Binaural Hearing and Systems for Sound Reproduction

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Recent developments in models of binaural hearing can be usefully adapted and extended to provide design tools for engineers engaged in the design of systems for sound reproduction. The particular focus of the work described is upon the development of models that give good statistical predictions of human sound localisation, based upon knowledge of the fluctuating acoustic pressures at the ears. Such models can be applied successfully to the prediction of stereophonic image localisation and reveal a number of important features of localisation relevant to audio system design. Developments will also be described in loudspeaker based systems for binaural reproduction that are finding their way into practical use. Binaural hearing models can be used to provide a preliminary evaluation of the performance of alternative designs. Finally, a brief review will be presented of multi-channel loudspeaker-based systems aimed at "full field" sound reproduction. Again, models of localisation provide some useful guidance for the designers of such systems.

1 Localisation of virtual sound images

1.1 Models of the auditory periphery

The human auditory periphery consists of outer, middle and inner ears and the model described here attempts to capture the main features of each. The head-related transfer function (HRTF) of an individual listener characterises the sound signals along the pathway from the source to the ear drum, and this may be measured and represented in computational models in the form of source position-dependent FIR filters [1]. The transfer function of the middle ear has been shown to roughly follow the inverted form of the equal-loudness contour [2], and this may be approximated by a bandpass filter [3]. The signal processing in the inner ear is considered in two parts. First, the tonotopic arrangement in the basilar membrane is often modelled with a bank of bandpass filters [4], where, in the current model, a fourth-order gammatone filterbank has been employed [5]. The neural transduction in the organ of Corti can be accounted for by a half-wave rectifier, lowpass filter, and a square-root compressor [6]. The former two devices reflect the characteristics of the generation of the neural impulses, e.g. the loss of high-frequency phase information, while the compressor approximates the input-output non-linearity [7].

1.2 Models of binaural processing

The auditory pathway continues to the binaural stage, where the neural firing patterns from the left and right channels are combined to produce an internal representation of sound localisation cues [8]. Most models of this type have employed the Jeffress coincidence detector [9] for the binaural processing, which attempts to explain the neural computation of interaural cross-correlation by axonal delay lines. The Jeffress model of a biological cross-correlator, however, requires additional devices to incorporate the influence of the interaural level difference [4], and the validity of such models is still a matter of debate, especially for mammalians [10].

The current model employs equalisation-cancellation (EC) networks for the binaural processor, which were suggested by Breebaart et al [3]. The neurophysiological background may be weaker than that for Jeffress' model [9], but the model provides a very efficient method for handling the ITD and ILD information simultaneously. There are two main distinctions from the Jeffress model. First, attenuation lines are annexed to the delay lines to induce the characteristic ILD as well as the characteristic ITD. Second, in place of the coincidence detector, the EC process allows subtraction of the neural signals between left and right channels. The output of the EC network in each auditory frequency band is a curved surface representing the excitation-inhibition (EI) cell activity patterns (EI patterns hereinafter) in the domain of characteristic ITD and ILD. The minimum of the patterns indicates the most probable ITD and ILD.

1.3 Models of central processing

As in the case of binaural processing, the neural mechanism operating at the higher decision-making level has yet to be fully understood. Therefore the models of central processing adopted hitherto have been based mainly on broadly accepted assumptions. For example, in models based on the Jeffress coincidence detector, the peak of the interaural cross-correlation or its centroid has been regarded as indicating the locus of the perceived sound image [4], where these measures have often been interpreted in relation to the ITD values corresponding to free-field stimuli [11]. With similar emphasis on the importance of free-field cues, a simple pattern matching process has been assumed in the current model. This is used to simulate central auditory processing. The target EI pattern is compared to the template EI patterns through cross-correlation. These template patterns are established in advance for the azimuthal directions chosen in accordance with the azimuthal resolution of the HRTFs. The output of this process is a probability function $\psi_f(\theta)$ that represents the similarity between the target and template EI patterns in a given frequency band as a function of azimuthal direction.

2 Stereophony

2.1 Two-channel stereophony

In conventional stereophony systems, the position of acoustic images is usually controlled by the amplitude ratio of the input signals to the transducers that are positioned to the listener's front with a 60deg angular aperture. The inter-channel gain ratio is ultimately related to the phase difference between listener's ears, and therefore, amplitude-panning is effective only at low frequencies, below approximately 700 Hz, above which the interaural phase difference becomes ambiguous [12]. Nevertheless, localisation at high frequencies can result from the interaural level difference resulting from the head-shadowing effect [13]. The results of a simulation using the model described above have been found to be consistent with the known characteristics of stereophony systems. As the stimulus frequency increases, the response angle deviates from the target image position, where the model predicts that, between 1 and 3 kHz, the position of the stereophonic image will be mainly overestimated by human subjects, with a maximum bias and variance observed around 2 kHz. Above 3 kHz, the bias reduces, reflecting the influence of the ILD, but the image position becomes ambiguous again around 6 kHz.

The model prediction for the performance of conventional stereophony has been partly verified in a series of listening tests [14]. The participants used an electromagnetic tracking device to report the position of stereophonic images, where the stimulus used was 1/3 octave band noise centred at 7 frequencies from 0.5 kHz to 6 kHz. As shown in Fig. 1, the results of the tests compare successfully with the predictions of the model. In particular, the existence of side images that are quite prominent at 2 kHz and 6 kHz is well predicted by the model. Also, both test and simulation results show that the distribution of front-back reversed responses, especially at frequencies above 2 kHz, does not symmetrically correspond to that of the responses in the correct frontal hemifield. The agreement between the model and subjective responses suggests that the simple pattern-matching procedure that is based on the importance of free-field localisation cues reflects well the actual decision-making processes in the central stage of auditory perception.

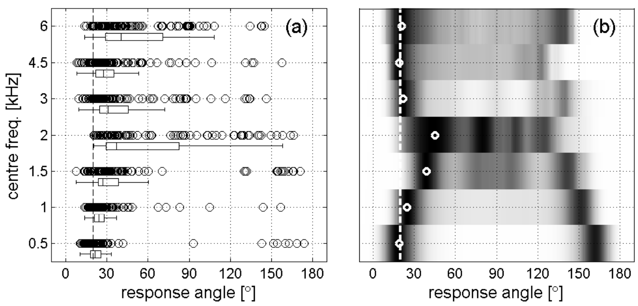

Fig. 1. (a) Subjective responses to stereophonic images created at 20° to the listener's right shown with box-plots (b) Model prediction, ψ_f for the test results shown in (a).

2.2 Multi-channel stereophony

Since two-channel stereophony was first invented in the early 1930's, many different types of multi-channel system have been designed to extend the range of the virtual acoustic field where phantom images can be effectively controlled. One of the simple techniques to create all-surround sound field with an arrangement of multiple loudspeakers may be to activate a pair of transducers in the vicinity of the target image position, and to adjust the inter-channel gain ratio. Nevertheless, the efficiency of such an amplitude-panning scheme is questionable, especially for loudspeaker arrangements to the listener's side. In this case the difference of the path-lengths from one receiver position to the transducer locations becomes indistinguishable between left and right ears. Therefore the interaural phase difference is less affected by the change in the inter-channel gain ratio. In an extreme case where two loudspeakers are positioned symmetrically to the frontal plane, it has been shown recently in listening tests [15] that the image position may not be controlled by the amplitude-panning scheme, and participants reported the position of the louder transducer as the locus of the acoustic image

3 Binaural Reproduction

3.1 Loudspeaker systems

Classical stereophony can be improved upon greatly in terms of the generation of virtual sound images by using binaural reproduction. This involves the very close replication of the signals at the listener's ears that would have been produced by a source at a given spatial position. As an outcome, the listener can perceive sound images with two more additional dimensions in addition to that achieved with multi-channel stereophony. Application of the model described above soon confirms the advantages that can be gained. Binaural reproduction over loudspeakers requires system inversion which is often referred to as cross-talk cancellation [16]-[24]. System inversion is the major factor leading to degradation of the quality of 3D sound reproduction but a simple and effective signal processing principle and loudspeaker design method has emerged which enables a "lossless" crosstalk cancellation process. This is provided by the so-called "Optimal Source Distribution" (OSD)[21] [23]. The principle of binaural reproduction over loudspeaker is to feed the listener's ears independently with the binaural signals that contain auditory spatial information. However, when loudspeakers are used for this purpose, each loudspeaker feeds its signal to both ears. There is a matrix of acoustic paths between the loudspeakers and the listener's ears, and this can be expressed as a matrix of transfer functions (the plant matrix). Independent reproduction of two signals (such as the binaural sound signals) at each ear of a listener can be achieved with two loudspeakers, by filtering the input signals with the inverse of the transfer function matrix of the plant.

3.2 The inverse filter matrix

In the free field case, with a model of two free field monopoles used to represent the loudspeakers, the inverse filter matrix **H** can be obtained from the exact inverse of the plant matrix and can be written as

$$\mathbf{H} = \frac{1}{1-g^2 e^{-2jk\Delta l}} \begin{bmatrix} 1 & -ge^{-jk\Delta l} \\ -ge^{-jk\Delta l} & 1 \end{bmatrix} \qquad (1)$$

where g is the ratio and Δl is the difference in path lengths between the two sources and the two points in space representing the ears of the listener. The magnitude of the elements of **H** show the necessary amplification of the desired signals produced by each inverse filter in **H**. The maximum amplification of the source strengths can be found from the 2-norm of **H** (denoted as $\|\mathbf{H}\|$) which is the largest of the singular values of **H**. These singular values are denoted by σ_o and σ_i [23] and $\|\mathbf{H}\| = \max(\sigma_o, \sigma_i)$ where σ_o corresponds to the amplification factor of the "out-of-phase" component of the desired signals and σ_i corresponds to the amplification factor of the "in-phase" component of the desired signals. $\|\mathbf{H}\|$ changes periodically and has peaks where the wavenumber k and the half-angular separation θ between the loudspeakers satisfy the relationship $k\Delta r\sin\theta = n\pi/2$ with even values of the integer number n. The singular value σ_o has peaks at n = 0, 4, 8, ... where the system has difficulty in reproducing the out-of-phase component of the desired signals, and σ_i has peaks at n = 2, 6, 10, ... where the system has difficulty in reproducing the in-phase component.

3.3 The Optimal Source Distribution

The Optimal Source Distribution (OSD) introduced the idea of a pair of conceptual monopole transducers whose direction varies continuously and a function of frequency while requiring n to be an odd integer number at all frequencies (except at very low frequencies). This relationship ensures that σ_o and σ_i are balanced and that the angular separation of the loudspeakers becomes smaller as frequency becomes higher. Under these circumstances it also follows that,

$$\mathbf{H} \approx \frac{1}{2}\begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \tag{2}$$

i.e., the cross-talk cancellation is nearly perfect simply as a result of a 90 degree phase change of cross-talk path in the inverse filter matrix without any change in amplitude response. The frequency response of the inverse filter is thus flat over all frequencies. Since the sound is always synthesised by constructive interference at all frequencies, there is no dynamic range loss or loss of quality compared to the case without system inversion. This means the system has a good signal to noise ratio and reduced distortion. The sound radiated by the OSD transducer pair is always smaller in directions other than those corresponding to the receiver, and is also smaller than the sound radiated by a single monopole transducer producing the same sound level at the ears. The system does not radiate excessive sound to the surrounding environment and is therefore robust to reflections in a reverberant environment. Furthermore, the radiation pattern becomes constant as a function of frequency and repeats periodically in the listening space. This offers the possibility of the perception of nearly correct binaural signals by multiple listeners [24]. The inverse filters have a flat frequency response so there is no coloration at any location in the listening room. When the listener is far away from the intended listening position, the spatial information perceived may not be ideal. However, the spectrum of the sound signals is not changed by the inverse filters and therefore a listener will continue to perceive correctly reproduced of sound. It has recently been recognised that the performance of the OSD can be improved with the addition of a third loudspeaker channel [24].

4 Full Sound Field Reproduction

4.1 A brief review of recent approaches

Audio systems which attempt the physical reconstruction or the synthesis of a desired sound field over an extensive area have increasingly been the subject of study. In general the reproduced sound field is obtained from the linear superposition of the sound fields generated by the single loudspeakers which constitute the system. A well known technique is "Wave Field Synthesis", which was first proposed by Berkhout [25], and which has subsequently been developed as described in references [26], [27], [28]. This technique has been derived from the Kirchhof-Helmholtz integral equation. This suggests that a generic sound field can be reconstructed using an ideally continuous distribution of monopole-like and dipole-like secondary sources arranged on the boundary $\partial\Omega$ of a spatial volume once the acoustic pressure of the target sound field and its normal derivative are known on $\partial\Omega$. This general formulation has been simplified in order to be adapted to systems composed of a finite number of only monopole-like loudspeakers, arranged in a two dimensional array. Another well known technique is Ambisonics, which was first introduced by Gerzon [29] and has then been further extended to High Order Ambisonics [30], [31], [32]. This technique relies on spherical harmonic analysis of the field to be reproduced for the calculation of the loudspeaker signals. Although the original formulation of Ambisonics was based on perceptual considerations, the work of Daniel et al. [32] has shown that High Order Ambisonics has the features of a sound field reconstruction technique. Other theories which rely on the spherical harmonic analysis of the sound field have been developed more recently [33], [34]. Some techniques have also been proposed, which are based on the solution of an inverse problem [35], [36], [37] and often require the numerical inversion of a frequency dependent matrix. Other original theories such as, among others, [38], [39], [40] and [41] are founded on different principles or are developed from a combination of those mentioned above. In the next section, the outline of an innovative sound field reconstruction theory is briefly reviewed. This method implies the solution of an inverse problem represented by an integral equation of the first kind, and the reader is referred to [42] for a more detailed presentation. The method presented has been developed using some of the analysis described in [43] and [44]. The method could be interpreted as a theoretical extension of the technique proposed in [35] to include the ideal case of an infinite number of secondary sources and an infinite number of receivers arranged on the boundary of the reproduction area. This method has also some common elements with the multi-channel least squares techniques which are largely used in the active control of sound and vibration [45] and with the simple source formulation [46].

4.2 A reconstruction method based on the solution of an integral equation

It is assumed that the target monochromatic sound field $p(\mathbf{x})$ is defined over a simply connected and bounded region of space and that the field is a solution of the homogeneous Helmholtz equation. The propagation is characterised by a wavenumber k and the time dependence $e^{j\omega t}$ is implicitly assumed. Assume also that the value of the acoustic pressure is known on the boundary $\partial\Omega$. As discussed in [42], the knowledge of $p(\mathbf{x})$ on $\partial\Omega$ is sufficient to define the sound field in the interior region Ω, as long as k is not one of the eigenvalues of the negative Laplacian on Ω (i.e. one of the resonance frequencies of a cavity with the same shape of Ω with pressure release boundaries). Let $\partial\Lambda$ be the boundary of a simply connected and bounded region of the space Λ that includes within it the region Ω. Assume that a continuous distribution of secondary sources is arranged on $\partial\Lambda$, and that the electroacoustic transfer function between each secondary source located at $\mathbf{y}$ and the point $\mathbf{x}$ can be represented by the free space Green function $g(\mathbf{y}\,|\,\mathbf{x})$. The reproduced sound field $\hat{p}(\mathbf{x})$ generated by the secondary sources can be described by the single layer potential

$$\hat{p}(\mathbf{x}) = (Sa)(\mathbf{x}) = \int_{\partial\Lambda} g(\mathbf{y}\,|\,\mathbf{x}) a(\mathbf{y}) dS(\mathbf{y}) \qquad (3)$$

where $a(\mathbf{y})$ is a complex valued function, which defines the driving signals of the secondary sources. In the ideal case of $\hat{p}(\mathbf{x})$ being a perfect reconstruction of the target sound field $p(\mathbf{x})$, then $a(\mathbf{y})$ should be the exact solution of the integral equation

$$p(\mathbf{x}) = (Sa)(\mathbf{x}) \quad \mathbf{x} \in \partial\Omega \ . \qquad (4)$$

This is an integral equation of the first kind and therefore constitutes an ill-posed inverse problem [43]. This means that it is not possible to compute an exact solution, but it is nevertheless possible to calculate an approximate solution following, for example, the procedure illustrated in [42]. A solution to (4) could be computed from

$$a(\mathbf{y}) = \sum_{n=1}^{N} \frac{1}{\sigma_n} a_n(\mathbf{y}) \int_{\partial\Omega} \overline{p_n(\mathbf{x})} p(\mathbf{x}) dS(\mathbf{x}) \ . \qquad (5)$$

where the over-bar denotes the complex conjugate. Equation (5) is derived from the singular value decomposition of the operator S, explained in more detail in [42] and [43]. The functions $p_n(\mathbf{x})$ and $a_n(\mathbf{y})$ are the singular functions S that constitute two sets of mutually orthogonal functions. The real and positive values σ_n are the singular values of the operator S and they can be ordered by decreasing magnitude. They accumulate, in general, at zero [43]. The reconstructed sound field corresponds, in the case of $a(\mathbf{y})$ defined as in (5), to the orthogonal projection of the target sound field $p(\mathbf{x})$ on the N-dimensional subspace identified by the range of the operator S. In this theoretical formulation N is generally infinite, but the finite number of secondary sources used in practical cases implies as a consequence that the number of (non zero) singular values is also limited and does not exceed the number of secondary sources. The roll off of the singular values implies that the solution (5) can be very unstable, with small values of σ_n potentially resulting in catastrophic amplification of the effect of data errors, typical of ill-conditioned problems [43], [45]. It is therefore possible to make the solution (5) more stable by applying a regularisation scheme, such as Tikhonov regularisation or spectral cut-off [43], [36].

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