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Band gaps in a phononic crystal constituted by cylindrical dots on a homogeneous plate

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Using the Finite difference Time Domain method (FDTD), we investigate the existence of absolute band gaps associated with a phononic crystal of finite thickness constituted by a periodical array of cylindrical dots deposited on a thin plate of a homogeneous material. We demonstrate the existence of a low frequency gap in the band structure of the phononic crystal plate which means that the acoustic wavelengths in the constituent materials are much larger than the lattice period. The opening of the gap is discussed as a function of the geometrical parameters of the structure, in particular the thickness of the homogeneous plate and the height of the dots. We show that the gap persists even if we change the materials constituting the plate and the dots. Besides, the band structure can exhibit one or more higher gaps whose number increases with the height of the cylinders. Finally, we discuss the condition to realize waveguiding through a linear defect inside the phononic crystal dots.

1 Introduction

Phononic crystals are heterogeneous materials constituted by a periodical repetition of inclusions in a matrix. Associated with the possibility of absolute band gaps in their band structure [1], these materials have found several potential applications, in particular in the field of waveguiding and filtering [2, 3] as well as in the field of sound isolation [4, 5]. In addition to bulk phononic crystals, recent works have dealt with the study of surface modes of semi-infinite two-dimensional phononic crystals [6-8]. A few works also started to investigate the dispersion curves of free standing plates of 2D phononic crystals [9-12]. In particular those works tend to show the existence of absolute band gaps in the dispersion curves of surface modes [7]. In contrast to previous works, we present in this paper a theoretical investigation of a new structure constituted by a periodical repetition of dots of cylindrical shape deposited on a homogeneous free standing plate (Fig. 1a). We use the Finite Difference Time Domain (FDTD) method to investigate the acoustic wave dispersion and unravel the conditions for the formation of absolute band gaps in the plate modes. More specifically, the behavior of the forming gap is discussed as a function of the geometrical and physical parameters. Finally, we show the possibility of confinement and waveguiding when a guide is created inside the phononic crystal by removing or modifying one row of dots.

2 Results and discussion

The phononic crystal is constituted by a square array of cylindrical dots deposited on a free standing plate as shown in Fig. 1a. The z axis of the Cartesian coordinates system (O, x, y, z) is chosen to be perpendicular to the plate and parallel to the cylinders. The lattice parameter a is taken equal to $1mm$ in the whole paper. The filling factor is defined as the ratio $\beta = \pi r^2/a^2$, where r represents the radius of the cylinders. The height of the cylinders is denoted by h and the thickness of the plate by e.

The materials that constitute the dots and the plate are taken to be respectively steel and silicon except if stated otherwise. The elastic constants and mass densities of the materials involved in the calculations are given in table 1. The Finite Difference Time Domain computation of the band structure is conducted on a unit cell of length a along x and y directions and length b along z direction. Along z (see dashed lines in Fig. 1a), the unit cell contains both the plate and the dot, as well as a thin layer of vacuum on the top of the cell in order to decouple the interaction between neighboring cells.

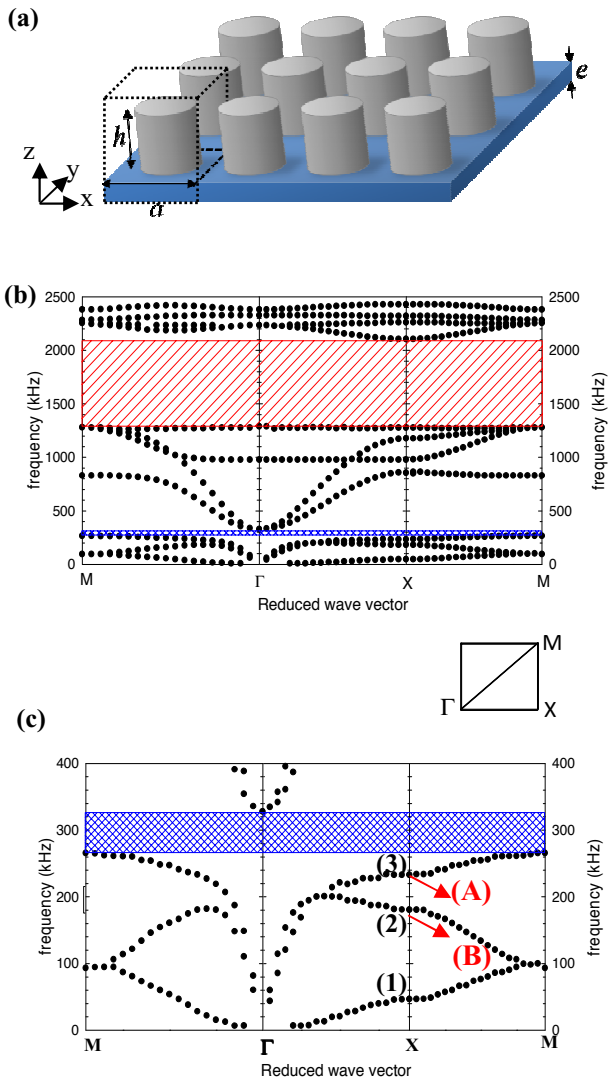

Fig. 1. (a) Phononic crystal made of a square lattice of finite cylinders deposited on a homogeneous plate. The dashed cube represents one unit cell of the periodic structure with dimensions (a, a, b). (b) Band structure in the frequency range $[0, 2500]kHz$ for steel cylinders on a silicon plate, calculated in the first irreducible Brillouin zone of the phononic crystal. The parameters are $a=1mm$, $h=0.6mm$ and $e=0.1mm$. (c) Magnification of (b) in the frequency range $[0, 400]kHz$.

Constant	Silicon	Steel	Tungsten	Aluminum	Epoxy
ρ (kg/m^3)	2331	5825	18700	2730	1142
C_{11} (N/m²)	16.57x10^{10}	26.4x10^{10}	50.23 x10^{10}	10.82x10^{10}	0.754x10^{10}
C_{12} (N/m²)	6.39x10^{10}	10.2 x10^{10}	20.27x10^{10}	5.12x10^{10}	0.458x10^{10}
C_{44} (N/m²)	7.962x10^{10}	8.10 x10^{10}	14.98x10^{10}	2.85x10^{10}	0.148x10^{10}

Table 1: Physical characteristics of the used materials: ρ is the density, C_{11}, C_{12} and C_{44} are the three independent elastic moduli of cubic structure.

Fig. 1b shows the calculated band structure for propagation in the *(x, y)* plane, along the high symmetry axes of the first Brillouin zone, in the frequency range *[0. 2500] kHz*, and magnified in Fig.1c for its lowest part (*[0, 400] kHz*). The following parameters are used: filling factor β=0.*564*, height of the cylinders *h=0.6 mm* and thickness of the plate *e=0.1mm*. A new feature with respect to usual phononic crystals is the existence of a low frequency gap, extending from *265 kHz* to *327 kHz*, where the acoustic wavelengths in all constituting materials are more than 10 times larger than the size of the unit cell. The occurrence of this gap is closely related to the choice of the geometrical parameters in the structure as discussed below. This result looks like as the low frequency gap in the so-called locally resonant materials [4, 5] where the opening of the gap results from the crossing of the normal acoustic branches with a flat band associated with a local resonance of the structure rather than from the Bragg reflections due to the periodicity of the structure. The band structure in Fig. 1b displays also a higher Bragg gap, around *2000 kHz*, which is in accordance with the period of the structure as usual. Finally, in the vicinity of the Brillouin zone center, the three lowest branches starting at Γ point are quite similar to those of a homogeneous plate. They respectively correspond to the antisymmetric Lamb mode (A_0), the shear horizontal mode (SH), and the symmetric Lamb mode (S_0). At the boundary *X* of the Brillouin zone, the three corresponding branches are labeled as #1, #2 and # 3.

We have studied in more details the behavior of the low frequency gap which is generated from the bending of both shear horizontal (branch #2) and symmetric Lamb mode (branch #3) of the plate. We first study the existence of this gap as a function of the parameters *e* and *h*, with a constant value of the filling factor: β=*0.564*. For *h*=0.6*mm*, the lowest dispersion curves move to higher frequencies when increasing *e* from *0.1* to *1.2 mm* and the gap closes for *e* exceeding 0.4*mm*. This result is due to a faster upward shift of branch #3 with respect to the other branches as sketched in Fig. 2a. This evolution leads to the closing of the gap in both directions of the Brillouin zone. On the other hand, for *e=0.1mm*, the dispersion curves move downwards when increasing *h* and the gap disappears when *h* exceeds *1.0mm*. As seen in Fig. 2b, this result comes from a slower downward shift of branch #3 with respect to the other branches. The central frequency of the gap depends on both parameters *e* and *h*: it increases either by increasing *e* or decreasing *h*. The opening of the gap is closely linked to the shift and bending of the branch #3 which is mostly dependent on the thickness of the plate than the height of the dots.

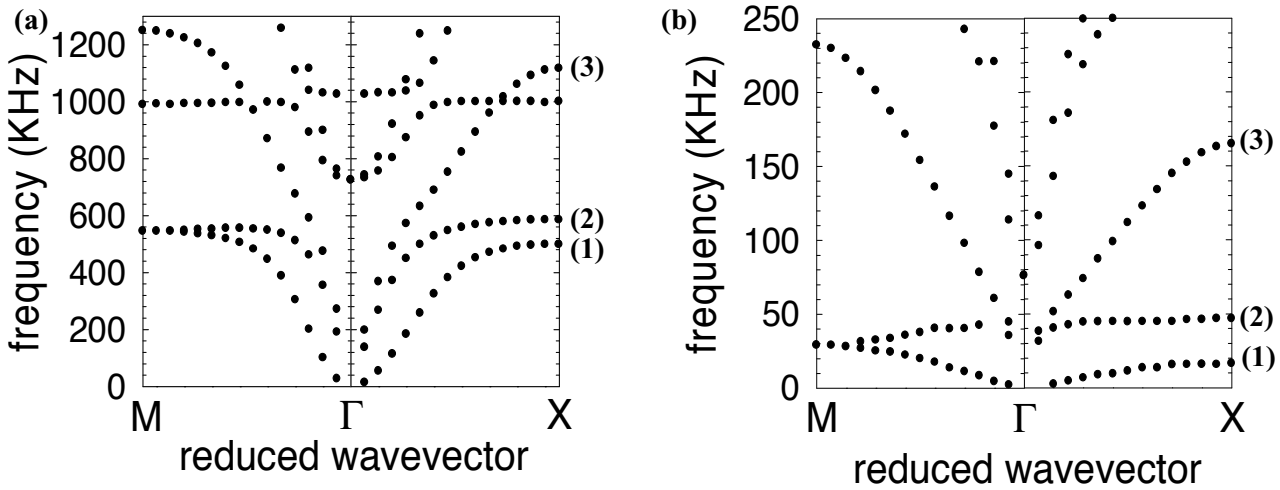

Fig.2. (a) Band structure of the model of Fig. 1a for steel cylinders on a silicon plate. In comparison with the geometrical parameters used for the calculation of dispersions curves in Fig. 1, we have changed in (a) the thickness of the plate (*e=1.2mm*) and in (b) the height of the dots *h=2.7mm*.

As reported previously, the gap is generated from the bending of the branches #2 and #3. We have investigated the distribution of the eigenmodes inside the unit cell for these two branches, at the points A and B of the dispersion curves in Fig. 1c (*h=0.6mm*, *e=0.1mm*). The displacement fields of the corresponding eigenmodes ($\boldsymbol{k_A} = \boldsymbol{k_B} = (\pi/a, 0, 0)$; $f_A = 233.0$ *kHz* and $f_B = 180.6$ *kHz*) have been calculated using the Finite Element method and are plotted in Fig. 3. For the point A (Fig. 3a), we clearly observe an oscillation of the dot in the x direction associated with a bendig of the plate. For the point B (Fig. 3b) we observe an oscillation of the dot in the z direction correlated to a strong bending of the plate. In both cases, the displacement fields are distributed in the dot as well in the plate, in agreement with the dependence of the branches #2 and #3 with both parameters *e* and *h*. Moreover, the stronger dependence of the branch #3 with the thickness of the plate observed in the previous section can be related to a higher deformation in the plate than in the dot.

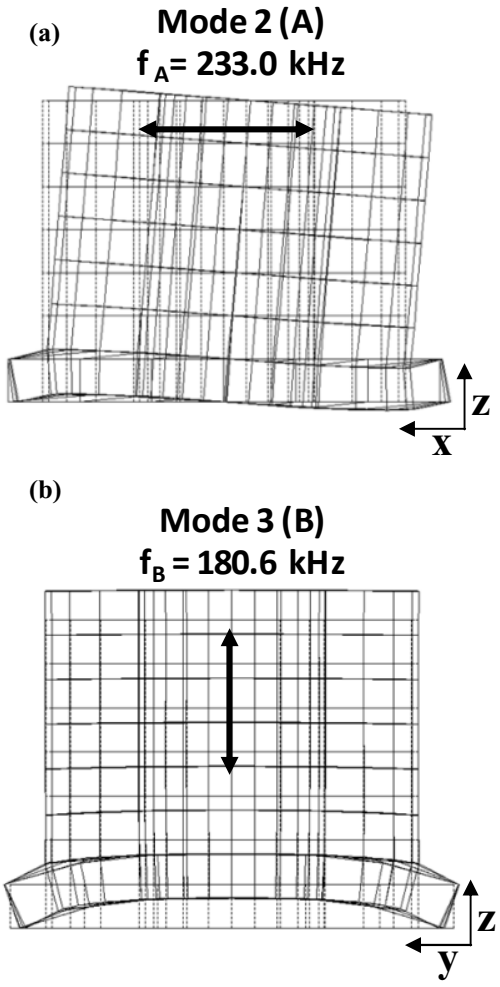

Fig.3. Displacements field of the eigenmode at the boundary of the Brillouin zone (*X*) inside one unit cell for *e=0.1 mm*, *h=0.6 mm* at the eigenfrequencies (a) *233.0 kHz* and (b) *180.6 kHz* (points A and B in Fig. 2.b). In those figures, the dashed lines correspond to the rest position of the structure.

We have also investigated the persistence of this gap against different combinations of the materials constituting the dot and the plate among a set of five materials (tungsten, steel, silicon, aluminum and epoxy). In Fig. 4a, we show the gap by changing the material of the plate when the dots are made of steel. Similarly, Fig. 4b displays the gap for various materials in the dots and the plate being made of silicon. One can notice the persistence of this gap even if the constituting materials are identical. This supports the origin of the gap as being related to the geometrical rather than physical parameters of the structure. On the other hand, the central frequency of the gap is very dependent upon the choice of the materials and happens at lower frequencies when we combine a high density material (tungsten) in the cylinders with a low density material (epoxy) in the plate.

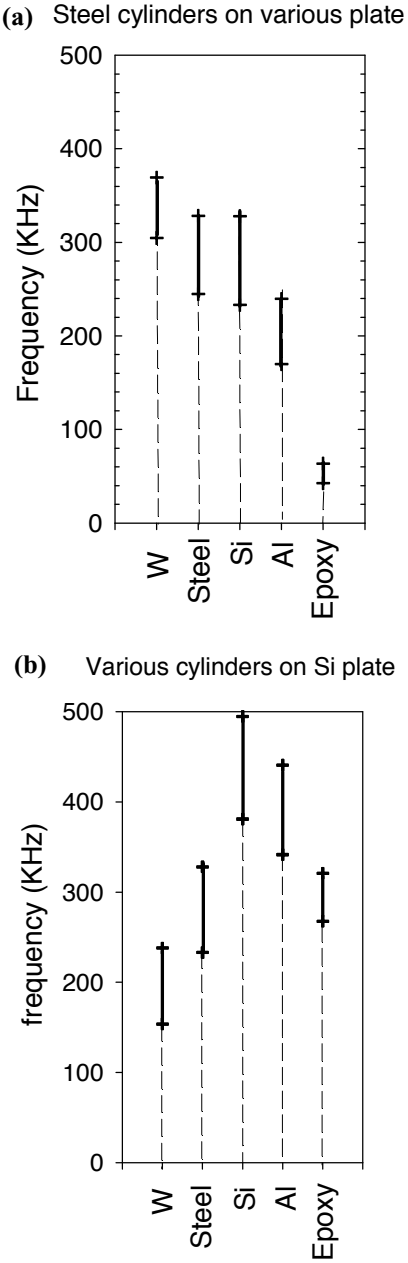

Fig.4. (a) Phononic Evolution of the lower frequency gap limits substituting the material of the cylinders or the plate by tungsten, steel, silicon, aluminum and epoxy. (a) Steel cylinders on the previous materials constituting the plate. (b) Various cylinders on silicon plate.

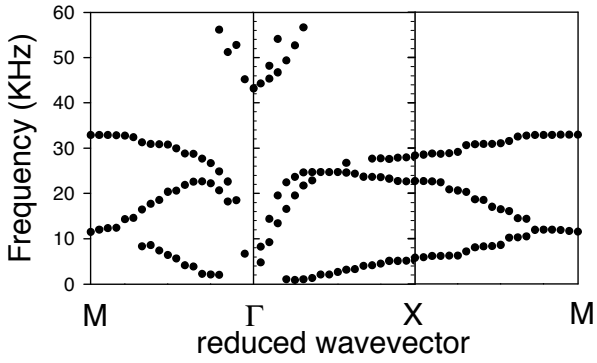

Fig.5. Band structure in the frequency range *[0, 60] kHz* of the model of Fig. 1a for tungsten cylinders on an epoxy plate, calculated in the first irreducible Brillouin zone of the phononic crystal. The lattice parameter is $a = 1$ *mm*, the height of the cylinders $h = 0.6$ *mm* and the thickness of the plate $e = 0.1$ *mm*.

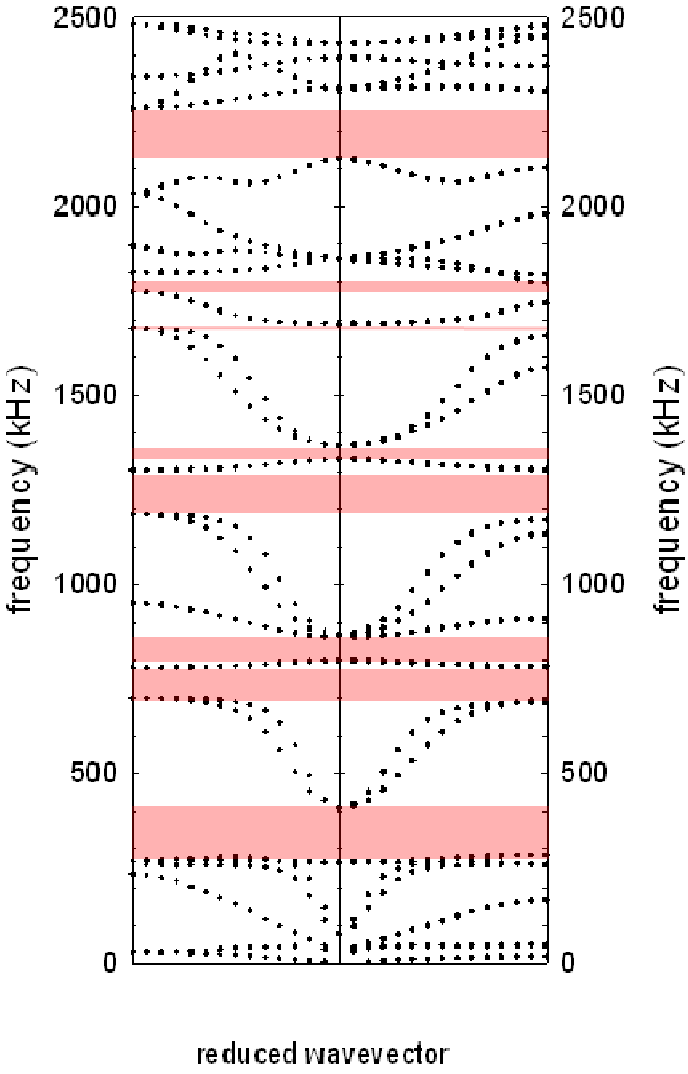

Fig.6. Band structure in the frequency range *[0, 2500] kHz* for steel cylinders on a silicon plate with the following parameters are $a = 1$*mm*, $h=2.7$*mm* and $e=0.2$*mm*.

We show in Fig. 5 the low frequency band structure of the phononic crystal made of tungsten (for dots and epoxy (for the plate). The gap extends now from *32.7 kHz* to *43.2 kHz*. It is worthwhile to notice that one can obtain a gap in the audible frequency range, around 2 kHz, by multiplying all the lengths in the structure by 20, which means a lattice period of *20mm*. Such solid system could then easily be used as acoustic insulator.

To investigate the possibility of more and/or higher frequency gaps, we present in Fig. 6 the band structure for the following parameters: $h=2.7$*mm*, $e=0.2$*mm* and $\beta=0.564$; in particular, the height of the cylinders is now taken to be much bigger than in the preceding case. One can notice the occurrence of several gaps whose number is dependent upon the height of the dots. It is interesting to remark that, up to a certain frequency range, the opening of the gaps results from the crossing of the normal acoustic branches with almost flat bands, which is similar to the case of locally resonant materials.

Finally, we have studied the case of a rectilinear waveguide created inside the phononic crystal dots. The geometrical parameters have been chosen as in Fig. 1b, i.e. $a=1$*mm*, $h=0.6$*mm* and $e=0.1$*mm* to ensure the largest forbidden gap. A supercell containing five unit cells in the y direction is considered in the FDTD calculation. The guide is created by removing one row of dots in the third unit cell, thus constituting a linear waveguide in the x direction. The width of the waveguide, δ, has been chosen as a variable parameter that insures a frequency translation of the waveguide modes [3]. Fig. 7a shows the band structure in the ΓX direction for the waveguide structure with $\delta = 1.05\ a$. It exhibits one new branch inside the gap (*[265.2, 327.9] kHz*). To show the confinement of this mode inside the waveguide, we focus on the point labeled C of the dispersion curve. The modulus of the displacement field associated with this mode is displayed on the maps of Fig. 7b and represented in three-quarter, top and lateral views. The map shows clearly that the acoustic displacement is confined in the plate and localized in the waveguide and does not leak out into the rest of the structure.

3 Conclusion

The purpose of this paper was to investigate, using the Finite Difference Time Domain method, the dispersion of the elastic waves of a periodic array of dots deposited on a plate. We put emphasis on the possibility of a low frequency gap and its existence conditions against various geometrical and physical parameters. This gap is generated by the bending of the two plate's modes, i.e. the shear horizontal (SH) and the symmetric Lamb mode (S_0). The central frequency of the gap depends on both geometrical parameters e, the thickness of the plate and h, the height of the dots. The opening of the gap is related to the bending of the symmetric Lamb mode (S_0) which is strongly sensitive to the thickness of the plate. The gap is always present, even if the constituting materials are the same that supports the origin of the gap as coming from the geometric design. However, the central frequency and the width of the gap are dependent upon the materials constituting the cylinders and the plate. More particularly in view of acoustic isolation, the value of the lowest gap could be lowered using a strong density material for the cylinders and a low density one for

the plate. We also showed the existence of higher gaps, especially by increasing the height of the cylinders. Finally, we show that plate modes can be guided inside a linear defect created by removing one row of dots. Similar studies should be performed for different shapes of the dots, hollow or coated cylinders, etc… Such system could found original application in the field of guiding and filtering waves as well as sound isolation inside vibration plate structures.

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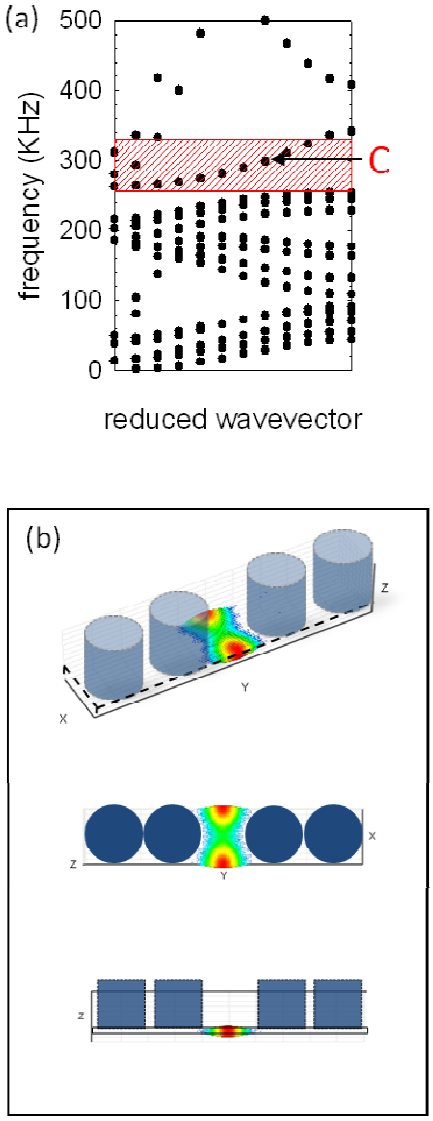

Fig.7. (color online): (a) Band structure along the ΓX direction of the Brillouin zone calculated with a supercell containing 1 x 5 unit cells for the phononic crystal dots on a homogeneous plate. The waveguide is formed by removing one row of dots in the third unit cell. The red hatched areas indicate the location of the absolute band gap of the perfect structure. The dispersion curve is calculated for a waveguide width of $\delta=1.05a$. (b) Maps of the modulus of the elastic displacement field for the waveguide mode, represented in three-quarter, top, and lateral views for the point C of the dispersion curve. The red color corresponds to the maximum displacement, whereas the blue color corresponds to the minimum.