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A maximum likelihood method for obtaining integrated attenuation from ultrasound transmission mode measurements

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The estimation of integrated ultrasound transmission parameters is important in ultrasound computed tomography and of late also in photoacoustic imaging. We derive and evaluate a maximum likelihood estimator for the measurements of integrated acoustic attenuation. This estimator is applicable to media like soft tissue. In soft tissue, the attenuation due to dissipative effects obeys a frequency power law. By measuring the propagation of transmitted ultrasound signals, the parameters that describe the attenuation can be estimated. In this paper a new method is introduced for estimating the attenuation of ultrasound media by means of transmission mode measurements. The method is based on analyzing the noise characteristics of the received signals and the formulation of a maximum likelihood estimator. The new estimator is compared to existing methods. Our new estimator is less restrictive on the input signal and attenuating medium and its performance is equal or better than existing estimators.

1 Introduction

The estimation of acoustic propagation parameters like attenuation and speed of sound are important factors in the fields of ultrasound tissue characterization and non-destructive material testing. An application in medical ultrasound is the reconstruction of distributions of the propagation parameters inside an object from projections[1]. Recently methods to measure ultrasound propagation parameters were also proposed by obtaining projections appropriately in photoacoustic imaging [2, 3, 4]. The more accurate these projections can be estimated, the more accurate the reconstructed images will be. The accuracy of the estimates depends on the signal to noise ratio of the measured signal, as well as on the performance of the estimator that is being used. In this paper we will focus on the formulation of an accurate estimator of the (projected) attenuation parameters. The accuracy will be presented in terms of the bias and variance of the estimators and the resulting root mean square error (rmse).

The ultrasound propagation estimators are used in the application of transmission mode measurements. Measuring in transmission mode means that an input signal is generated at one side of the object and a distorted version of this signal is measured at the opposite side of the object. The distortion of the input signal is caused by the ultrasound propagation parameters of the object. The measured signal is thus a function of the input signal and the unknown ultrasound propagation parameters. The estimation task is now defined as finding an estimate of the unknown ultrasound propagation parameters from the measured signal. This requires the formulation of a measurement model that describes the relation between the parameters and the measured signal.

We will refer in this paper to two existing ultrasound propagation parameter estimators. These two estimators will be briefly introduced and their performance will be compared to our newly formulated estimators.

2 The ultrasound propagation parameters

The propagation of an ultrasound signal through an ultrasound medium consists of two contributions. First, there is a change in amplitude of the signal and secondly there is a time delay which corresponds to the time the signal travels through the medium. These amplitude and time delay effects are material properties and are position dependent in inhomogeneous materials. Besides being material dependent, the amplitude and time delay can also depend on the frequency of the input signal. In a very general form, we write the dependence of the measured signal on the input signal and the medium parameters as:

$$Y(f) = \exp\left[\int\left(-\alpha(f,\boldsymbol{r}) - j2\pi\frac{f}{c(f,\boldsymbol{r})}\right)\mathrm{d}\boldsymbol{r}\right]X(f) \tag{1}$$

where $\alpha(f,\boldsymbol{r})$ is a frequency and position dependent medium property describing attenuation and $c(f,\boldsymbol{r})$ is a frequency and position dependent medium property describing speed of sound. For the problem of estimating the ultrasound propagation parameters, we are not interested in the position dependency of the medium properties. We will only look at the final result after propagation over a certain path through in the object. Also we will confirm our study in this paper to the estimation of the attenuating properties only and for now ignore the time delay properties. The position independent, projected or integrated attenuation function is then the function we want to estimate:

$$a(f) = \int \alpha(f,\boldsymbol{r})\mathrm{d}\boldsymbol{r} \tag{2}$$

This function is frequency dependent, but can very well be parametrized. Attenuation can be represented by frequency power laws for a wide variety of materials[5] as:

$$a(f) = a_0|f|^y \tag{3}$$

where a_0 and y are material dependent parameters. The parameter y typically varies between 0 and 2, for soft tissue it is $y = 1$ and for water it is $y = 2$. We will now continue with the assumption that the object will have a power factor of $y = 1$, such as is the case for soft tissue. Besides the attenuation due to dissipative effects in the object, there can also be attenuation due to reflection. This occurs at the boundaries of the object and at transitions from one type of material to another one and it is a frequency independent attenuation.

The relation between the unknown parameters and the measured signal is now defined, but requires knowledge of the input signal. A measurement of the input signal can be obtained by performing a reference measurement where the object is removed from the transmission mode setup as described in [6]. The reference measurement will then approximately be a time delayed version of the input signal with negligible attenuation and no dispersion when a suitable reference medium like distilled water is used. Both the object measurement and the reference measurement are a function of the input signal $X(f)$:

$$Y(f) = \exp\Big[-a_r - a_0|f| - j2\pi\tau_o(f)\Big]X(f) \quad (4)$$

$$Y_w(f) = \exp\Big[-j2\pi\tau_w\Big]X(f) \quad (5)$$

so that we can express the object measurement as a function of the reference measurement:

$$Y(f) = \exp\Big[-a_r - a_0|f| - j2\pi\tau(f)\Big]Y_w(f) \quad (6)$$

The ultrasound propagation parameters are now:

a_r	Attenuation due to reflection
a_0	Linearly with frequency increasing attenuation constant
$\tau(f)$	Time delay function between the object measurement and the reference measurement

Both the object and reference measurements will be available as sampled data in the time domain. A realistic model for noise on the measurements is additive Gaussian white noise. The reference measurement is taken without an attenuating object so that we can assume it is a noise free observation. In the frequency domain, the additive Gaussian white noise will manifest itself also as additive Gaussian white noise due to the linearity and orthogonal properties of the FFT transform. This noise is present on both the real part and on the imaginary part of the FFT transformed signals.

3 Ultrasound propagation parameter estimators

There are several solutions in finding an estimate of the ultrasound propagation parameters from the measured object and reference signals. We will discuss here two existing algorithms, which both operate on the magnitude of the FFT transformed signals and finally we will describe our maximum likelihood estimator.

3.1 Spectral shift estimator

The spectral shift estimator[7] is based on the fact that the input signal has a Gaussian distribution in the frequency domain. After propagating a Gaussian modulated signal through a medium with a linear with frequency increasing attenuation function, an output signal will result which still has a Gaussian distribution in the frequency domain. This signal will have the same bandwidth but a lower center frequency. The amount of down shift in center frequency is a measure for the attenuation constant a_0:

$$a_0 = \frac{\Delta f}{\sigma_w^2} \quad (7)$$

The bandwidth or variance of the Gaussian σ_w^2 is calculated on the magnitude of the FFT of the measured object signal instead of the power of the measured object signal as was done by Kuc [7]. The reason for this, is that we will estimate the spectral shift also on the magnitude signals and not on the power signals, since this gives better estimation performance.

Besides finding the value for a_0 we can also estimate the reflection coefficient a_r. To do so, we use the two relations describing the original Gaussian spectrum and the shifted Gaussian spectrum:

$$|Y_w(f)| = K\exp\left[-\frac{1}{2\sigma_w^2}(f-f_c)^2\right] \quad (8)$$

$$|Y(f)| = K\exp\left[-\frac{1}{2\sigma_w^2}(f-(f_c-a_0\sigma_w^2))^2\right]\times \exp\left[-a_r - f_c a_0 + \frac{1}{2}a_0^2\sigma_w^2\right] \quad (9)$$

where now a_0 is a known (estimated) parameter. Using the estimated value of a_0 we can find an estimate of the parameter a_r. To do so we find a least squares estimate of the term $\exp\left[-a_r - f_c a_0 + \frac{1}{2}a_0^2\sigma_w^2\right]$ and calculate the estimate of a_r by inverting the function.

The performance of the estimator actually depends on the performance of estimating the frequency shift in the frequency domain. We have implemented two approaches for estimating the center frequency of the object signal.

Estimating the first moment One approach is based on calculating the center frequency using the weighted sum of the frequency components. The idea behind the approach is that the magnitude of the FFT transformed object signal can be seen as a probability density function (pdf) and calculating the first moment gives us the mean of the pdf. The weights were chosen to be equal to the magnitude of the FFT transformed object signal and normalized to a sum of one. To avoid a biased estimate, only a selection of frequency components was used, so that on both sides of the mean an equal number of frequency components.

Matched filter with a Gaussian template The other approach was based on correlating a Gaussian template with the magnitude signal over a range of frequency shifts. The frequency shift that generates the highest correlation value was chosen as the center frequency of the Gaussian in the object signal.

After extensive simulations it was clear that the matched filter approach lead to superior performance in terms of rmse than the first moment approach.

3.2 Least squares fit

Another commonly used estimator is based on fitting a straight line through the logarithm of the division of the magnitude of the object signal with the magnitude of the reference signal. If we take the logarithm after division we get a linear function:

$$\ln\left(\frac{|Y(f)|}{|Y_w(f)|}\right) = -a_r - a_0|f| \quad (10)$$

and fitting this function to the data in a least squares sense will give us the estimated a_r and a_0 values. The measurements $|Y(f)|$ are noisy and care should be taken when the signal to noise ratio (SNR) becomes too low. A too low SNR will degrade the performance of this estimator, since no weighting of the data is performed.

We will propose a solution based on this least squares fit with weighting in the next section on the maximum likelihood estimator.

3.3 Maximum likelihood estimator

A maximum likelihood estimator finds an estimate of the unknown parameters $\boldsymbol{x}$ by maximizing the likelihood function of measurements $\boldsymbol{z}$ of these unknown parameters:

$$\boldsymbol{x}_{ML} = \arg\max_{\boldsymbol{x}} p(\boldsymbol{z}|\boldsymbol{x}) \tag{11}$$

In this section we will look at an estimator that operates on transformed measurements, by applying the logarithm on the magnitude of the measured FFT signals. Applying a log function to the FFT transformed signals will linearize the exponential function to a linear function, as was observed in the least squares fitting solution. However, after applying the logarithm and magnitude function to the measurements, the noise on the measurements will be transformed non-linearly and not be Gaussian anymore. For larger values of the SNR, a first order Taylor expansion gives a good representation for the behavior of the function around the uncertainty domain. Consequently, for larger values of the SNR we can deal with the noise distribution as additive Gaussian noise due to the linear behavior.

Suppose we use a set of n frequency components in the estimation $\boldsymbol{f} = [f_1 \dots f_n]^T$. Our log magnitude measurements, which are linear functions of the parameters $\boldsymbol{x}$ will be represented in the following vector:

$$\boldsymbol{z} = [z_1 \dots z_n]^T \tag{12}$$

We obtain the measurements $\boldsymbol{z}$ by the following non-linear transforms on the measured object signal:

$$z_i = h_i(\boldsymbol{y}_i) = \ln\left(\frac{\sqrt{y_{\mathrm{R},i}^2 + y_{\mathrm{I},i}^2}}{|Y_w(f_i)|}\right) \tag{13}$$

where the complex quantity $\boldsymbol{y}_i = y_{\mathrm{R},i} + j y_{\mathrm{I},i}$ represents the complex FFT object measurement $Y(f_i)$. The noise on $\boldsymbol{y}_i$ is additive Gaussian with a covariance of:

$$P_{yy} = \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \tag{14}$$

so all frequency components have the same noise covariance, and the real and imaginary parts of each component have the same noise variance. The resulting measurements, which are linear in the unknown parameters, are given by:

$$z_i = -a_r - a_0 f_i \tag{15}$$

We will model the noise on the transformed measurements $\boldsymbol{z}$ also as additive Gaussian noise. The variance of this noise will be calculated by linearization of the non-linear measurement transformation functions. For each of the elements z_i we can calculate the resulting variance from the linearized measurement transform function:

$$\sigma_{z_i}^2 = (\nabla h_i)^T \begin{bmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_y^2 \end{bmatrix} \nabla h_i \tag{16}$$

Where ∇h_i are the gradients of the measurement transform functions:

$$\nabla h_i = \frac{1}{y_{\mathrm{R},i}^2 + y_{\mathrm{I},i}^2} \begin{bmatrix} y_{\mathrm{R},i} \\ y_{\mathrm{I},i} \end{bmatrix} \tag{17}$$

The resulting variances of the elements z_i can now be calculated:

$$\sigma_{z_i}^2 = \frac{\sigma_y^2}{||\boldsymbol{y}_i||^2} \tag{18}$$

This expression was obtained by evaluating the gradients at the measured values of $\boldsymbol{y}_i$. The full covariance matrix of the measurements $\boldsymbol{z}$ after combining the individual variances is then given by:

$$P_{zz} = \begin{bmatrix} \sigma_{z_1}^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{z_n}^2 \end{bmatrix} \tag{19}$$

The frequency components that we use as measurements in the estimation procedure have to be carefully selected based on the SNR. A too low SNR will result in the fact that the original FFT measurement $\boldsymbol{y}_i$ can not be transformed reliably to a measurement z_i with the modeled additive Gaussian noise. The selection procedure we use is defined as finding frequency components $\boldsymbol{f}$ from the whole set of available components $\boldsymbol{f}_{\mathrm{total}}$ from the FFT measurement, which have a high enough SNR to participate as reliable measurements:

$$\boldsymbol{f} = \left\{ f_i : f_i \in \boldsymbol{f}_{\mathrm{total}} \wedge \frac{||\boldsymbol{y}_i||}{\sigma_y} > \mathrm{SNR}_{th} \right\} \tag{20}$$

Also here, the selection is based on the actual measured values of $\boldsymbol{y}_i$. The threshold SNR_{th} can be set to a value as low as 1.

Our parameter vector $\boldsymbol{x}$ consists of the attenuation coefficients:

$$\boldsymbol{x} = [a_0, a_r]^T \tag{21}$$

The measurement vector will consist of the log magnitude measurements $\boldsymbol{z}$ and the linear relation between $\boldsymbol{z}$ and $\boldsymbol{x}$ is given in vector notation by:

$$\boldsymbol{z} = H\boldsymbol{x} \tag{22}$$

with

$$H = \begin{bmatrix} -f_1 & -1 \\ -f_2 & -1 \\ \vdots & \vdots \\ -f_n & -1 \end{bmatrix} \tag{23}$$

The resulting likelihood function is than given by the multivariate Gaussian pdf:

$$p(\boldsymbol{z}|\boldsymbol{x}) = \frac{1}{|2\pi P_{zz}|} e^{-\frac{1}{2}(\boldsymbol{z}-H\boldsymbol{x})^T P_{zz}^{-1} (\boldsymbol{z}-H\boldsymbol{x})} \tag{24}$$

The parameters are now calculated by finding the maximum of the likelihood function. Finding the maximum of a Gaussian probability density function comes down to finding the minimum of the corresponding Mahalanobis distance:

$$\boldsymbol{x}_{ML} = \arg\min_{\boldsymbol{x}} (\boldsymbol{z} - H\boldsymbol{x})^T P_{zz}^{-1} (\boldsymbol{z} - H\boldsymbol{x}) \tag{25}$$

The minimization of this quadratic function can simply be calculated by setting the gradient of the function to zero, resulting in the linear system:

$$H^T P_z z^{-1} H\boldsymbol{x} = H^T P_z z^{-1} \boldsymbol{z} \tag{26}$$

which can be solved by inverting the linear system.

When a solution has been found to this minimization problem, we can iteratively improve our solution. The calculation of the covariance matrix $P_z z$ and selection of frequency components $\boldsymbol{f}$ can both be improved by using predicted measurements based on a previous estimate of the parameters, rather than the noisy measurements themselves. Thus instead of using measurements $||\boldsymbol{y}_i||$ in the calculation of P_{zz} and the selection $\boldsymbol{f}$ we will use the predicted measurements $||\boldsymbol{y}_i^{(k)}||$ which can be calculated from the estimated parameters at step k, $\boldsymbol{x}^{(k)}$ as:

$$||\boldsymbol{y}_i^{(k)}|| = |Y_w(f_i)| \exp\left[-x_1^{(k)} f_i - x_2^{(k)}\right] \tag{27}$$

Using the more accurate predicted covariance matrix $P_{zz}^{(k)}$ and selection of frequency components $\boldsymbol{f}^{(k)}$ we will estimate a new parameter vector $\boldsymbol{x}^{(k+1)}$. This process will be repeated until the estimated parameter vector converges. Conversion is determined by calculating the norm of the difference between two subsequent iterative solutions. When this difference is small enough: $||\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}|| < \epsilon$, the process is stopped and a final solution is obtained.

3.4 Cramer-Rao Lower Bound

In terms of estimator efficiency, it is useful to look at the Cramer-Rao lower bound (CRLB). This bound gives the theoretically lowest possible variance for any unbiased estimator. If we look at the FFT transformed measurements, which have additive Gaussian noise on both the real and imaginary parts, we can find an expression the CRLB. To do so, the real and imaginary parts are explicitly formed. Both the real and imaginary parts are a nonlinear function of the parameters of which we can calculate Jacobian matrices. If we combine both Jacobian matrices and know that the variance on the real and imaginary parts was σ_y^2 for all frequency components, the expression for the fisher information matrix is given by[8]:

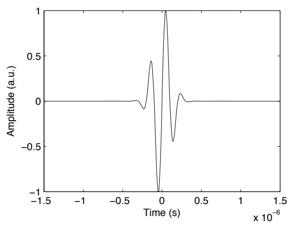
$$I = \frac{J^T J}{\sigma_y^2} \tag{28}$$

where J is the combined Jacobian of the real and imaginary parts of the FFT measurements. The minimum attainable covariance for any unbiased estimator is then given by:

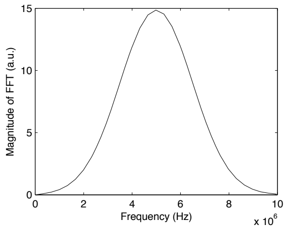
$$P_{xx} = I^{-1} = \sigma_y^2 (J^T J)^{-1} \tag{29}$$

4 Results

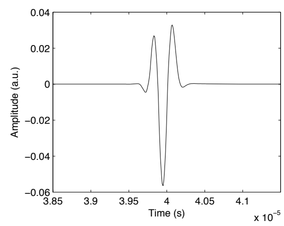
In order to compare the different ultrasound propagation estimators, we ran a set of Monte-Carlo simulations with different propagation distances/attenuation constants. As input signal, we used a Gaussian modulated pulse with a center frequency of 5 MHz and a bandwidth of 1.5 MHz. The input signal is displayed in Fig. 1a and the magnitude of the FFT of the input signal in Fig. 1b. To illustrate the effects of propagating an ultrasound signal through a highly attenuating medium we also show a propagated signal in Fig. 1c and the magnitude of its FFT in Fig. 1d. The propagated signal clearly has a different shape and a much lower amplitude than the input signal.



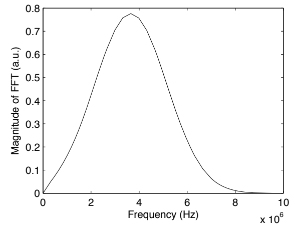
(a) Input signal



(b) Magnitude of FFT of input signal



(c) Noise free propagated object signal



(d) Noise free magnitude of FFT of object signal

Figure 1: Input signal and a propagated signal for a highly attenuating medium ($a_0 = 0.6$ Np/MHz)

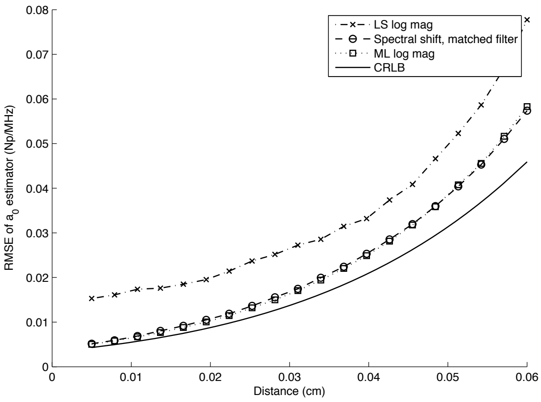
To simulate realistic ultrasound propagation media, we use an attenuation coefficient of $\alpha_0 = 0.1$ Np/MHz /cm and vary the propagation distance from 0.5 cm to 6 cm. A noise free propagated object signal is calculated for each of the simulated propagation distances. A set of 60,000 noisy propagated signals per distance were generated by adding zero mean Gaussian noise to the noise free propagated signals. The amount of noise was chosen to have a SNR of 100. This means that the maximum amplitude of the input (or reference) signal in the time domain divided by the standard deviation of the noise in the time domain will be equal to 100. After propagating the input signal through an attenuation object of a certain distance, the SNR will drop and hence the rmse will increase. Statistics (bias and variance) of the estimators are then calculated from the set of 60,000 samples. The rmse was calculated from the estimated bias and estimated variance:

$$\text{rmse} = \sqrt{\text{bias}^2 + \text{variance}} \tag{30}$$

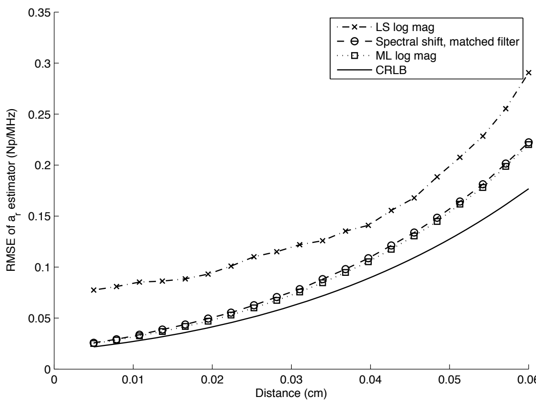
The resulting rmse as a function of propagation distance of each of the estimators is displayed in Fig. 2, together with the CRLB.

5 Conclusion

We have developed a new estimator for estimating integrated ultrasound attenuation parameters from ultrasound transmission mode measurements. The estimator performs equally well as the existing spectral shift estimator and better than the existing least squares fit



(a) Attenuation estimation results for a_0. True values are given by $a_0 = 10 \cdot \text{Distance}$ (Np/MHz)



(b) Reflection estimation results for a_r. The true value is given by $a_r = 0.36$ (Np)

Figure 2: rmse values for all estimators on the estimation of both attenuation and reflection

estimator. However, it does not have the limitations that the spectral shift estimator has. These limitations are the constraints that the input signal should have a Gaussian magnitude distribution in the frequency domain and that the attenuation function should be a linear function of frequency. In our new estimator, different attenuation functions can be implemented in a similar manner by just changing the value of y in the measurement model and no assumption is made about the frequency magnitude distribution.

Our estimator has a rmse which is still above the minimum possible Cramer-Rao lower bound. Currently we are working on a new estimator which is expected to have a performance much closer to the Cramer-Rao lower bound.

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