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Adaptive predictive feedback control of circular plate vibrations

Lucyna Leniowska and Pawel Kos

University of Rzeszow, Rejtana 16, PL35-310 Rzeszow, Poland
lleniow@univ.rzeszow.pl

Adaptive predictive controller, consists of an on-line identification technique coupled with a control scheme, is used in this paper for a plate vibration suppression. It is assumed, that the system to be regulated is unknown and the controller schemes presented have ability to identify and suppress the plate vibrations with only an initial estimate of the system order. The choice of structure is motivated by its representative nature. This configuration has also been studied by the authors both analytically and experimentally, using several kind of controllers (PID, PI2D, fuzzy, LQR). There are two fundamental steps involved in the closed-loop system. The first step is to identify a mathematical model. The second step is to use the identified model to design a controller. One drawback of this approach is that, the control signal is fed to the actuator after updates of the control law expression, which always leads to some delay. In order to align better the updating process, the authors introduce the prediction of plant output with established error convergence. The one-step ahead system output prediction is calculated from the recursive formulas of the interpolation functions chosen. Simulation are included and discussed.

1 Introduction

The main aim of the control system designed for plates is to cancel its vibrations and related acoustic radiation as much as possible. There are many control strategies that could be developed for the considered structure [1, 2]. Problem of suppressing vibration of circular plate has been also studied by the authors using several kind of controllers (PID, PI2D, fuzzy, LQR) [3-7] which parameters were statically calculated using MATLAB/SIMULINK simulation tools. It is well known that most control law design methods require an explicit mathematical model of the system to be controlled. Derivation of the models for planar structures with the point or surface mounted sensors/actuators can be guided into two ways. The first approach consists of modeling the fluid-acoustic-structural dynamics in the form of the partial differential equations derived from physical principles and it was also used by the authors [4, 5]. The second way in which a model can be established is *system identification* and it is used here. Process of designing the controller involved making preliminary tests which results were used to identify considered system. Using selected identification method one could received values of unknown system model parameters. They were applied in algorithm for reducing vibration of plate. Described procedure has limited accuracy because parameters are appointed only once and used for all calculations. However, computational complexity of this algorithm is low, that's why it could be executed on low performance PC computers.

Development of IT technologies gives opportunities to use more advanced techniques. One of them, which involves on-line adapting of controller is considered in this paper. Adaptive control is a set of techniques for the automatic, on-line adjustment of control-loop regulators designed to maintain a given level of system performance. In this method process of identification of the system and updating of the adapter parameters is integral part of algorithm. As a rule, the adaptive control algorithm can be seen as a combination of two algorithms:

- an identification algorithm using measurement for system model,
- a control law computation algorithm for determination, at each instance, the adaptive controller parameters and the control to be applied to the system.

Application of its is possible if PC computer works under a real time operating system. For this reason the designed adaptive controller was implemented on the RTAI-Linux platform. Moreover, in order to align better the updating process, the authors introduce the prediction of plant output with established error convergence. The one-step ahead system output prediction is calculated from the recursive formulas of the interpolation functions chosen. Finally, simulations results obtained for the considered plate are presented.

2 Plant description

An active vibration control system is proposed for suppressing the small amplitude vibration of circular plate (Fig 1.). An experimental set-up consists of a hard-walled cylinder with a thin metallic plate at one end. Primary excitation is provided by a low frequency loudspeaker installed centrally at the bottom of the cylinder. The vibration of the plate are measured by the application of strain sensors. Intelligent materials such as 2-layer piezo disk elements are used as the actuators.

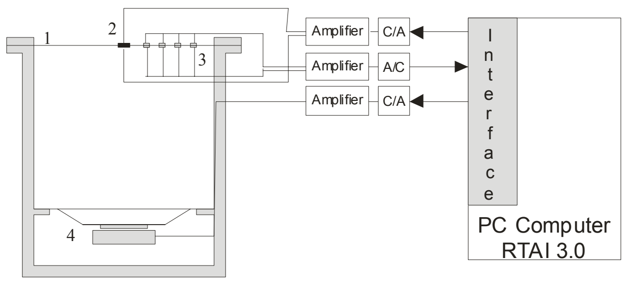

Fig. 1. Research position
1 – circular plate, 2 – PZT elements, 3 – strain sensors, 4 - loudspeaker

Process of reducing vibration is controlled by PC computer with the Real Time Application Interface (RTAI) installed on it. This is an extension of Linux operating system which guarantees deterministic response time, because all of real time applications are build as kernel modules and if they run, they always have higher priority than ordinary Linux process. Linux task is treated as *idle task*, that means that they are executed only if there is no RTAI Linux tasks. Described way of running real time applications significantly increase rate of executing them and give a chance to use more advance algorithm such as shown on Fig. 2.

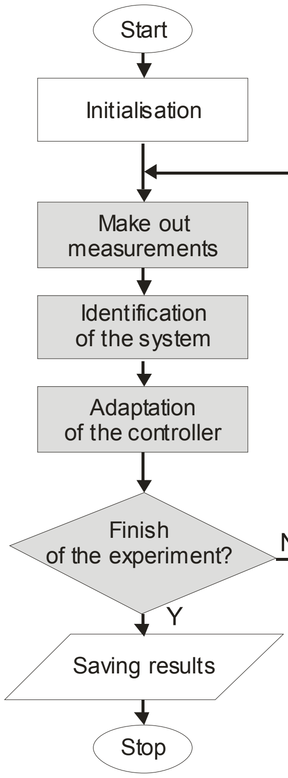

Fig. 2. Algorithm used in experiment

The main loop of chosen procedure involves 3 fundamental steps: taking measurements from strain sensors, identifying a mathematical model and designing a controller.

3 Identification of the system

The determination of the structure of a parametric system model is an important first step before going on to design an adaptive control algorithm. The capabilities of the adaptive control depends on the faithfulness with the model represents the system and its behaviour. If the set of actuators and sensors are located at discrete points of the structure, they can be treated separately. But the distinctive feature of smart structures commonly used for vibration control is that the actuators/sensors are distributed and often integrated with the structure, which makes separate modelling impossible. Moreover, in the case of plates, the vibration of the structure can be coupled with the surrounding medium.

Greater attention has been focused on the identification of process models in recent years, as a part of the controller design methods that rely on an explicit process model. Unfortunately, theoretical models provided excellent agreement with real systems and them dynamics are quite difficult to obtain. In these situations models should be identified based on actual process data. In this paper the identification of Auto-Recursive eXtensive (ARX) model is considered.

3.1 Linear ARX model

In ARX models the current output is considered to be a function of past values of both the input and output. Thus the ARX model takes the form [1, 8]:

$$y(k)+a_1y(k-1)+\ldots+a_ny(k-n)= b_1u(k-d)+\ldots+b_my(k-d-m+1)+\eta(k) \quad (1)$$

where *y(k)* represents the output at time *k*, *u(k)* represents the input at time *k*, *n* is the number of poles, *n* is the number of zeros plus 1, *d* is the number of samples before the input affects the system output, and *η(k)* is the white-noise disturbance.

Reorganizing Eq. (1). one could get equation for calculating model output *y(k)*:

$$y(k)=\varphi(k-1)\theta, \quad \forall k \quad (2a)$$

where:

$$\varphi(k-1)=[-y(k-1),\ldots,-y(k-n), u(k-d),\ldots,u(k-d-m+1)] \quad (2b)$$

$$\theta=[a_1,\ldots,a_n,b_1,\ldots,b_m]^T \quad (2c)$$

Vector φ represents the output observed from real process, θ - unknown parameters of model. Then real process output could be obtain as:

$$y(k)=\varphi(k-1)\theta+\eta(k), \quad \forall k \quad (3)$$

and *η(k)* is unmeasured noise.

The ARX model structure can be given also by the following equation [7]:

$$A(q^{-1})y(k)=B(q^{-1})u(k)q^{-d}+\eta(k), \quad (4)$$

and after simplifying we get:

$$y(t)=\frac{B(q^{-1})}{A(q^{-1})}u(k)q^{-d}+\frac{1}{A(q^{-1})}\eta(k), \quad (5a)$$

where

$$A(q^{-1})=1+a_1q^{-1}+\ldots+a_nq^{-n}, \quad (5b)$$

$$B(q^{-1})=b_1q^{-1}+\ldots+b_mq^{-m}, \quad (5c)$$

Considered ARX model is presented on Fig 3.

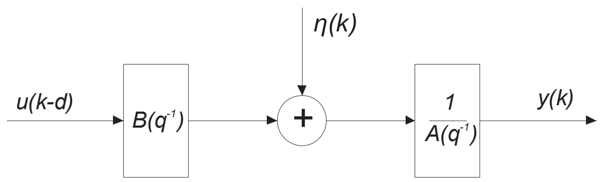

Fig. 3. ARX Model Structure

3.2 ARX model with *d*-step prediction

Often for developing of the adaptive controllers one uses method with *d*-step prediction presented below:

$$y(k+d)=\varphi(k)\theta, \quad \forall k \quad (6a)$$

$$\varphi(k) = [-y(k),...,-y(k-n+1), \quad u(k),...,u(k-m+1)] \tag{6b}$$

$$\theta = [a_1,...,a_n,b_1,...,b_m]^T \tag{6c}$$

The main aim of identification process is to obtain vector θ which will minimize difference between output of the process *y(k)* and result of the model for each step *k*. In literature, there are discussed many methods of calculating the model. Authors focused on procedure used for an algorithm with prediction proposed originally by L. LJUNG, T. SÖDERSTRÖM [8].

$$\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k-1) + \frac{\alpha(k)\boldsymbol{P}(k-d-1)\boldsymbol{\varphi}(k-d)}{1+\boldsymbol{\varphi}^T(k-d)\boldsymbol{P}(k-d-1)\boldsymbol{\varphi}(k-d)}\varepsilon(k) \tag{7a}$$

$$\boldsymbol{P}(k-d) = \boldsymbol{P}(k-d-1) + -\frac{\alpha(k)\boldsymbol{P}(k-d-1)\boldsymbol{\varphi}(k-d)\boldsymbol{\varphi}^T(k-d)\boldsymbol{P}(k-d-1)}{1+\boldsymbol{\varphi}^T(k-d)\boldsymbol{P}(k-d-1)\boldsymbol{\varphi}(k-d)} \tag{7b}$$

$$\varepsilon(k) = y(k) - \boldsymbol{\varphi}(k-d)\boldsymbol{\theta}(k-1) \tag{7c}$$

$$\alpha(k) = \begin{cases} a, & |\varepsilon(k)| > \delta\sqrt{1-a}, \\ 0, & |\varepsilon(k)| \le \delta\sqrt{1-a}, \end{cases} \tag{7d}$$

$$\boldsymbol{P}_0 = \rho \boldsymbol{I} \tag{7e}$$

Equations presented in this paragraph were used for building algorithm for obtaining a model of considered process.

3.3 Result of system identification

Working and precision of this algorithm depends on chosen values of several parameters: $0<a<1$, δ and $\rho>0$. Accuracy depends also on numbers *n* and *m*. This two parameters are important for developing adaptive controller which will updates control law using data from identification process.

In this paper authors considered model with parameters *m* and *n* equals *2*. Fig. 4 and Fig 5. shows example results of identification process of considered system.

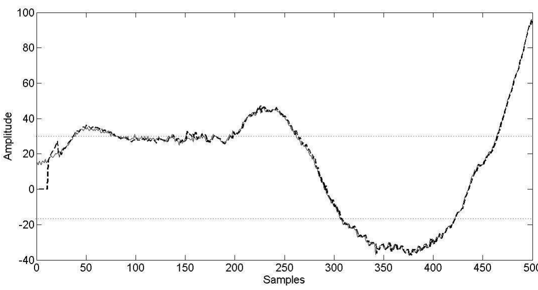

Fig. 4. Example of identification process for 500 samples
'—— ' output of real process, '----' output of model

As it can be seen on Fig. 4, after several initial steps algorithm starts to work very well: result of calculation is comparable with measured output of system. Finally, in this paper authors considered system described by discrete transfer function as follows:

$$y(k) - 1.13284y(k-1) + 0.13377y(k-2) = 0.00276u(k-1) - 0.00199u(k-2) \tag{8}$$

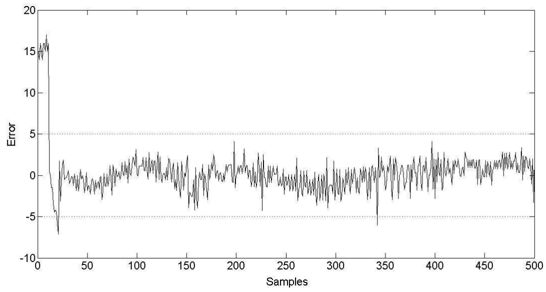

Fig. 5. Error of identification process

Error of identification process (Fig. 5) is limited and constitute a small part of real output. Result of conducted examination shows also that accuracy of method increase when sampling rate of process increase either. It is possible only after using more advance software like real time operating system (for example RTAI).

4 Adaptive predictive controller

Adaptive controller uses result of identification process to establish control law. Authors have chosen PID structure as a regulator which reduce vibration of the plate. Generally scheme of designed system can be presented as Fig. 6.

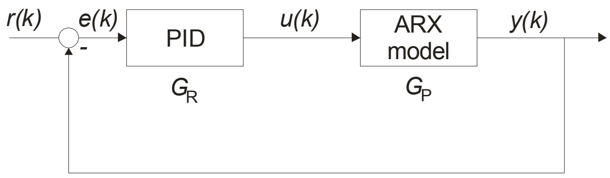

Fig. 6. Block diagram of PID controller

Main task of PID controller is minimizing of error signal *e(k)* which is calculated as difference between desired setpoint *r(k)* and output of system. In considered case, where adapter should reduce vibration of plate, the most expected value of signal *r(k)* equals *0*. That means that regulator PID should reduce signal *y(k)* (*e(k)=-y(k)*).

It was assumed, that the system in analysis is represented by *m=2* order discrete transfer function, as showed in Eq.(9)

$$G_P = \frac{B(z^{-1})}{A(z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \tag{9}$$

The technique used to find the controller parameters is the pole placement. The purpose of the method is to design the controller so that all poles of the closed-loop system assume prescribed values. We seek a controller:

$$G_R = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + ... + q_n z^{-n}}{1 + p_1 z^{-1} + ... + p_n z^{-n}} \tag{10}$$

of order n satisfying the polynomial Diophantine equation [1]:

$$a(z)p(z)+b(z)q(z)=d(z) \qquad (11)$$

where $d(z)$ denotes a designed polynomial of $m+n$ order which moved the system roots to some predefined location.

In theory, if the system is controllable, the poles and zeros can be placed anywhere to improve close system performance. It can be done analytically by solving the linear system of equation with $(m+n)\times(m+n)$ non-singular Sylvester matrix:

$$\begin{bmatrix} a_0 & 0 & & 0 & b_1 & 0 & \cdots & 0 \\ a_1 & a_0 & \ddots & \vdots & b_2 & b_1 & \ddots & \vdots \\ \vdots & a_1 & \ddots & 0 & \vdots & b_2 & \ddots & 0 \\ a_m & \vdots & \ddots & a_0 & b_m & \vdots & \ddots & b_1 \\ 0 & a_m & \ddots & a_1 & & b_m & \ddots & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_m & 0 & 0 & \cdots & b_m \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_m \\ q_0 \\ \vdots \\ q_m \end{bmatrix} = \begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ d_{m+n} \end{bmatrix} \qquad (12)$$

In practice, there are numerical troubles for solving a higher-order of Diofantine equation. Moreover, a low-order controller is generally preferred for physical implementation reasons, so we took into account a PID controller described as follows:

$$G_R = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 + p_1 z^{-1} + p_2 z^{-2}} \qquad (13)$$

As a result of solving linear system equations (12) of four order one should get unknown parameters of PID controller: q_0, q_1, q_2, p_1, p_2, which determines output control signal $u(k)$:

$$\begin{aligned} u(k) = q_0 y(k) + q_1 y(k-1) + q_2 y(k-2) + \\ - p_1 u(k-1) - p_2 u(k-2) \end{aligned} \qquad (14)$$

5 Simulations results and conclusions

The system identification was performed with own software on PC computer working under RTAI-Linux operating system. Before implementation of adaptive control law design on real plant the simulations in MATLAB was performed. In order to determine the dynamics of the plate system, the obtained model was first subjected to a sinusoidal signal with constant amplitude and frequency. Figures below shows result of simulation of described object for two different input signals (for close- and open- loop system).

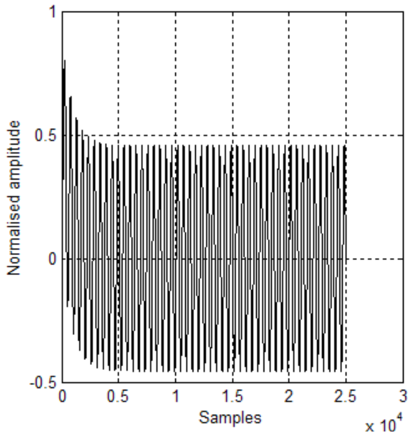

Fig. 7. Response of open-loop system for *sin* signal

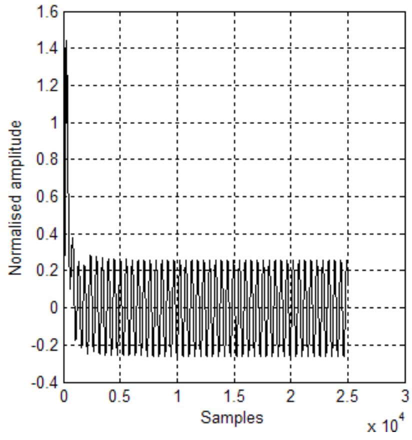

Fig. 8. Response of close-loop system for *sin* signal

Comparing the plate responses for the open and close-loop systems, it can be seen that the obtained controller was damped the output signal very well, more than 60%.

Next, the forcing function used (excitation) was a *chirp* signal as depicted in Fig. 9. It can be seen that the uncontrolled plate response vibrates significantly while the controller causes that plate vibrations have been reduced very well again (Fig. 10). However, there is one disadvantage of such control - a steady-state error appears in close-loop system (Fig. 10).

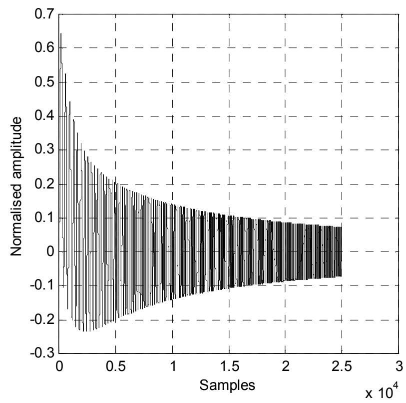

Fig. 9. Response of open-loop system for *chirp* signal (50-300Hz)

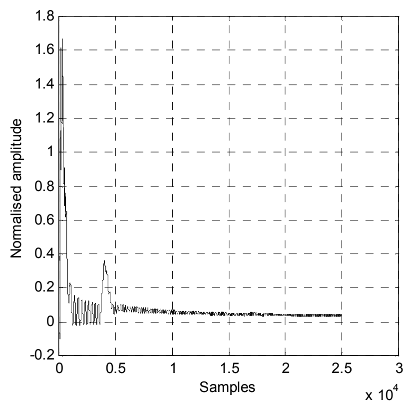

Fig. 10. Response of close-loop system for *chirp* signal (50-300Hz)

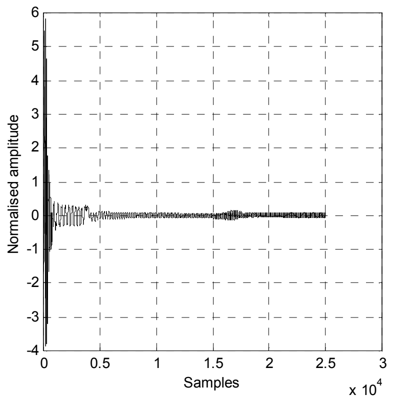

Fig. 11. Control signal generated by adaptive predictive controller in case when *chirp signal* (50-300Hz) as excitation is applied

Figure 11 shows a plot of the control signal generated by controller in the close-loop system. It can be seen that the maximum of such control occurs during the several initial steps of chirp excitation, and next, the control signal is rather adjusted. Finally, results of simulation shows that designed controller suppress the plate vibrations significantly even if identified model is only of second order.

6 References

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