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Generation and Evolution of Cavitation in Magma under Decompression Waves

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It is well known that a compressed magma contains enough large amount of dissolved water (up to 5-7% of weight fraction). During eruption under decompression wave a magma becomes oversaturated. Dissolved gas comes out from solution forming a separate phase (bubbles). If the liquid magma has a high viscosity, the gas is not able to expand very easily, pressure is increasing inside of bubble, and they burst explosively fragmenting a liquid magma into clots. The solidified clots become bombs or volcanic ash. Development of these processes, as was proved, depends on four factors: an amount of gas dissolved in a magma, a viscosity of magma, the rate of magma decompression and the number of nucleation sites. The paper presents the full system of the equations for the study of the initial stage of explosive eruption based on the kinetic theory of phase transformations. The problem of nucleation frequency in a liquid magma as well as the dynamics of cavitation development was numerically studied, taking into account the gravity, special model of diffusive zones and dynamically changing viscosity.

1 Introduction

According to data of [1] silica-rich magma is noticeably colder, it can be by ten orders of magnitude more viscous than basalt magma, and its eruption has an explosive character. The volcano itself is actually a hydrodynamic system consisting of the magma chamber and a vertical channel filled by high-pressure hot magma, separated from a crater or free part of channel by a diaphragm (plug or dome). This is a typical scheme of a hydrodynamic shock tube in which the working sections are both the high-pressure section where the initial stage of cavitation development behind the front of the decompression wave is considered and the low-pressure chamber where the process of liquid spreading is studied. Magma possesses unique physical and chemical properties. The most important of them are the presence of dissolved substances, such as water whose concentration can reach 5–7% (wt.), and the high viscosity, which varies from 10^2 to 10^{12} Pa $\cdot$ sec depending on the concentration of the dissolved gas and crystallites contained therein. Among many papers dealing with this issue, let us note the series of investigations [2, 3] with an attempt to simulate eruption dynamics in the general formulation. A large class of papers involves consideration of particular processes that accompany this phenomenon. These are papers on growth dynamics of a single bubble in a viscous gas-saturated melt [4, 5, 6] and associated thermal effects, mechanism of magma solidification under decompression, etc. Nevertheless, despite significant efforts undertaken to study this phenomenon, there are still many issues that require special consideration. In particular, the initial stage of eruption, when cavitation nuclei are generated in magma under rapid unloading, has not been adequately studied. The present paper deals with modeling of this process in heavy magma.

2 Formulation of the Problem.

A vertical column of a gas-saturated magma melt of height H in the gravity field is adjacent to the magma chamber at the bottom and is separated by a diaphragm from the ambient medium on the top (atmospheric pressure is denoted by p_0). The initial pressure in magma in the column–chamber system corresponds to the pressure of magma in the chamber with allowance for hydrostatics: $p_i(z) = p_{\rm ch} - \rho_0 g z$, where ρ_0 is the magma density and $p_{\rm ch}$ is the pressure at $z = 0$. We assume that the gas dissolved in magma has initially an equilibrium concentration $C^{\rm eq}(p) = K_{\rm H}\sqrt{p}$ where $K_{\rm H}$ is Henry's constant and correspondingly, $C_i(z) = C^{\rm eq}(p_i(z))$.

At the initial time ($t = 0$), the diaphragm confining the melt is broken, the surface $z = H$ becomes free, and a rarefaction (decompression) wave starts propagating vertically downward over a magma and results in spontaneous nucleation and growth of gas bubbles in the melt volume. The pressure in the magma chamber (at the boundary $z = 0$) is retained constant during the entire process. The process considered is described by the one-dimensional system (1)-(4) which includes the conservation laws and kinetic equations:

$$\rho_t + (\rho v)_z = 0, v_t + v v_z = -\frac{1}{\rho} p_z - g + \frac{1}{\rho}\Big(\mu v_z\Big)_z,$$

$$\mu = \mu^* \exp(\frac{E_\mu(C)}{k_{\rm B}T}), \tag{1}$$

$$p = p_0 + \frac{\rho\, c^2}{n}\Big\{\Big(\frac{\rho}{\rho_0(1-k)}\Big)^n - 1\Big\},$$

$$J = J^* \exp(\frac{-W_*}{k_{\rm B}T}), \tag{2}$$

$$R\ddot{R} + (\frac{3}{2})\dot{R}^2 = \frac{p_{\rm g} - p}{\rho} + \frac{4\nu\dot{R}}{R}, \quad (\frac{4}{3})p_{\rm g}R^3 = (\frac{m_{\rm g}}{M})k_{\rm B}T,$$

$$\frac{dm_{\rm g}}{dt} = 4\pi R^2 \rho D\Big(C_r\Big)_R. \tag{3}$$

$$X_D(t) = 1 - \exp\Big(-\int_0^t J(\tau)\, v_D(t-\tau)\, d\tau\Big),$$

$$N_b(t) = \int_0^t J(\tau)\,(1 - X_D(\tau))\, d\tau. \tag{4}$$

Here, v, ρ, p, and μ are the mean velocity, density, pressure, and viscosity of the medium, where $E_\mu(C) = E^*_\mu(1 - k_\mu C)$ is the activation energy, [7], the equation of two-phase liquid state and the frequency of spontaneous nucleation J, the Rayleigh equation, the state equation of ideal gas and diffusion equation.

The volume concentration k of gas phase is determined on the basis of the results obtained in [8]. The pressure $p_{\rm g}$ is determined by the diffusion gas flow from the supersaturated melt to the bubble.

As it follows from Fig. 1 the dissolved gas concentration C(r) in the melt decreases with approaching

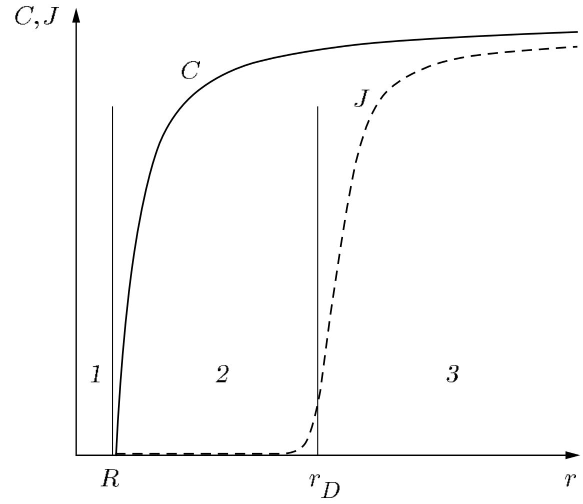

Figure 1: Dissolved gas concentration (C) and nucleation frequency (J) vs. coordinate r:zone (1)-bubble,(2)-diffusion layer,(3)-nucleation region.

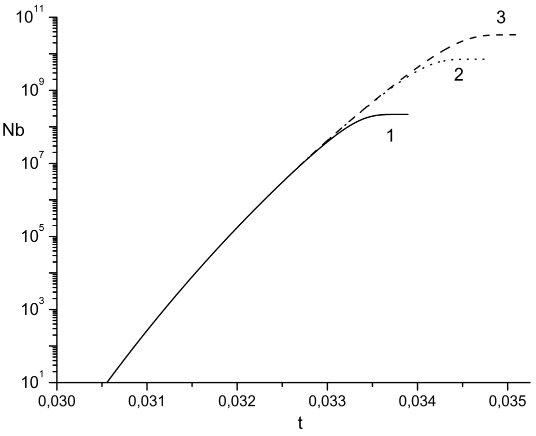

Figure 2: Dynamics of nuclei formation Nb m^{-3} vs.t, sec, for r_D values - 1/50 (1), 1/10 (2), 1/2 (3)

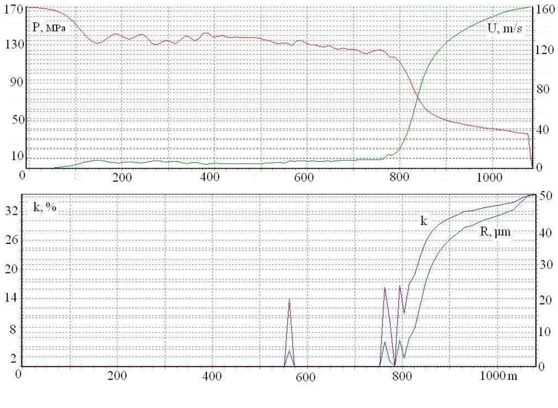

Figure 3: Fields of pressure $p(z)$, mass velocity $u(z)$ as well as distributions of bubble radii $R(z)$ and concentration $k(z)$ for $t \approx\ 0,5\ s$;

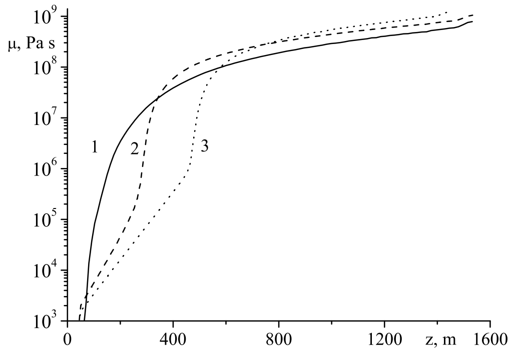

Figure 4: Distribution of viscosity μ vs. z for t = 3.1, 4.6, 6.2s

the growing bubble, and, hence, the notion of a diffusion boundary layer forming around the bubble can be introduced. Because of the strong dependence of the nucleation frequency J on supersaturation, we can assume in the first approximation that nucleation of bubbles occurs only outside the diffusion layer Fig. 1, domain 3. The diffusion-layer thickness was determined as $r_D = J(r_D)/J(r \to \infty)$. This approach allows one to write the expression for the total volume of diffusion layers $X_D(t)$ around the bubbles per unit volume of the melt and the number of bubbles N_b formed in a unit volume during the time t, Eq 4. As it follows from Eq 4, function $X_D(t)$, the bubble nucleation is terminated when the diffusion layers of the growing bubbles completely cover the entire volume ($X_D \to 1$). According to the estimates [8], the characteristic time of nucleation is significantly smaller than the time of the entire gas-release process. It is clear that N_b depends on r_D values: Fig. 2 presents this function dynamics vs. t, sec for three values of r_D - 1/50 (1), 1/10 (2), 1/2 (3). The results presented below correspond to case (2).

Knowing the dependence of the number of bubbles formed in magma on time and their growth rate, we can find the volume fraction κ of bubbles generated in a unit volume of the melt: $\kappa = \frac{4\pi}{3} \int\limits_0^t \dot{N}_b(\tau) R^3(t-\tau)\, d\tau$. Taking into account that the volume of the medium increases in the course of bubble growth, we obtain the relation $k = \kappa/(1+\kappa)$. Here we considered more complicated and obviously more real situation when crystallites can also play the role of cavitation nuclei. For that, after termination of homogeneous nucleation, new nuclei (as "former" crystallites) are introduced in the magma flow. Their density $N_*(b)$ was changed in the diapason $10^{12} \to 10^{15}\ m^{-3}$. For comparison: the number of potential nuclei in magma melt (according to the dissolved gas mass) was about 10^{26}, m^{-3}.

3 Calculation Results.

The calculations were performed for the following parameters of volcanic system: $H = 1$ km - magma-column height, $p_{\rm ch} = 170$ MPa - pressure and $T = 1150$ K - temperature in a magmatic chamber. The magma parameters are : $\rho_0 = 2300$ kg/m^3, $K_{\mathrm{H}} = 4.33 \cdot 10^{-6}\ Pa^{-1/2}$, $D = 2 \cdot 10^{-11}\ m^2/sec$, $E^*_\mu = 5.1 \cdot 10^{-19}$ J, $k_\mu = 11$.

According data of Fig. 2 the homogeneous nucleation has terminated about 0,033 - 0,035 s. Fig. 3 shows the pressure fields over the column height approximately in 0.562 s after diaphragm fracture. We can see that the precursor of decompression wave reaches coordinate $z = 100\ m$, its front has the oscillating profile as a result of cavitation development influence and the main decompression front moving with the sound velocity in cavitating liquid reaches the coordinate $z = 800\ m$ only (Fig. 3, see also distribution $u(z)$). The second interesting effect of "crystallite - nuclei" influence is the disintegration of cavitation area on the separate zones which later can join (Fig. 3, see $k(z)$ and $R(z)$. Finally, the viscosity distributions $\mu(z)$ presented on Fig. 4 for three values of instants in time interval $3 - 6$s show that in the large part of magma column (from 600 to 1200 m) the viscosity distribution can be considered as close ones.

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