Acoustics'08 Paris
June 29-July 4, 2008
www.acoustics08-paris.org

A novel modelling approach for sound propagation analysis in a multiple scatterer environment

Bert Van Genechten, Bart Bergen, Bert Pluymers, Dirk Vandepitte and Wim Desmet

K.U.Leuven - Dept. of Mechanical Engineering, Celestijnenlaan 300B - bus 2420, 3001 Heverlee, Belgium
bert.vangenechten@mech.kuleuven.be

The Wave Based Method (WBM) is an alternative deterministic prediction method for steady-state acoustic problems, which is based on an indirect Trefftz approach. It uses wave functions, which are exact solutions of the underlying differential equation, to describe the dynamic field variables. As a result, the WBM does not require a dense element discretization, as opposed to the commonly used element based prediction techniques. The relatively smaller system and the absence of pollution errors make the WBM very suitable for the treatment of mid-frequency problems, where the element-based methods are no longer feasible due to the large computational cost associated with the fine discretizations needed to retain accurate results.
This paper introduces a new modelling concept for the efficient treatment of radiation and scattering problems when multiple distinct objects are involved. Each object is assigned to a separate level, where it is treated using existing wave based modelling techniques. An adapted weighted residual formulation links the multiple levels, yielding a multi-level wave based model describing the entire problem.

1 Introduction

Underwater acoustics, utilizing SONAR technology, is by far the most commonly used technique for detection, assessment, and monitoring of underwater physical and biological characteristics and objects which may be floating in the water, lying on the seafloor, or buried below the sediment. In some applications, such as low frequency detection and classification of structurally complex objects immersed in a fluid, signal processing techniques must be aided by a-priori model based knowledge of the target echo. Because objects of interest in practical applications often consist of a configuration of multiple geometrically complex objects with a detailed internal structure, mathematical techniques capable of dealing with generic geometries, and capable of coupling different physical domains (i.e. the solid and the fluid domains) must be used. Moreover, since the frequency range of interest is very wide, the development of efficient numerical modelling techniques for the study of underwater scattering in the frequency domain are of great interest.

Both the Finite Element Method (FEM) and the Boundary Element Method (BEM) are well established deterministic CAE tools which are commonly used for the analysis of real-life structural-acoustic problems. The FEM (1) discretizes the entire problem domain into a large but finite number of small elements. Within these elements, the dynamic response variables are described in terms of simple, polynomial shape functions. Because the FEM is based on a discretization of the problem domain into small elements, it cannot inherently handle unbounded problems. An artificial boundary is needed to truncate the unbounded problem into a bounded problem. Special techniques are then required to reduce spurious reflection of waves at the truncation boundary. Three strategies are applied to this end: absorbing boundary conditions, infinite elements or absorbing layers (2). The BEM (3) is based on a boundary integral formulation of the problem. As a result, only the boundary of the considered domain has to be discretized. Within the applied boundary elements, some acoustic boundary variables are expressed in terms of simple, polynomial shape functions, similar to the FEM. Since the boundary integral formulation inherently satisfies the Sommerfeld radiation condition, the BEM is particulary suited for the treatment of problems in unbounded domains.

However, since the simple shape functions used in both the FEM and BEM are no exact solutions of the governing differential equations, a very fine discretization is required to suppress the associated pollution error (4) and to obtain reasonable prediction accuracy. The resulting large numerical models limit the practical applicability of these methods to low-frequency problems (5; 6), due to the prohibitively large computational cost.

The Wave Based Method (WBM) (7) is an alternative deterministic technique for the analysis of vibro-acoustic problems. The method is based on an indirect Trefftz approach (8), in that the dynamic response variables are described using wave functions which exactly satisfy the governing differential equation. In this way no approximation error is made inside the domain. However, the wave functions may violate the boundary and continuity conditions. Enforcing the residual boundary and continuity errors to zero in a weighted residual scheme yields a small matrix equation. Solution of this matrix equation results in the contribution factors of the wave functions used in the expansion of the dynamic field variables. The WBM has been applied successfully for many steady-state structural dynamic problems (9), interior acoustic problems (10) and interior and exterior vibro-acoustic problems (11). It is shown that, due to the small model size and the enhanced convergence characteristics, the WBM has a superior numerical performance as compared to the element based methods. As a result, problems at higher frequencies may be tackled, making it an attractive technique for studying underwater scattering phenomena and sonar applications.

This paper discusses a new modelling concept for acoustic radiation and scattering problems, particularly suited for the treatment of problems involving different distinct objects (scatterers). The main idea is to assign each object in the problem to a particular model level. In this level, the scattering on this particular object is treated using existing wave based modelling techniques for unbounded problems. The general problem is then composed by combining the different levels through an adapted weighted residual formulation.

The first part of this paper briefly addresses the general acoustic problem setting and the WB modelling techniques used for the models in each level. A second part is devoted to the discussion of the new multi-level modelling concept. Finally, the new method is applied to an underwater scattering problem, in order to illustrate its potential and validate the accuracy of the method.

2 Problem description

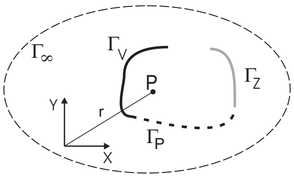

Figure 1: A 2D unbounded acoustic problem

Consider a general 2D unbounded acoustic problem as shown in figure 1. The steady-state acoustic pressure inside the problem domain is governed by the inhomogeneous Helmholtz equation:

$$\nabla^2 p(\boldsymbol{r}) + k^2 p(\boldsymbol{r}) = -j\rho_0\omega\delta(\boldsymbol{r}, \boldsymbol{r_q})q \qquad (1)$$

with ω the circular frequency and $k = \omega/c$ the acoustic wave number. The acoustic fluid is characterised by the density ρ_0 and the speed of sound c. The fluid is excited by a cylindrical acoustic volume velocity source q. The problem boundary Γ is constituted of 2 parts: the finite part of the boundary, Γ_b, and the boundary at infinity, Γ_∞. Based on the three types of commonly applied acoustic boundary conditions, the finite boundary can be further divided in three non-overlapping parts: $\Gamma_b = \Gamma_v \cup \Gamma_p \cup \Gamma_Z$. If we define the velocity operator $\mathcal{L}_v(\bullet)$ as:

$$\mathcal{L}_v(\bullet) = \frac{j}{\rho_0\omega}\frac{\partial \bullet}{\partial n}, \qquad (2)$$

we can write the boundary condition residuals:

$$\boldsymbol{r} \in \Gamma_v : \quad R_v = \mathcal{L}_v(p(\boldsymbol{r})) - \overline{v}_n(\boldsymbol{r}) = 0\ , \qquad (3)$$

$$\boldsymbol{r} \in \Gamma_p : \quad R_p = p(\boldsymbol{r}) - \overline{p}(\boldsymbol{r}) = 0\ , \qquad (4)$$

$$\boldsymbol{r} \in \Gamma_Z : \quad R_Z = \mathcal{L}_v(p(\boldsymbol{r})) - \frac{p(\boldsymbol{r})}{\overline{Z}_n(\boldsymbol{r})} = 0\ , \qquad (5)$$

where the quantities $\overline{v}_n$, $\overline{p}$ and $\overline{Z}_n$ are, respectively, the imposed normal velocity, pressure and normal impedance. At the boundary at infinity Γ_∞ the Sommerfeld radiation condition for outgoing waves is applied. This condition ensures that no acoustic energy is reflected at infinity and is expressed as

$$\lim_{|\boldsymbol{r}|\to\infty}\left(\sqrt{\boldsymbol{r}}\Big(\frac{\partial p(\boldsymbol{r})}{\partial|\boldsymbol{r}|} + jkp(\boldsymbol{r})\Big)\right) = 0\ . \qquad (6)$$

Solution of the Helmholtz equation (1) together with the associated boundary conditions (3), (4), (5) and (6) yields a unique acoustic pressure field $p(\boldsymbol{r})$.

2.1 The wave based method

The Wave Based Method (WBM) (7) is a numerical modelling method based on an indirect Trefftz approach for the solution of steady-state acoustic problems in both bounded and unbounded problem domains. The field variables are expressed as an expansion of wave functions, which inherently satisfy the governing equation, in casu the Helmholtz equation (1). The degrees of freedom are the weighting factors of the wave functions in this expansion. Enforcing the boundary and continuity conditions using a weighted residual formulation yields a system of linear equations whose solution vector contains the wave function weighting factors.

Partitioning into subdomains
When applied for bounded problems, a sufficient condition for the WB approximations to converge towards the exact solution, is convexity of the considered problem domain (7). In a general acoustic problem, the acoustic problem domain may be non-convex so that a partitioning into a number of convex subdomains is required.

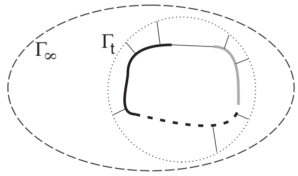

Figure 2: A WB partitioning of the 2D unbounded problem

When the WBM is applied for unbounded problems, an initial partitioning of the unbounded domain into a bounded and an unbounded region precedes the partitioning into convex subdomains (10). Figure 2 illustrates the principle. The unbounded acoustic problem domain is divided into two non-overlapping regions by a truncation curve Γ_t. The unbounded region exterior to Γ_t is considered as one acoustic subdomain.

Acoustic pressure expansion
The steady-state acoustic pressure field $p^{(\alpha)}(\boldsymbol{r})$ in an acoustic subdomain $\Omega^{(\alpha)}$ ($\alpha = 1 \dots N_\Omega$, with N_Ω the number of subdomains) is approximated by a solution expansion $\hat{p}^{(\alpha)}(\boldsymbol{r})$:

$$\hat{p}^{(\alpha)}(\boldsymbol{r}) = \sum_{w=1}^{n_w^{(\alpha)}} p_w{}^{(\alpha)}\Phi_w^{(\alpha)}(\boldsymbol{r}) + \hat{p}_q^{(\alpha)}(\boldsymbol{r}). \qquad (7)$$

The wave function contributions $p_w^{(\alpha)}$ are the weighting factors for each of the selected wave functions $\Phi_w^{(\alpha)}$. For a complete description of the functions $\Phi_w^{(\alpha)}$ the reader is referred to PLUYMERS (10).

Wave based model
The function expansions used guarantee compliance with the Helmholtz equation inside the domain and the Sommerfeld radiation condition at infinity. The boundary conditions and subdomain continuity are enforced by means of a weighted residual formulation. This yields a square system that is solved for the unknown wave function contributions. For a detailed description of the system matrices, the reader is referred to PLUYMERS (10).

3 A multi-level concept in Wave Based Modelling

The WBM has shown to be efficient in modelling 2D acoustic radiation problems (11). However, when multiple acoustic scatterers are present, the method's efficiency tends to deteriorate. This is due to the fact that the circular truncation line Γ_t, whose interior domain is modelled using a set of wave function expansions for bounded subdomains, needs to enclose all the scatterers at once. As a result many unbounded wave functions need to be included in the model in order to accurately couple the spatial resolution of the expansions in the bounded and unbounded subdomains. Moreover, since all the scatterers are included within a single circle, a complex partitioning of the interior is often needed in order to satisfy the requirement of using convex bounded subdomains.

To remedy this, the concept of multi-level modelling is introduced. The main idea of the multi-level WBM approach is to consider the multiple objects in the problem as different 'levels' of the problem. In every level, the scattering of one particular object is studied. The incident field for this problem is the scattered field from the other objects and the external excitations (plane wave or point source). Since this incident field is the result of a scattering calculation in the other levels, all the calculations have to be carried out simultaneously. A weighted residual formulation links the levels, yielding one system for the coupled problem composed of all the levels. This system can be solved for all the unknown weighting factors. Using these factors, the scattering field can be calculated in each level. The total resulting pressure field is then composed from the fields of all levels. This procedure is illustrated in figure 3.

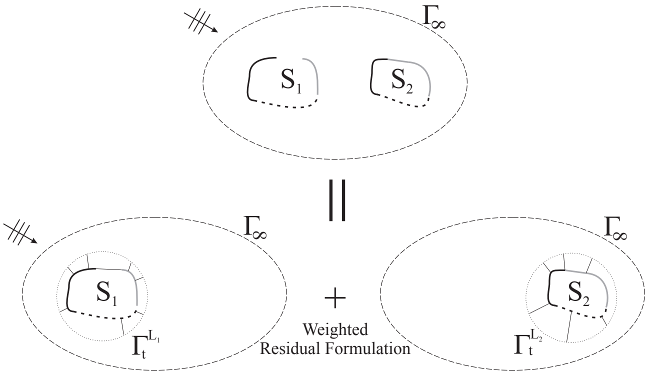

Figure 3: Graphic representation of the multi-level modelling concept

When looking at the different levels, it is clear that there are parts of the problem for which the levels geometrically overlap. More precisely, this overlap takes place in the unbounded part of the wave models of the different levels. Consequently, the pressure field in this unbounded part of the total problem will be described as a summation of the fields present in each level:

$$p_{ub,total} = \sum_{i=1}^{n_\lambda} \boldsymbol{\Phi_w^{Li}} \cdot \boldsymbol{p_w^{Li}}. \tag{8}$$

where n_λ is the number of different levels in the problem. Each of the sets $\Phi_w^{(Li)}$ is a complete set for the Neumann problem on its associated truncation circle Γ_t^{Li}.

With the function set chosen, the wave model for this domain can now be constructed by enforcing the boundary conditions through the weighted residual formulation. The residuals of the boundary conditions are now evaluated using the new, combined wave function set. Similarly, continuity conditions can be applied using the same types of residuals. This continuity can be used to couple the multi-level unbounded wave set with bounded domains in the appropriate level, in the same way as a coupling would be set up between a conventional unbounded and bounded wave domain. The bounded domain in each of the levels can then further be modelled using the conventional wave based domain division techniques and function sets. If the test functions used in the weighted residual are random functions, then the residuals on the boundary and continuity conditions are forced to zero in an integral sense, resulting in a numerical solution for the physical problem. To obtain a numerical model which can be solved, these test functions are written as a randomly weighted sum of certain basis functions t_a:

$$\tilde{p}^{(\bullet)}(\boldsymbol{r}) = \sum_{a=1}^{n_a^{(\bullet)}} \tilde{p_a}^{(\bullet)} t_a^{(\bullet)}(\boldsymbol{r}) = \boldsymbol{t}^{(\bullet)}(\boldsymbol{r}) \tilde{\boldsymbol{p}}_{\boldsymbol{w}}{}^{(\bullet)}, \tag{9}$$

with $\tilde{p}_a$ random weighting factors. The choice of the basis functions needs to be such that this basis is rich enough to be combined to any field on the boundary considered, but may vary for different parts of the boundary. When integrating the boundary of a conventional (bounded) domain, an expansion in terms of the same basis functions as used to describe the acoustic variables can be used (like in de the widely used Galerkin approach). For the multi-level unbounded domain, an alternative selection of test functions is proposed. Since the unbounded basis functions Φ_w^{Li} for each of the levels are chosen such that they can accurately approximate any field on the associated truncation Γ_t^{Li}, this set will suffice as basis for the test functions on this part of the boundary.

4 Numerical validation example

This section discusses a numerical underwater acoustics example which illustrates the applicability of the proposed concept for complex scattering calculations. The configuration studied is shown in figure 4. To tackle this problem with the WBM, the problem domain is partitioned in 10 bounded subdomains. Each object is considered in a separate level, yielding in total 5 levels in the model.

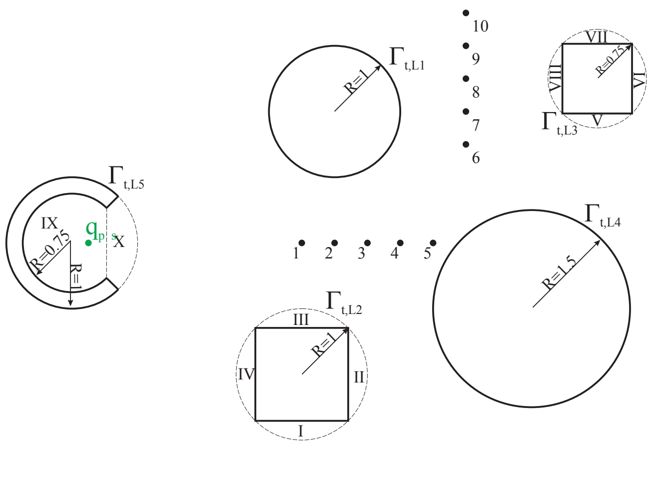

Figure 4: Problem definition

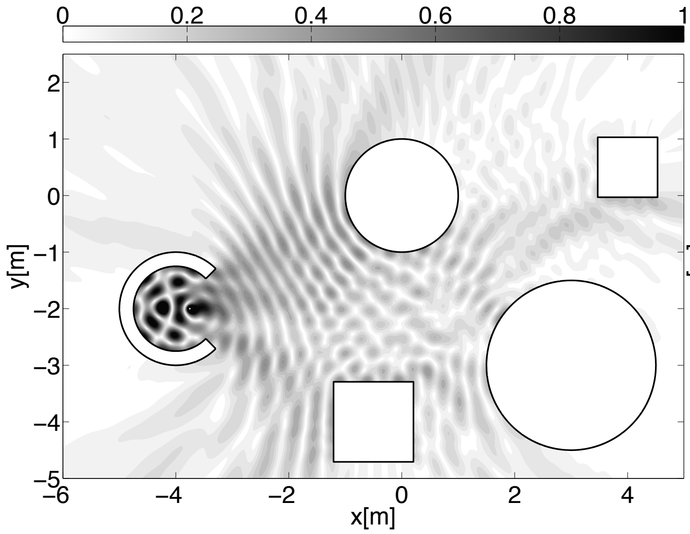

Figure 5: Pressure Amplitude [Pa], $3000Hz$

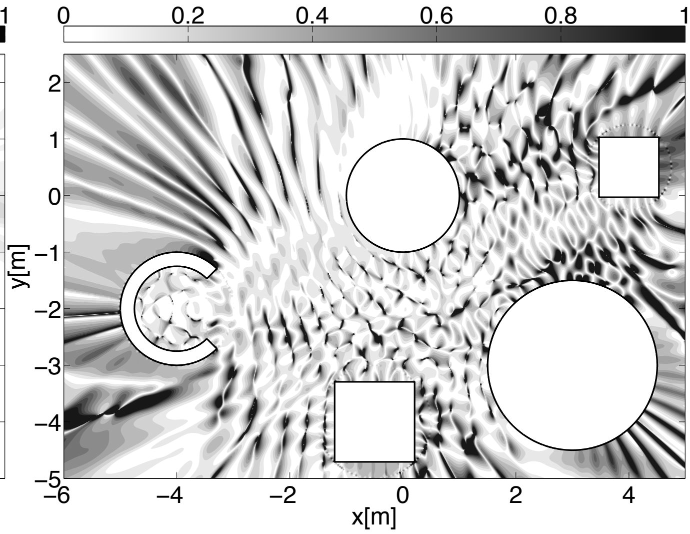

Figure 6: Relative pressure amplitude error [%], $3000Hz$

The acoustic fluid is water ($c = 1500m/s, \rho_0 = 1000kg/m^3$). The system is excited by a point source in the c-shape with amplitude $q = 1$.

Several indirect variational BEM models are built for this problem using *LMS/Sysnoise Rev5.6* . The mesh details are given in table 1. The number of DOF's in the wave model used to solve this problem varies with the frequency, and ranges between 1225 and 2347 and the model is constructed such that the calculation time to calculate a frequency response function (FRF), consisting of 900 frequency lines between 3000 and $7500Hz$ is the same as the time needed by the most coarse BE model. All calculations are performed on a 2.66GHz Linux-based Intel Xeon system.

	element size	♯ DOF	calculation time [s]	ϵ_{av} [dB]
BEM	$2.5mm$	27484	540000	/
	$5mm$	13536	83160	0.7460
	$7.5mm$	9160	27450	1.9382
	$12.5mm$	5498	6030	5.2144
WBM	/	1225-2347	5760	0.7431

Table 1: Model information

The contours in figure 5 show the amplitude of the acoustic pressure field due to the acoustic point source at a frequency of $3000Hz$, calculated using the WBM. In figure 6, the relative error of this pressure amplitude with respect to the results from the most detailed BEM calculation is plotted. It is observed that the errors remain well within the range of $0 - 1\%$, except at the pressure nodal lines, where the error calculation itself is inaccurate due to almost-zero division. The errors are also higher along the coupling arcs between bounded domains and the multi-level unbounded domain. These coupling errors do however not influence the prediction accuracy in the rest of the problem domain.

Figure 7 compares the frequency response function of the acoustic pressure amplitude in response point 6 indicated in figure 4 for the BEM reference model and the WB model. The good match between the WBM and the BEM reference is evident from this figure. A more precise comparison is made in figure 8 where the prediction errors (averaged over all the response points in figure 4) for three BEM models and one WBM model are shown. It is clear that the WBM result (bottom figure) is far more accurate than the one obtained by a BEM model with the same calculation time (top figure). As shown in the two middle figures, the BEM mesh needs to be refined twice in order to obtain the same overall average prediction accuracy as the WBM calculation. This is also indicated by the frequency averaged prediction errors ϵ_{av} in table 1. This results in an average computation time of $92.4s$ per frequency. It can be concluded that for this validation example the WBM efficiency is better by a factor of about 14. This clearly illustrates the advantageous properties of the proposed modelling concept.

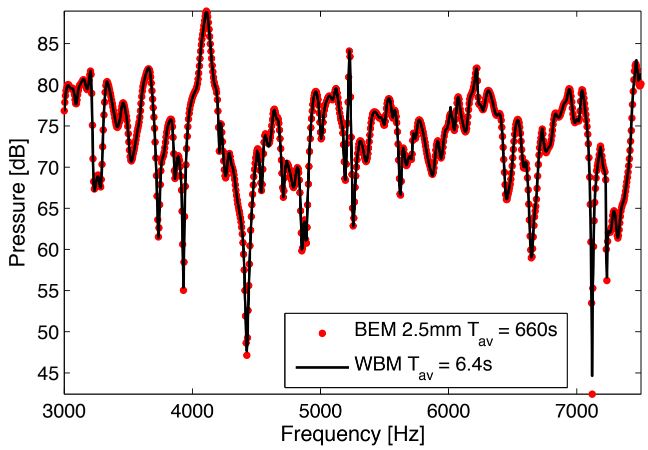

Figure 7: Pressure amplitude FRF [dB re 2.10^{-5}Pa] comparison: WBM and BEM reference

5 Conclusion

This paper discusses a new modelling concept, particulary suited for the treatment of scattering problems involving multiple objects. The main idea of the approach is to consider the multiple objects in the problem as different 'levels' of the problem. Each level considers the scattering on one particular object, using existing WBM techniques for bounded and unbounded problems. A

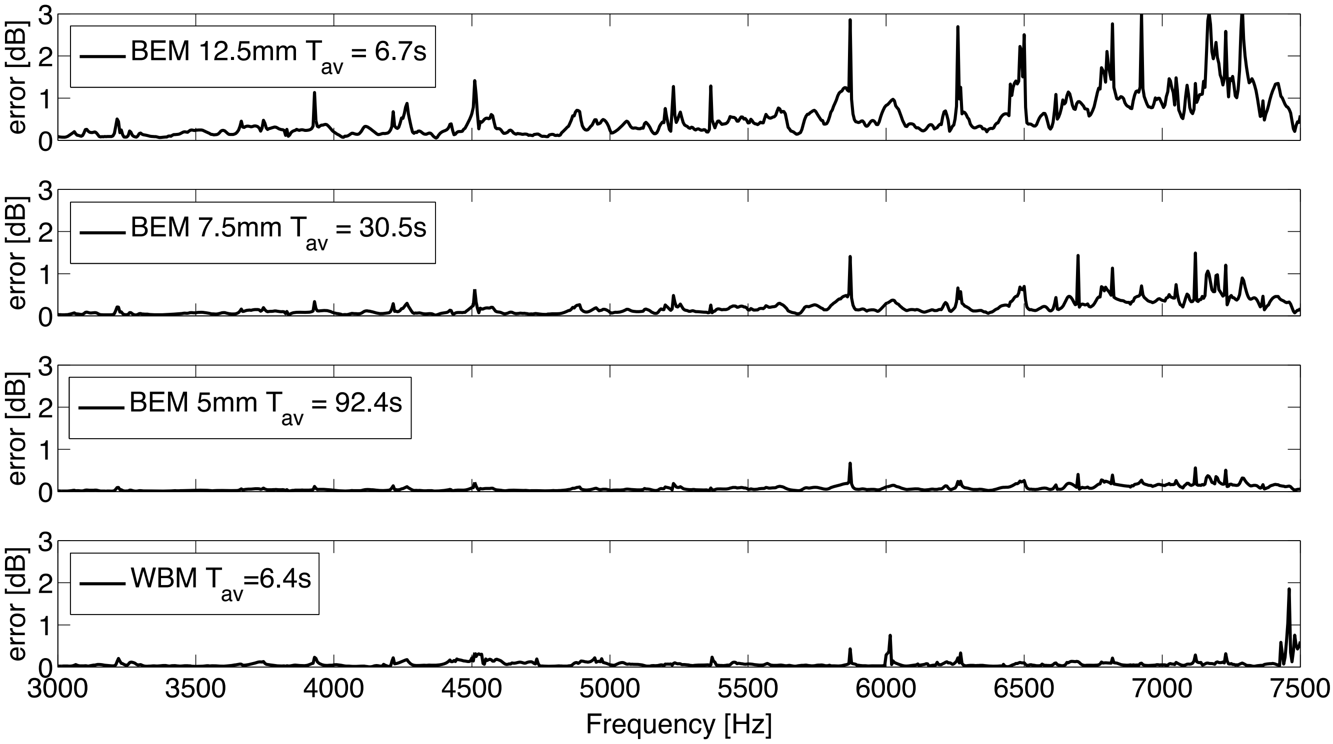

Figure 8: Pressure amplitude error [dB re 2.10^{-5}Pa] comparison: WBM and BEM models

special compound wave function set for the unbounded part and an adapted weighted residual formulation link the different levels together, yielding a single multi-level system, describing the entire problem.

The new method is validated on a numerical example, indicating both the excellent accuracy and the superior numerical performance as compared to the BEM. This reduction in computational load, combined with the lack of pollution errors, makes the WBM particulary suited for the treatment of multiple scatterer problems in an extended frequency range, as compared to the element based methods.

Acknowledgements

The research of Bart Bergen is funded by a Ph.D. grant of the Institute for the Promotion of Innovation through Science and Technology in Flanders (IWT-Vlaanderen) and Bert Van Genechten is a Doctoral Fellow of the Fund for Scientific Research - Flanders (F.W.O.), Belgium.

References

[1] O. C. Zienkiewicz, R. L. Taylor, and J. Z. Zhu. "The Finite Element Method - Vol. 1: Its Basis & Fundamentals". *Butterworth-Heinemann, 6th edition,* 2005.

[2] L. L. Thompson. "A review of finite-element methods for time-harmonic acoustics". In *Journal of the Acoustical Society of America*, 119:1315–1330, 2006.

[3] O. Von Estorff. "Boundary Elements in Acoustics: Advances and Applications". *WITpress,* 2000.

[4] P. Bouillard and F. Ihlenburg. "Error estimation and adaptivity for the finite element method in acoustics: 2D and 3D applications". *Computer Methods in Applied Mechanics and Engineering*, 176:147–163, 1999.

[5] R. Freymann. "Advanced Numerical and Experimental Methods in the Field of Vehicle Structural-Acoustics". *Hieronymus Buchreproduktions GmbH, München,* 2000.

[6] S. Marburg. "Six boundary elements per wavelength: is that enough?" *Journal of Computational Acoustics*, 10:25–51, 2002.

[7] W. Desmet. "A wave based prediction technique for coupled vibro-acoustic analysis". *KULeuven, division PMA,* PhD. thesis 98D12, 1998.

[8] E. Trefftz. "Ein Gegenstück zum Ritzschen Verfahren". In *Proceedings of the 2^{nd} International Congress on Applied Mechanics, Zurich, Switzerland*, pages 131–137, 1926.

[9] C. Vanmaele, D. Vandepitte, and W. Desmet. "An efficient wave based prediction technique for plate bending vibrations". *Computer Methods in Applied Mechanics and Engineering (CMAME)*, 196:3178 – 3189, 2007.

[10] B. Pluymers, B. Van Hal, D. Vandepitte, and W. Desmet. "Trefftz-based methods for time-harmonic acoustics". *Archives of Computational Methods in Engineering (ARCME)*, DOI: 10.1007/s11831-007-9010-x:343–381, 2007.

[11] B. Pluymers, W. Desmet, D. Vandepitte, and P. Sas. "On the Use of a Wave Based Prediction Technique for Steady-State Structural-Acoustic Radiation Analysis". *Journal of Computer Modeling in Engineering & Sciences*, 7(2):173–184, 2005.