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Scattering of seismic waves by a fracture zone containing randomly distributed frictional cracks

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An investigation is conducted how the geometrical properties of a crack distribution in a fault zone and the frictional characteristics of the crack surface are reflected in the attenuation and dispersion of incident seismic waves. All cracks are assumed to be either aligned or randomly oriented. The crack width is assumed to obey a power law distribution, according to seismological knowledge. The crack surface is assumed to be stress-free, or to undergo viscous friction. When the crack distribution is statistically homogeneous, the calculated dispersion and attenuation exhibit that the variance of crack size affects in different way the coherent wave. The analysis on the effect of the friction shows that the crack scattering decreases as the viscosity increases, which is expected since for high viscosity, the crack faces remain almost welded to each other. The results obtained in this work will be applicable to the state close to the occurrence time in large earthquakes.

1 Introduction

In the present paper, we investigate the attenuation and dispersion of seismic waves propagating through a zone of randomly distributes frictional cracks. We employ the idea of mean wave formalism developed in [1, 2]. The crack width is assumed to obey a power law distribution [3]. The distribution of cracks is assumed to be homogeneous. All cracks are assumed to be either aligned or randomly oriented. We also assume that the crack surface is stress-free, or undergoes viscous friction Newtonian-type. To deal with cracks under high confining pressure, the friction-free boundary conditions will be more realistic than the stress-free boundary conditions (which we considered in our previous works [4, 5]), for the following two reasons. The first is the existence of fluid in the earth's crust. The existence of fluid inside a crack cavity might obstruct the entire coherence of the crack surface. Such fluid could be highly viscous like, for example, the composite of water and gouge. The second reason is the visco-elastic response of contacting solid material to seismic waves under high confining pressure. The assumption subject to viscous friction for the crack surfaces was also adopted in [6, 7]. However, apart from [4, 5, 7], the authors of the other cited works assumed the low crack density. It is, however, thought that the crack density and the crack interaction are high in the epicentral region of imminent large earthquakes. Hence the assumption of high crack density is required to study the state immediately before the occurrence of large earthquakes. We propose in this paper a theoretical study which takes into account a relatively high crack interaction. The boundary integral equation formulation applied in [7, 8] can rigorously treat the multiple crack interactions for arbitrary wave numbers. This method, however, faces the difficulty of core memory and cpu-time limits in dealing with large numbers of cracks.

It has been observed experimentally and verified analytically that multiple scattering by cracks generates a coherent wave that is described by a complex-valued and frequency-dependent wave number. In this paper, we have been able to express the effective wave number in closed form, from which both real and imaginary parts (thus, dispersion and attenuation) can be obtained without difficulty. This formula formally requests the knowledge of the angular shape function for a frictional crack, and is relevant for even moderate densities of cracks. In the following, the effect of the variance of crack width and the influence of the viscosity on the scattering attenuation and dispersion is investigated. In addition, the low- and high-frequency limits are analytically derived.

2 Multiple scattering formulation

We consider N cracks uniformly distributed, and N is assumed to be very large. Each crack has a very thin cavity filled up by Newtonian fluid with viscosity coefficient η. The mean thickness of cavity, ℓ, is thought to be negligibly small, though its value affects the strength of viscous friction acting on the crack surface. The crack width $2a$ is assumed to obey a power law density distribution,

$$p(a) = Ca^{-\gamma}, \quad a_{\min} \leq a \leq a_{\max}, \tag{1}$$

where $a_{\min}$ and $a_{\max}$ are the lower and upper limits of the distribution a, γ is a positive constant, and C is a normalization factor given by [3]

$$C = \begin{cases} (1-\gamma)/\left(a_{\max}^{1-\gamma} - a_{\min}^{1-\gamma}\right), & \gamma \neq 1, \\ \ln^{-1}\left(a_{\max}/a_{\min}\right), & \gamma = 1. \end{cases} \tag{2}$$

Let u^{inc} denote the incident displacement in the absence of any crack. The total displacement field u is the sum of the incident displacement u^{inc} and the scattered displacements corresponding to each of the N cracks. Thus, one has

$$u_P = u_P^{inc} + \sum_{j=1}^{N} T_P^j u_j^E, \tag{3}$$

where P and j denote the observation point and the location of the jth crack, respectively. In Eq. (3), u_j^E represents the wave effectively incident on the jth crack, and the term $T_P^j u_j^E$ is the wave scattered by the jth crack due to the excitation of u_j^E, which, in turn, is expressed as

$$u_j^E = u_j^{inc} + \sum_{\substack{i=1,\\ i\neq j}}^{N} T_j^i u_i^E. \tag{4}$$

Eqs. (3) and (4) form the basic equations which describe the displacement field exactly, though formally. The operator T_P^j determines the scattering property of the jth crack. The stochastic property of the crack is assumed to be described by the location of the center of the crack, the crack width and the crack orientation. In addition, it is assumed to be independent of that of the other cracks. Then, we have the configurational average of the total displacement, $\langle u_P \rangle$, as

$$\langle u_P \rangle = u_P^{inc} + \frac{n_0}{N}\sum_{j=1}^{N}\int T_P^j \left\langle u_j^E \right\rangle_j p\left(\theta_j\right) p\left(a_j\right) d\boldsymbol{\xi}^j d\theta_j da_j, \tag{5}$$

where $p(\cdot)$ is the probability density function, $\boldsymbol{\xi}^j$ is the location of the center of the jth crack, $2a_j$ is the width of the jth crack, and n_0 is the number density of the crack distribution. The quantity $\left\langle u_j^E \right\rangle_j$ represents the exciting displacement acting on the jth crack averaged over all possible configurations of all the other scatterers. Here, we

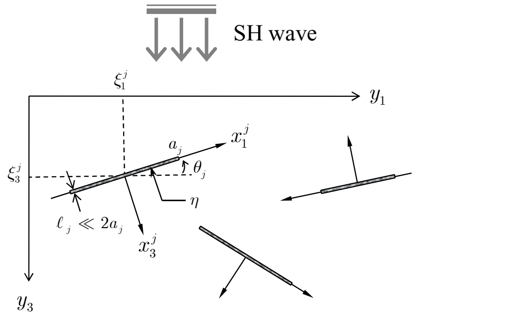

Fig.1 Global and local coordinate systems. The cavity of all cracks is filled with fluid with viscous coefficient η .

adopt the approximation, first introduced by Foldy [9]

$$\left\langle u_j^E \right\rangle_j \simeq \lim_{P\to j} \left\langle u_P \right\rangle, \text{ as } N\to\infty, \tag{6}$$

and reduce Eq. (5) to the following form of an integral equation

$$\left\langle u_P \right\rangle = u_P^{inc} + \frac{n_0}{N}\sum_{j=1}^{N}\int T_P^j \left\langle u_j \right\rangle p\left(\theta_j\right) p\left(a_j\right) d\boldsymbol{\xi}^j d\theta_j da_j . \tag{7}$$

Next, we represent $T_P^j\left\langle u_j\right\rangle$ in explicit form. We define the local coordinate system $\left(x_1^j, x_3^j\right)$ attached on the j th crack ($j = 1,2,\ldots,N$) and the origin is assumed to be at the center of the crack. The x_1^j axis coincides with the crack faces. The angle between the x_1^j and y_1 axes is given by θ_j , and the center of the crack is located at $\boldsymbol{\xi}^j = \left(\xi_1^j, \xi_3^j\right)$ in the coordinate system $\left(y_1, y_3\right)$, as shown in Fig. 1. As in [1], we seek a coherent displacement field $\left\langle u_P\right\rangle$ in the form

$$\left\langle u_P \right\rangle = \mathrm{e}^{\mathrm{i}Ky_3} = \mathrm{e}^{\mathrm{i}K\left(x_1^j\sin\theta_j + x_3^j\cos\theta_j + \xi_3^j\right)}, \tag{8}$$

where K is referred to as the effective wave number. Here the time factor $\exp(-\mathrm{i}\omega t)$ is omitted for brevity. The wave front of the effective plane wave is assumed to parallel the y_1 axis. According to the representation theorem, $T_P^j\left\langle u_j\right\rangle$ can be described in terms of the displacement discontinuity across the crack faces [10], Δu_j , generated by the incidence of $\left\langle u_j\right\rangle$. If we take into account the form of Eq. (8), we have [2]

$$T_P^j\left\langle u_j^T\right\rangle = \frac{K}{4}\cos\theta_j \mathrm{e}^{\mathrm{i}K\xi_3^j}\frac{\partial}{\partial x_3^j}\int_{-a_j}^{a_j}\Delta u_j(s) H_0^{(1)}\left(k_T r_0\right) ds , \tag{9}$$

where $k_T = \omega / c_T$, $r_0^2 = \left(x_1^j - s\right)^2 + (x_3^j)^2$, c_T is the shear wave speed of the matrix, and $H_0^{(1)}$ is the Hankel function of order zero. If we apply $\partial^2/\partial(x_1^j)^2 + \partial^2/\partial(x_3^j)^2 + k_T^2$ on both sides of Eq. (9), combining the resulting expression with Eqs. (7) and (8), and using $d\boldsymbol{\xi}^j = d\xi_1^j d\xi_3^j = dx_1^j dx_3^j$, we obtain [2]

$$\begin{aligned} k_T^2 - K^2 &= -n_0 K^2 \\ &\int_{-\pi/2}^{\pi/2}\cos^2\theta p(\theta)\int_{a_{\min}}^{a_{\max}} p(a) B\left(k_T,\ell,\eta,a,\theta\right) da\, d\theta, \end{aligned} \tag{10}$$

where $a_{\min}$ and $a_{\max}$ are the lower and upper limits of the distribution of a , and

$$B\left(k_T,\ell,\eta,a,\theta\right) = \int_{-a}^{a}\Delta u(s)\,\mathrm{e}^{-\mathrm{i}Ks\sin\theta} ds . \tag{11}$$

Stochastic property of each crack was assumed to be identical, so that the subscript j could be discarded in the Eq. (10). Note that the complex-valued number B given by Eq. (11) depends on k_T , but not on K . It can be shown that the quantity B is related to the forward scattering shape function $f(\theta,\theta)$ for a crack of width $2a$, mean thickness

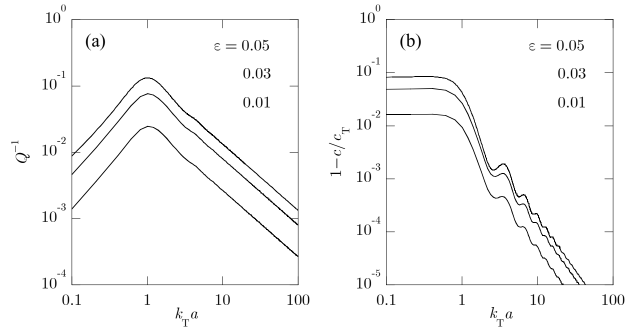

Fig.2 Dependence of (a) Q^{-1} and (b) c on ε . All cracks have equal width. $\theta' = 0°$ and $\sigma = 0.25$.

ℓ , and viscosity coefficient η , excited by an incident plane wave of wave number k_T at an arbitrary angle θ from the normal of the crack [4]

$$B\left(k_T,\ell,\eta,a,\theta\right) = \frac{4\pi f(\theta,\theta)}{k_T^2\cos^2\theta} . \tag{12}$$

Substituting Eq. (12) into Eq. (10) we obtain the following simple and explicit formula for the effective wave number

$$K^2 = k_T^2 \Bigg/ \left(1 - \frac{4\pi n_0\left\langle f(0)\right\rangle}{k_T^2}\right), \tag{13}$$

where $\left\langle f(0)\right\rangle$ is the average forward shape function given by

$$\left\langle f(0)\right\rangle = \int_{-\pi/2}^{\pi/2} p(\theta)\int_{a_{\min}}^{a_{\max}} p(a) f(\theta,\theta)\, da\, d\theta . \tag{14}$$

Note that the expression (13), which solves Eq. (10) posed initially in [2], is a new result. Let $\left(r_P, \theta_P\right)$ denote the polar coordinates defined by $r_P^2 = x_1^2 + x_3^2$ and $\cos\theta_P = x_3 / r_P$. Next, we calculate the angular shape function $f\left(\theta_P,\theta\right)$ for a frictional crack insonified by an incident wave $u^{inc} = u_0\exp\left(\mathrm{i}k_T\left(x_1\sin\theta + x_3\cos\theta\right)\right)$, where u_0 is an amplitude factor. The solution of the one-crack problem cannot be expressed in closed-form. However, it can be expressed in terms of a dislocation density, which is a solution of a singular integral equation [11]. For a frictional crack with a cross-section located along the segment $x_3 = 0$, $\left|x_1\right| < a$, in the $\left(x_1, x_3\right)$ plane, the dislocation density b satisfies the two equations,

$$\int_{-a}^{a} b(v,\theta,\sigma)\left[\frac{1}{v - x_1} + \tilde{S}\left(v - x_1;\sigma\right)\right] dv = g\left(x_1\right), \tag{15}$$

$$\int_{-a}^{a} b(v,\theta,\sigma)\, dv = 0 , \tag{16}$$

for $\left|x_1\right| < a$. In Eq. (15), the functions $\tilde{S}$ and g are given by

$$\tilde{S}(x;\sigma) = \int_0^\infty\left(\frac{\beta}{\zeta} - 1\right)\sin(\zeta x)\, d\zeta - 2\mathrm{i}\pi k_T\sigma\mathrm{H}(x) , \tag{17}$$

$$g(x) = \mathrm{i}\pi u_0 k_T\cos\theta\,\mathrm{e}^{\mathrm{i}k_T x\sin\theta} , \tag{18}$$

where $\mathrm{H}(\cdot)$ is a unit step function and β is defined by $\beta^2 = \zeta^2 - k_T^2$, with $\mathrm{Im}\,\beta \leq 0$. The dimensionless parameter $\sigma = c_T\eta / \mu\ell$ represents the strength of the friction, and μ denotes the rigidity of the matrix. When $\sigma = 0$, the crack surface is stress-free. In the far field, the scattered displacement in the presence of a crack takes the form

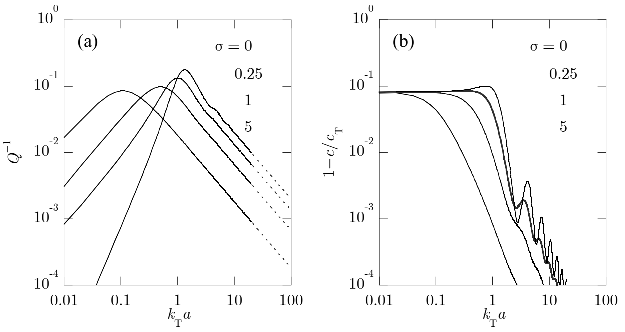

Fig.3 Dependence of (a) Q^{-1} and (b) c on σ . All cracks have equal width. $\varepsilon = 0.05$ and $\theta' = 0°$.

$$T_P u^{inc} \simeq u_0 \sqrt{\frac{2\,\mathrm{i}\,\pi}{k_T r_P}}\, \mathrm{e}^{\mathrm{i} k_T r_P}\, f\left(\theta_P, \theta\right), \tag{19}$$

where the angular shape function $f\left(\theta_P, \theta\right)$ has the following general form [4]

$$f\left(\theta_P, \theta\right) = \frac{\cot\theta_P}{2\pi u_0} \int_{-a}^{a} b(v, \theta, \sigma)\, \mathrm{e}^{-\mathrm{i} k_T v \sin\theta_P}\, dv\,. \tag{20}$$

Then, the forward ($\theta_P = \theta$) scattering shape function can be obtained from Eq. (20), once the dislocation density has been computed from Eqs. (15)-(16).

3 Wave attenuation and dispersion

Observe that $\operatorname{Im} K^2$ and $\operatorname{Im}\langle f(0)\rangle$ in Eq. (13) have the same sign. It can be verified numerically that $\operatorname{Im}\langle f(0)\rangle$ is positive for all values of k_T , θ , a and σ . It follows then, that K^2 lies in the upper complex plane. Calculating from Eq. (13) the complex root of K^2 that lies in the first quadrant, one finds that

$$\operatorname{Re} K = \frac{q k_T^2}{\sqrt{2}\left|k_T^2 - 4\pi n_0 \left\langle f(0)\right\rangle\right|}, \tag{21}$$

$$\operatorname{Im} K = \frac{4\pi n_0 \operatorname{Im}\left\langle f(0)\right\rangle k_T^2}{\sqrt{2} q\left|k_T^2 - 4\pi n_0 \left\langle f(0)\right\rangle\right|}, \tag{22}$$

where the real-valued quantity q is defined by

$$q = \left(\operatorname{Re}\left(k_T^2 - 4\pi n_0 \left\langle f(0)\right\rangle\right) + \left|k_T^2 - 4\pi n_0 \left\langle f(0)\right\rangle\right|\right)^{1/2}. \tag{23}$$

The symbols Re and Im stand for the real and imaginary parts of a complex number, respectively, and the vertical bars denote the complex modulus. Since the attenuation coefficient Q^{-1} and the wave speed c are expressed in the forms [12]

$$Q^{-1} = 2\frac{\operatorname{Im} K}{\operatorname{Re} K},\ \frac{c}{c_T} = \frac{k_T}{\operatorname{Re} K}, \tag{24}$$

Eqs. (21)-(24) yield the expressions for Q^{-1} and c as functions of k_T .

We assume two models for the crack orientation, that is, the aligned crack model and the randomly-oriented crack model. All cracks are assumed to be aligned in the former model, so that $p(\theta) = \delta\left(\theta - \theta'\right)$, where $\delta(\cdot)$ denotes the Dirac function, and θ' is the angle between the crack faces and the wave front of the coherent plane wave $\langle u_P \rangle$. The orientation of each crack is assumed to be random in the later model, and the distribution for the crack orientation is thought to be homogeneous, so that $p(\theta) = 1/\pi$. Calculated results are shown in Figs. 2-6.

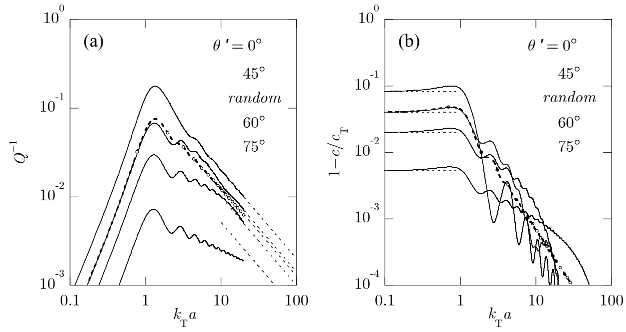

Fig.4 Dependence of (a) Q^{-1} and (b) c on θ' . All cracks have equal width. $\varepsilon = 0.05$ and $\sigma = 0$. Broken curves show the asymptote of Q^{-1} at high frequency, and the static value of c . The case of randomly-oriented cracks is also plotted as (-o - o).

The attenuation coefficient Q^{-1} and the wave speed c are functions of the crack concentration $\varepsilon = n_0 a^2$ and the dimensionless frequency $\tilde{\omega} = k_T a$, when all cracks have the same width $2a$; or of $\varepsilon = n_0 a_{\max}^2$, $\tilde{\omega} = k_T a_{\max}$, $\delta = a_{\min} / a_{\max}$ and γ , when the crack width has the power law density distribution (1); Q^{-1} and c are also functions of the dimensionless friction parameter σ and the orientation angle θ' (when the crack orientation is not random).

Fig. 2 illustrates the effect of Q^{-1} and c on ε . An increase in the crack concentration ε produces larger (smaller) values for the wave speed (attenuation) for the entire range of frequencies, as can be observed in Fig. 2.

When the frequency approaches infinity, for any value of ε , σ , and θ' , the attenuation coefficient oscillates around a constant value, and the wave speed approaches the speed in the matrix. For all frequencies and crack densities considered, the wave speed c of a cracked medium is always smaller than c_T of the uncracked medium. In Figs. 3-6, the broken curves show the static and geometrical limits of the wave speed and the attenuation, respectively, which are derived in the next section.

Fig. 3 shows the effect of Q^{-1} and c on σ . As σ increases, the peak value of Q^{-1} becomes broader and the peak frequency decreases monotonically. The dependence on σ of c , denotes that the corner of the curve moves to low-frequency range as σ increases. The figure denotes that the wave length at which the most intensive scattering is observed becomes longer as the viscous friction becomes stronger. The effect of the friction may be to suppress the slips along the cracks especially in the high frequency range.

Fig. 4 illustrates the effect of Q^{-1} and c on θ' , when the crack surface is stress-free. The figure denotes that the peak frequency at which Q^{-1} takes the peak value is almost insensitive to θ' . As the cracks are aligned more obliquely to the incident wave, Q^{-1} (c) values become smaller (larger). Results corresponding to the case of randomly-oriented cracks are also given in Fig. 4. The essential advantage of the assumption of random crack orientation is that one can treat the cracked medium as a macroscopically isotropic medium which is much easier to handle than the orthotropic one (when cracks are aligned). Observe that at low frequencies, Q^{-1} and c for randomly-oriented cracks, are almost identical to the corresponding values of Q^{-1} and c for the

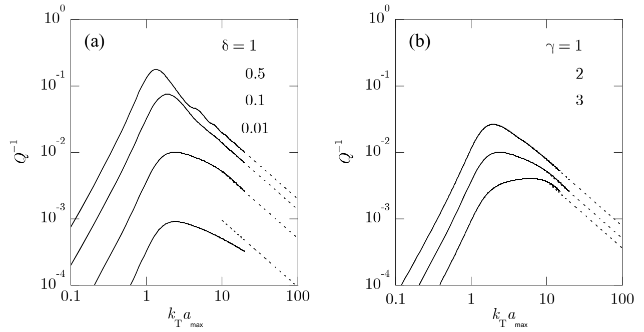

Fig.5 Effect of δ and γ on Q^{-1}. (a) Dependence on δ with $\gamma = 2$. (b) Dependence on γ with $\delta = 0.1$. We chose $\sigma = 0$ and $\theta' = 0°$. Broken curves show the asymptote of Q^{-1} at high frequency.

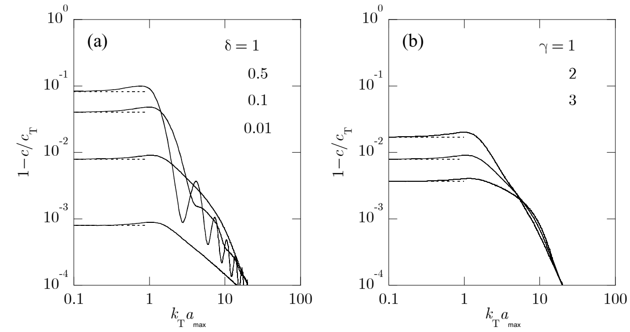

Fig.6 Effect of δ and γ on c. (a) Dependence on δ with $\gamma = 2$ is assumed. (b) Dependence on γ with $\delta = 0.1$. We chose $\sigma = 0$ and $\theta' = 0°$. Broken curves show the static value of c.

case of cracks inclined at $45°$.

The effects of γ and δ on Q^{-1} and c are now inspected. Fig. 5 shows the dependence of Q^{-1} on γ and δ. The attenuation Q^{-1} is larger for smaller γ and/or larger δ. This occurs because the rate of the number of larger cracks increases as γ decreases and/or δ increases for a fixed $a_{\max}$: larger cracks will contribute more to the attenuation. The peak frequency at which Q^{-1} takes the peak value tends to increase, and Q^{-1} has a broader peak as γ increases and/or δ decreases.

Fig. 6 illustrates the dependence of c on γ and δ. The form of all curves show the same general features. The wave speed is observed to be smaller for smaller γ and/or larger δ. This occurs because smaller γ and/or larger δ cause more resistance to the propagation of the coherent wave as stated above in the interpretation of the effect of γ and δ on Q^{-1}. The increase rate of c seems to be considerably dependent on the model-parameters in the high-frequency range, which contrasts with the behavior of Q^{-1} in Fig. 5. However, we notice that the ripples in the high frequency-range, on both curves of Q^{-1} and c, disappear as δ decreases.

4 Analytical limits

The low- and high-frequency limits of the wave speed and the attenuation coefficient corresponding to a system of frictional cracks are obtained in this section in analytical form.

4.1 Low-frequency limits

The limit of the angular shape function $f(\theta_P, \theta)$ given by Eq. (20), when $\tilde{\omega} = k_T a$ approaches zero, can be calculated in closed-form by using the method presented in [13]. Then, using this expression in Eqs. (14), (21)-(23) one finds the following results when all cracks have equal width

case 1 : $\pi\varepsilon < \cos^{-2}\theta'$

$$\frac{c}{c_T} = \left(1 - \pi\varepsilon\cos^2\theta'\right)^{1/2} + O\left(\tilde{\omega}^2 \ln\tilde{\omega}\right), \qquad (25)$$

$$\alpha a = \frac{\pi^2 \varepsilon \cos^2\theta' \tilde{\omega}^3}{16\left(1 - \pi\varepsilon\cos^2\theta'\right)^{3/2}} + O\left(\tilde{\omega}^4\right), \quad (\sigma = 0), \qquad (26)$$

$$\alpha a = \frac{h(\sigma,\theta')\cos^2\theta'}{\left(1 - \pi\varepsilon\cos^2\theta'\right)^{3/2}} O\left(\tilde{\omega}^2\right), \quad (\sigma \neq 0); \qquad (27)$$

case 2 : $\pi\varepsilon > \cos^{-2}\theta'$

$$\frac{c}{c_T} = \frac{16\left(\pi\varepsilon\cos^2\theta' - 1\right)^{3/2}}{\pi^2 \varepsilon \tilde{\omega}^2 \cos^2\theta'} + O(\ln\tilde{\omega}), \qquad (28)$$

$$\alpha a = \frac{\tilde{\omega}}{\left(\pi\varepsilon\cos^2\theta' - 1\right)^{1/2}} + O\left(\tilde{\omega}^3 \ln\tilde{\omega}\right), \quad (\sigma = 0), \qquad (29)$$

$$\alpha a = \frac{h(\sigma,\theta')\cos^2\theta'}{\pi\varepsilon\cos^2\theta' - 1} O(\tilde{\omega}), \quad (\sigma \neq 0), \qquad (30)$$

where $h(\sigma,\theta')$ is a linear combination of σ and $\sin\theta'$, and all cracks are aligned. When the cracks are randomly oriented, one infers that

case 1 : $\pi\varepsilon < 2$

$$\frac{c}{c_T} = \left(\frac{2 - \pi\varepsilon}{2}\right)^{1/2} + O\left(\tilde{\omega}^2 \ln\tilde{\omega}\right), \qquad (31)$$

$$\alpha a = \left(\frac{2}{2 - \pi\varepsilon}\right)^{3/2} \frac{\pi^2\varepsilon}{32}\tilde{\omega}^3 + O\left(\tilde{\omega}^4\right), \quad (\sigma = 0), \qquad (32)$$

$$\alpha a = \left(\frac{2}{2 - \pi\varepsilon}\right)^{3/2} \tilde{h}(\sigma) O\left(\tilde{\omega}^2\right), \quad (\sigma \neq 0); \qquad (33)$$

case 2 : $\pi\varepsilon > 2$

$$\frac{c}{c_T} = \left(\frac{\pi\varepsilon - 2}{2}\right)^{3/2} \frac{32}{\pi^2\varepsilon\tilde{\omega}^2} + O(\ln\tilde{\omega}), \qquad (34)$$

$$\alpha a = \left(\frac{2}{\pi\varepsilon - 2}\right)^{3/2} \tilde{\omega} + O\left(\tilde{\omega}^2 \ln\tilde{\omega}\right), \quad (\sigma = 0), \qquad (35)$$

$$\alpha a = \frac{2\tilde{h}(\sigma)}{\pi\varepsilon - 2} O(\tilde{\omega}), \quad (\sigma \neq 0), \qquad (36)$$

where $\tilde{h}(\sigma)$ is the mean value of $h(\sigma,\theta')$ over the range $[0,\pi)$. Note that $\alpha = \operatorname{Im} K$ represents the spatial attenuation. Observe that, as expected form physical ground, the wave speed in the limit as $\tilde{\omega}$ approaches zero is independent of the viscosity. For $\pi\varepsilon > \cos^{-2}\theta'$ and $\pi\varepsilon > 2$, Eqs. (28) and (34), respectively, show that the velocity tents to infinity as the frequency approaches zero, which is not physically acceptable. Thus, we conclude that the results of this paper are valid for small values of the crack density not greater than $1/\pi\cos^2\theta'$ for the aligned crack model, and less than $2/\pi$ for the random crack model. A conclusion similar to ours is obtained in [14], where the viscosity is zero and crack faces are parallel to the incident wave front. The foregoing analysis can of course be reiterated for the case when the crack width has the power law distribution (1). Results are not reported here. However, we observed that the results for the wave speed are identical to the first order in the crack density to the corresponding limits given in [2].

4.2 High-frequency limits

The high-frequency asymptote of $f(\theta_P, \theta)$ can be calculated in a manner similar to that for the low-frequency limit. In this limit, one finds that, when all cracks have equal width and orientation angle θ', the attenuation is given by

$$\alpha(a) \simeq \frac{2n_0 a \cos\theta'}{1+2\sigma} - \frac{n_0 a\sigma \sin\theta'}{(1+2\sigma)^2}, \tag{37}$$

whereas, for randomly-oriented cracks

$$\alpha(a) \simeq \frac{4n_0 a}{\pi(1+2\sigma)}. \tag{38}$$

When the crack width obeys the power law distribution (1), one infers

$$\begin{aligned} \alpha &\simeq \alpha(a_{\max})\frac{\delta-1}{\ln\delta}, && \text{for } \gamma = 1; \\ &\simeq \alpha(a_{\max})\frac{\delta\ln\delta}{\delta-1}, && \text{for } \gamma = 2; \\ &\simeq \alpha(a_{\max})\frac{1-\gamma}{2-\gamma}\frac{1-\delta^{2-\gamma}}{1-\delta^{1-\gamma}}, && \text{for } \gamma \neq 1,2. \end{aligned} \tag{39}$$

The limits (37) and (38) agree with results of [4] where the viscosity is zero.

5 Discussion and conclusion

The effect of random crack distribution on the attenuation and dispersion of seismic waves is theoretically investigated.

The strength of the viscosity can be naturally neglected for low frequency. It means that, when the incident wavelength is much longer than the crack width, the cracks do not hinder the incident wave, whether they contain a viscous fluid or not.

When the viscosity increases, the wave speed increases. At the same time, the attenuation decreases and approaches zero. This shows that the crack scattering decreases as the viscosity increases, which is expected, since for high viscosity the crack faces remain almost welded to each other.

In the high-frequency limit, the wave speed approaches the speed in the matrix, and the attenuation coefficient reaches constant values. The limit of the attenuation is a function of the crack density and viscosity, when all cracks have the same width; but also on δ, γ, when the crack width has the power law distribution. The orientation angle appears in the limit only when the cracks are not randomly oriented.

When all cracks have the same width $2a$, we notice that there appear characteristic ripples in the high-frequency range, on curves of Q^{-1} and c. They reflect the periodic fluctuation of the integrated displacement discontinuity across each crack. This is interpreted as the variation of the interference pattern between the diffracted fields originated from both of the crack tips. When the crack width obeys the power law distribution stated above, due to various scales of the crack, no appearance of such ripples are observed on curves of Q^{-1} and c. The ripples with various wavelengths, which are related to the width of each crack, might be smoothed through the configurational averaging.

In the low-frequency limit, we find that the numerical results are valid for values of crack density less than $1/\pi\cos^2\theta'$ for the aligned crack model, and less than $2/\pi$ for the random crack model, independently of the viscosity.

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