Modeling the ultrasonic scattering in trabecular bone

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The paper describes the computer simulations allowing to investigate the properties of the ultrasound pulse-echo signal, as it is received on the transducer surface after scattering in trabecular bone. A computer simulation model to provide the verification of theoretical results was developed. It can be also used to yield an ideal environment in which, the effects of various parameters (scatterer mechanical and geometrical properties, scatterer' concentration), the shape of incident wave and experimental conditions influencing the scattering of ultrasonic waves in trabecular bone structure can be examined individually. The results proved that the computer simulation has a particular relevance in studying scattering in cancellous bone which may be approximated as a collection of cylindrical trabeculae.

1   Introduction

Ultrasonic examinations of soft tissues, based on the analysis of scattered ultrasonic signal were successfully applied to characterize and to differentiate the tissues [1, 2]. Similarly, signals that have been scattered in trabecular bone contain information about the properties of the bone structure. Therefore, the scattering-based ultrasonic methods potentially enable the assessment of microscopic structural of bone.

Many investigations have been focused on the measurements and calculations of the backscattering coefficient for trabecular bone and the dependency of that coefficient on the frequency [3-8]. It has been demonstrated that using the backscattering model it is possible to estimate some microstructural characteristics from experimental signals measured in vitro for calcanal samples [9,10]. Chaffai et al. [11] revealed a significant correlation between broadband ultrasonic backscatter (BUB) and the density and microstructure of the human calcanal bone in vitro. Also, ability of ultrasound backscattering to predict mechanical properties of trabecular bone was shown by Hakulinen et al. [12] who proved for the bovine bone samples that integrated reflection coefficient (IRC) and BUB are highly linearly correlated with Young's modulus, ultimate strength and yield stress.

Theoretical studies of ultrasonic scattering by trabecular bone were performed by Wear [8]. The model of a bone, proposed by Wear, consisted of a random space-distribution of long identical cylinders with a diameter much smaller than the wavelength, aligned perpendicularly to the acoustic beam axis. Therefore the scattering by trabecular bone was modelled as scattering of a plane wave by elastic cylinders.

In this paper, a more realistic computer simulation is introduced. Experimental conditions met during ultrasonic investigations of a heel bone were taken into consideration. To describe a cancellous bone the Wear's scattering model was applied with modifications that allowed for changes of mechanical and geometrical properties of individual trabeculae as well as for their spatial density variation. The simulation procedure also enabled considering the groups of scatterers with varying mean values e.g. the thick and thin trabeculae of cancellous bone. Also, the real interrogating pulses were considered, thus the pulse shape, the emitted field structure and the frequency transfer function of the transmitting-receiving transducer were applied in simulations.

The simulated echoes were used to determine the frequency dependent backscattering coefficient for varying parameters of the bone models.

2   Theoretical background

Experimental results show that in the range of diagnostic frequencies the trabecular bone absorption losses are large compared to scattering losses. Thus the amplitude of the backscattered signal, A, corresponding to the given depth z, can be described as

$$A(z) = A_0 \cdot \alpha_s(f) \cdot e^{-2\alpha_a(f) \cdot z} \qquad (1)$$

where A_0 is the initial amplitude, f is the frequency, $\alpha_s(f)$ is the scattering coefficient and $\alpha_a(f)$ is the absorption coefficient.

The coefficient $\alpha_s(f)$ increases with frequency. For the sound wavelength λ large compared to scatterer diameters (Rayleigh scattering region) the power scattering coefficient varies with the fourth or third power of interrogating frequency and with the six or four power of diameter for the spherical and cylindrical scatteres respectively. The absorption coefficient $\alpha_a(f)$ also increases linearly with frequency. Thus for the spectra of broadband echoes the high frequency components are backscattered with larger intensity compared to low frequency components due to absorption and at the same time the $exp(-2 \alpha_a(f)z)$ term in Eq.(1) results in stronger attenuation of higher frequency components. When studying experimentally the scattering properties of materials the effect of attenuation must be compensated. However in modelling, non attenuating material properties are usually assumed.

One scatterer located at distance z_i from the transducer, would contribute to the spectrum of the RF-echo R in the following way:

$$R_i(f) = P(f) \cdot T(f, z_i) \cdot H(f, z_i) \cdot B_i(f) \qquad (2)$$

where $P(f)$ is the applied pulse spectrum, $T(f, z_i)$ is the transducer transfer function for ith scatterer, $H(f, z_i)$ describes material transfer function for ith scatterer and $B_i(f)$ is the backscatter transfer function for ith scatterer.

Note that the transducer transfer function T consists of both, the frequency transfer function and the function describing diffraction patterns of the transmitted field. The material transfer function includes the variation of the amplitude and phase of the RF-echo spectrum components due to attenuation and velocity dispersion. For non dispersing, non attenuating media H - function describes the wave-phase variation introduced by the two-way travel-path of the echo from the z_i-scatterer only.

When Born approximation is applied the multiple scattering is neglected and the total scattering field can be calculated from a linear superposition of all the fields independently scattered by each individual scatterer. Thus the echoes

spectrum is formed by a summation of all scaterrers contributions on the surface of the transducer:

$$R_i(f) = \sum R_i = \sum_i P(f) \cdot T(f, z_i) \cdot M(f, z_i) \cdot B_i(f) \qquad (3)$$

3   Simulations

3.1   Experimental conditions

The important effort was made to introduce to the simulations the results of the experimental conditions occurring during insonification of the cancellous bone. Usually, in *in vivo* experiments, a single, weak focusing spherical transducers are used. Most often the heel bone properties are studied as this bone is easily accessible and it consists of the cancellous bone mainly.   The average direction of trabeculae arrangement in this bone is perpendicular to the interrogating ultrasonic beam axis. The bone is usually placed in water at some distance from the trancducer. The inspected area is situated in a focal zone of the transducer thus the distance between the transducer and scaterring trabeculae is much bigger then the size of trabecula. The transducer emits short ultrasonic sine-pulse that is scattered in the bone and the backscattered waves are recorded. The centre frequency of the pulse does not exceed 1MHz due to high attenuation of ultrasounds in cancellous bone. At this frequency the wavelength is several times bigger then the trabecula thickness. Nevertheless, for higher spectrum components of the transmitting pulse the long wavelength assumption is not valid and exact solutions for scattering should be applied.

Usually the effects of multiple scattering are negligible as the elastic mean free path of highly porous trabecular bone corresponding to 1MHz frequency is relatively long (approximately 1cm [6]) and the high absorption in bone tends additionally to reduce multiple scattering. Also, for the time being, experiments do not suggest strong multiple scattering in trabecular bone.

3.2   Backscatter transfer function of scatterer

In the cancellous bone model the assumption of scatterers (trabeculae) represented by the long, elastic cylinders with diameters much smaller that the wavelength, made of material with mechanical properties of bone tissue was made. The complex backscattering coefficient for trabecula was calculated on the basis of the theory of scattering of ultrasound from elastic cylinder [13] and depended on the cylinder diameter d, mechanical properties of the cylinder and immersion liquid and on frequency of the probing wave. It was assumed that the scatterer was located at the distance from transducer much grater then the scaterrer's size. Thus for the far field assumption only the 180° angular scattering (backscattering) was considered.

3.3   Interrogating pulse and transducer transfer function

The simulation procedures were carried out with a transmitted pulse identical to the pulse emitted by a real transducer. The simulated transducer corresponded to a experimentally–used   transducer   in   the   ultrasonic

densitometry field. The centre frequency was 1MHz with 70%, -10 dB bandwidth, diameter $2a$ of 15 mm and the focal length f_0 equalled to 50 mm. The pulse shape was measured with the hydrophone (Sonic 804-201, Sonora Medical Systems, USA) placed in focus. The transducer frequency transfer function was determined as the ratio of the amplitude spectra of the emitted ultrasonic pulse, measured with the hydrophone close to the transducer surface and of the electric pulse exciting the transducer.

Also a 2D cross-section of pressure distribution in focal zone ($z = f_0 \pm 30mm$) was recorded using the procedures described in [14]. The rotational symmetry in respect to Z axis was assumed for the emitted field. For each scatterer the coefficient x_i that reflected the value of pressure amplitude of the incident wave in the scatterer's position described by r and z coordinates was calculated. Even though the transducer pressure field was measured in water, it reflected to a large extent the field structure in the bone as the wave velocities in water and in trabecular bone are similar.
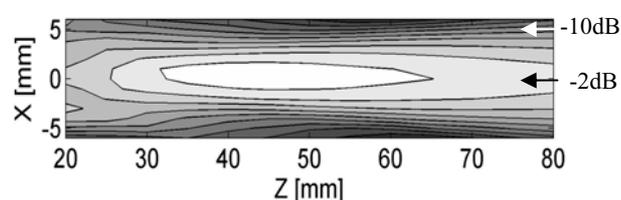
Fig.1   Pressure field distribution in focal zone of the transducer applied in simulations

3.4   Modelling the structural properties of trabecular bone

In the computer simulations the trabecular bone was modelled as a collection of scatterers (long, elastic cylinders aligned perpendicularly to the ultrasound beam axis) randomly distributed in water. The 1D geometry model was applied as a direct consequence of the pure, 180° angular scattering (backscattering) assumption. Note that the condition of normal incidence allows for non parallel arrangement of cylinders. The coordinates that define the locations of scatterer on Z-axis were random (uniformly distributed). Along with Z-axis coordinate, also the random value r (ranging from 0 to a_0 , a_0 – acoustic beam half-width in focal zone) for each cylinder was generated that was describing the distance of the scatterer from the Z axis. This r variable was used to include the influence of the emitted field structure of interrogating pulses.

Mean values and standard deviations for mechanical parameters and diameters of the trabeculae were determined experimentally by means of scanning acoustic microscope [15], or were adopted in accordance with published data.

Variations of the structural properties of cancellous bone were modelled by changing the mean value and variance of cylinders' diameters. The mechanical properties of the bone tissue were assumed to be constant and equal to 3900m/s, 1900m/s and 2000kg/m³ for the longitudinal and transverse waves velocity and density, respectively.

Thickness distributions of trabeculae are right-skewed as reported in experimental studies [16,17]. The published trabecula thickness distributions fit the Gamma distribution

well. Therefore, trabecula thickness values in the bone model were Gamma distributed.

In this approach it was assumed that the decreasing the bone porosity or density resulted mainly from the decrease of trabeculae thickness while the spatial trabeculae density remained unchanged. The constant spatial number density (amount of scatterers per volume unit) of trabeculae $\rho = 2.6/mm^3$ was used for all simulations. Assumption of a constant length and diameter of trabeculae, equal to 4 mm and 0.12 mm, respectively resulted in porosity 88%.

Three bone models were considered. The first model assumed that the bone consisted of identical cylinders. The second model allowed for variation of cylinders thickness within the population (gamma distributed with the thickness SNR = mean to standard deviation ratio = 2.7) and the last model assumed existence of two populations of cylindrical scatterers significantly differing in average diameters and thickness SNR = 2.7 and 3.2 for simulating thick and thin trabeculae, respectively [18]. For the *one population* models three values of the cylinders' thickness (or mean thickness) were considered namely d ($<d>$) = 0.05mm, 0.12mm and 0.2mm. For the *two population* model the mean thicknesses of cylinders equalled to 0.05mm and 0.033mm, 0.12mm and 0.08mm, 0.2mm and 0.13mm were assumed for modelling thick $<d>$ and thin $<d1>$ trabeculae respectively [18]. The spatial number density of thin trabeculae was assumed to be equalled $\rho = 7.8 /mm^3$.

3.5 Modeling the signal from the transducer

The signal that is received on the transducer surface after scattering in trabecular bone structure was simulated in the following way: Every cylinder (trabecula) was considered as a secondary source of an ultrasonic wave. A spectrum of every elementary pulse scattered from a cylinder was obtained as a product of the emitted pulse spectrum and the complex backscattering coefficient of the scatterer. Each backscattering coefficient was modify by field coefficient x_i which depended on the position of scaterrer and emitted field structure. According to Eq.(3) the receiving transducer signal was simulated by superposing all the elementary scattered pulses, taking into account the phase differences that result from various locations of individual cylinder. The spectrum of the simulated signal was limited in accordance with the frequency transfer function of transducer. The value of a complex, instantaneous signal amplitude A_j calculated for the ultrasonic pulse spectrum P scattered in the bone at the depth equaled to $v \cdot j \cdot \delta t/2$ (were δt is a sampling interval) is,

$$A_j = \sum_m \sum_k R_{m,k}(d_m, f_k) \cdot x_m \cdot \sqrt{\frac{-i \cdot v}{f_k \cdot L_m}} \cdot P_k \times \qquad (3)$$

$$\times \exp(i \cdot 2 \cdot \pi \cdot f_k \cdot \frac{2 \cdot L_m}{v}) \cdot \exp(i \cdot 2 \cdot \pi \cdot f_k \cdot j \cdot \delta t)$$

were $R_{m,k}$ is the backscattering coefficient for m-cylinder and k-spectral component and L describes the distance between the scatterer and transducer.

Every simulation consisted of one RF-line, 512 samples long corresponding to the bone structure depth of 35 mm and sampled at the rate of 10 MHz. In simulations this choice is somehow arbitrary but in practical measurements

it corresponds approximately to the thickness of calcaneus bone. The assumption of 1500m/s sound velocity v in trabecular bone was made.

Fig.2 Exemplary RF-echo simulated for the bone model consists of identical scatterers, 1MH centre pulse frequency, d=0.12mm

4 Frequency-dependent backscattering coefficient of simulated signals

The backscattering coefficient is a basic parameter extracted from the pulse echo investigation of TB. The potential sensitivity of this coefficient to the structural changes in trabecular matrix that occur in a process of osteoporosis can be studied in the computer experiments. Also, the properties of backscattered coefficient derived from simulated signals may validate to some extend the simulation procedures.

Analysis of the simulated backscattered signals was performed to determine the frequency dependence of the backscatter coefficient. The backscattering coefficient was determined following the substitution method proposed by Ueda and Ozawa [19]. Its value can be calculated from the simulated (measured) spectra of echoes which are scattered by a trabecular bone and by the plane reflector (calibration spectrum). Each backscattering coefficient presented in this paper was calculated in the following way: the plane reflector echo was simulated by placing in the focus of the transducer field one scatterer only with the backscattering coefficient equal to 1. Forty-eight RF-echoes from the trabecular bone were simulated, each for the new random scatterers location. These echoes were truncated by the rectangular window 256 samples long (25.6 μs) centred symmetrically within the echo. Windowed signals were Fourier transformed by FFT algorithm to calculate the power spectrum. The power spectra were summed up to get the average power spectrum of the random structure of scattered signal. The frequency-dependent coefficient μ_{bs} was then computed, applying volume compensation correction factor and assuming non-attenuating medium described by the formula given in [19]. Next the values of backscattering coefficient were least-squares fit to power-low function ($\mu_{bs}(f) = A f^n$) over the bandwidth 0.8MHz to 1.3MHz.

5 Results

For the considered bone models and cylinders' thicknesses an increase of the backscattered coefficient with frequency is evident (Fig.3). Also a strong dependence of this coefficient on cylinders diameters is apparent. For example

the backscattered coefficient calculated at 1MHz frequency using *one population* bone model consisted of varying scatterers is equal to $1.1 \cdot 10^{-3}$ and $1.6 \cdot 10^{-2}$ [arbitrary units] for $<d>$= 0.05mm and 0.12mm, respectively. In the bone models the volume of the cylinders directly define the bone porosity. Thus, as in experimental study the higher backscattered coefficients are associated with denser bones. The frequency dependence of the backscattered coefficient can be characterized by the exponent (*n*) of the power-law function fit. For the Rayleigh scattering on cylinder ($d<<\lambda$) the power backscattering coefficient depends on the frequency cubed. Calculated coefficients for the wave scattered from a single cylinder show the frequency dependence with the power of 2.815 to 1.308 for the cylinder thickness ranging from 0.05mm to 0.2mm, respectively (Table 1). Thus the requirements for Rayleigh scattering are not fulfilled for the US frequencies and trabeculae thicknesses used in the simulation.

For the bone model consists of identical scatterers values of the *n* coefficient are close to the results calculated for a single cylinders (Table 1). The variation of the cylinders diameters flatten the frequency dependence for $<d>$=0.05mm and 0.12mm. An increase of n for relatively thick trabeculae (cylinders $<d>$ = 0.2mm) could be explained by "resonance scattering" effects that might occur for high frequency components of interrogating pulse and thick cylinders.

When the two populations of scatterers consisted of the thick and thin cylinders were considered the value of *n* increased and for the case of the smaller diameters ($<d>$=0.05 and $<d1>$=0.033mm) the frequency dependence was almost cubed.

5   Conclusions

The results proved that the computer simulation is a useful tool for gaining a better understanding of the scattering of ultrasonic waves in biological tissue and has a particular relevance in studying scattering in cancellous bone which may be approximated by a random collection of cylindrical trabeculae.

A computer simulation model can be used to provide verification of theoretical results. It can be used also to yield an ideal environment in which the effects of various parameters (scatterer mechanical and geometrical properties, scatterers' concentration), the shape of incident wave and experimental conditions on the scattering of

ultrasonic waves in trabecular bone structure can be examined individually.
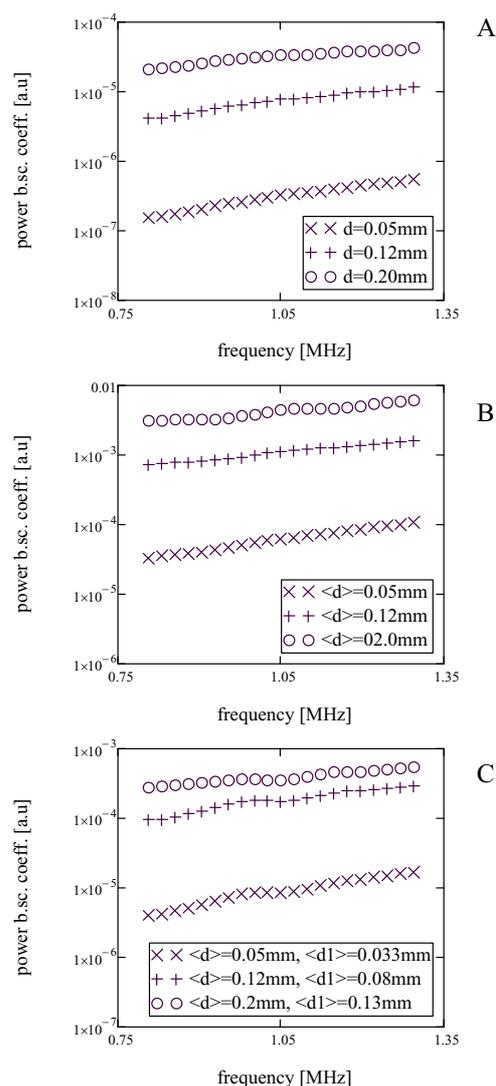
Fig.3 Power backscattered coefficients calculated from simulated RF-echoes for A/ identical scatterers bone model, B/ varying scatterers bone model and C/ model consists of two populations of scatterers.

Table 1. The exponents *(n)* of the power-law function fit together with the coefficient describing the goodness of fitting of the frequency-depended backscattered coefficient determined for the single elastic cylinders and tree bone models.

Cylinders diameters	Elastic cylinder	Identical scatterers bone model	Varying scatterers bone model	2 populations bone model, $<d1> = 0.66 <d>$
d or $<d>$ = 0.05 mm	2.815 (0.921)	2.663 (0.997)	2.587 (0.998)	2.978 (0.988)
d or $<d>$ = 0.12 mm	2.249 (0.934)	2.148 (0.993)	1.789 (0.991)	2.306 (0.976)
d or $<d>$ = 0.20 mm	1.388 (0.964)	1.350 (0.970)	1.603 (0.996)	1.416 (0.958)

The proposed bone model can be easily modified. The cylindrical scatterers can be replaced by spherical ones. Also the mixed model consisting of several populations of cylindrical and spherical scatterers can be easily constructed. The last one could be of particular interest as the very small trabeculae seems to be better approximated by the spherical scatterers than the cylindrical ones. Also using the mixed model of the trabecular bone potentially the better approximation of the frequency depended backscatter to the experimental data could be achieved as such a model allows for the frequency dependences exceeding the third power.

Acknowledgments

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