound radiation pattern plays a significant role in determining the best position to put the microphones.

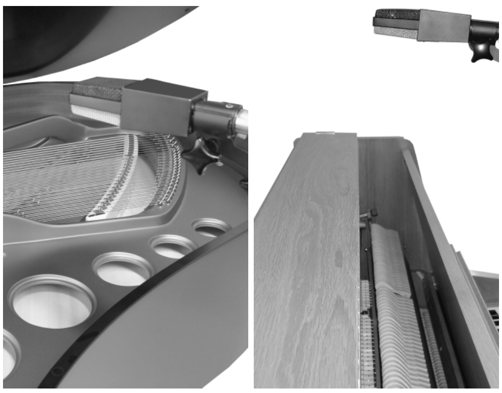

Fig.3 Recording setup where four fully assembled grand pianos as well as four fully assembled upright pianos were used

As far as grand pianos are concerned, a usual position is on the bend and a typical type of microphone is that with a large diaphragm capacitor. As far as upright pianos are concerned, placing a microphone for recording purposes is a difficult task as upright pianos do not efficiently radiate sound as grand pianos do. One of the usual techniques is to place the microphones above the open top or in front of the withdrawn section at the lower front part of the piano.

The recordings performed involve four fully assembled grand pianos as well as four fully assembled upright pianos of different brands. The sample rate and dynamic level used are 44.1 kHz and forte, respectively. One microphone was used and was placed in the middle of the grand piano bend as well as in the middle of the open upright piano top at a distance of approximately 0.25 m from the soundboard for both cases.

4 Comparison of the sound of grand and upright pianos using wavelets

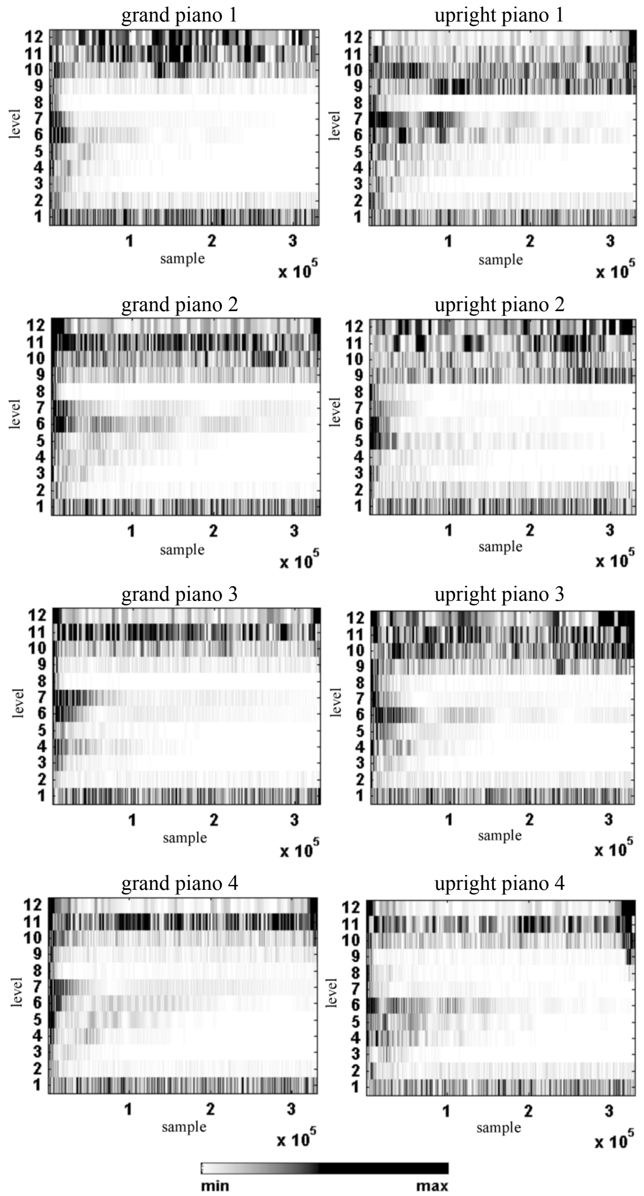

Fig.4 Discrete Wavelet Transform (DWT) of C_4 played by eight pianos and analyzed with Daubechies order 40 wavelet

There are significant differences between Fourier and wavelet analysis. The fact that wavelets are localized both in scale via dilations as well as in time via translations whereas Fourier basis functions are localized only in frequency can be proven advantageous in many cases. Moreover, functions with sharp spikes and discontinuities can be represented with fewer wavelet basis functions, which makes them appropriate for data compression.

Wavelet analysis uses a fully scalable window for the observation of a signal, which is shifted along the signal many times. For every new cycle a slightly shorter or longer window is used. Then the corresponding spectrum is calculated and the results of this calculation are time-scale representations of different resolutions.

Fourier analysis has the shortcoming that the time information is lost in the frequency domain and whereas Short-Time Fourier Transform (STFT) keeps also time information, the observation window used is the same for all frequencies. As a result, wavelet analysis is more appropriate for processing burst-like signals such as piano tones as it keeps time information and uses a variable sized observation window. Moreover, wavelet analysis is capable of highlighting different attributes of a signal.

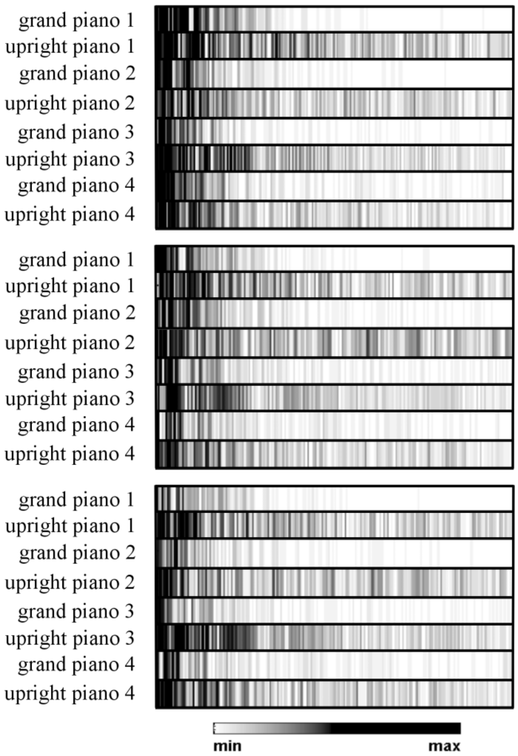

Fig.5 The first second of the DWT at level 8 of C_4 played by eight pianos and analyzed with Daubechies order 40, Symlet order 10, and Coiflet order 5 wavelets

By observing the DWT of C_4 with onset and decay played by four grand and four upright pianos as well as analyzed with Daubechies order 40 wavelet it can be concluded that with the use of level 8 it is possible to distinguish between a grand and an upright piano as the corresponding DWT seems to fade out much faster for a grand than for an upright piano. Other piano tones near C_4 have produced similar results. Symlet order 10 wavelet as well as Coiflet order 5 wavelet have been used in the same way and have produced similar results.

5 Conclusions

The sound of four grand pianos was compared with that of four upright pianos with the use of wavelets. The Discrete Wavelet Transform (DWT) was used for such a comparison that resulted in the classification of the sound of grand and upright pianos. It was found that the DWT of C_4 at level 8 seems to fade out much faster for a grand than for an upright piano. This analysis was performed with Daubechies order 40, Symlet order 10, and Coiflet order 5 wavelets.

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