A computational study is carried out to predict rough-wall boundary layer noise and elucidate noise generation mechanisms. As a first step, the sound radiation from a single hemispherical roughness element in a turbulent boundary layer at $Re_\theta=7500$ is investigated. The roughness height is 3.6% of the boundary layer thickness, or 95 wall units. The flow field is computed by large-eddy simulation. The velocity statistics show reasonable agreement with the experimental data measured at Virginia Tech. Acoustic calculations are performed based on the Curle-Powell integral solution to the Lighthill equation. The sound radiation is dominated by unsteady drag dipoles and their images in the wall, with the spanwise dipole of similar magnitude or stronger compared to the streamwise dipole. The viscous contribution to the drag dipole is negligible relative to the pressure contribution. Numerical experiments are performed to isolate the roles of vortex shedding and diffraction of convected hydrodynamic pressure by the roughness element; both are shown to be important noise source mechanisms. The effects of roughness height, upstream wake, and multiple roughness elements are discussed as well.