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## **Coupling transfer matrix method to finite element method for the analysis of hollow body networks with passive or reactive elements**

Fabien Chevillotte<sup>a</sup>, Raymond Panneton<sup>a</sup>, Hakim Bougrab<sup>a</sup>, Christophe Chaut<sup>b</sup> and Jean-Luc Wojtowicki<sup>b</sup>

<sup>a</sup>Groupe d'Acoustique de l'Université de Sherbrooke, 2500, Boul. de l'Université,  
Département de génie mécanique, Sherbrooke, QC, Canada J1K-2R1

<sup>b</sup>Henkel Technologies, Acoustics Center, 58203 Cosne sur Loire, France  
[raymond.panneton@usherbrooke.ca](mailto:raymond.panneton@usherbrooke.ca)

This work shows how to couple transfer matrix method to finite element method with a view to analyze the acoustic response of an automotive hollow body network with a minimum of memory requirements and computational time. A hollow body network is made up from a series of elongated fluid partitions similar to waveguides. These fluid partitions are generally separated by sealing parts. In the proposed hybrid model, the elongated fluid partitions are modeled using one-dimensional fluid finite elements, and the sealing parts using transfer matrices. After discretization of the acoustic pressure and application of the variational principle, the global finite element matrix system is obtained, where only the nodal pressures in the fluid partitions remain. During this process, each transfer matrix has been converted into a kind of admittance matrix, where no additional degrees of freedom are necessary to take into account the sealing parts into the finite element model. The so called TM-FEM method is applied on a simple hollow body network and compared to experimental measurements. Good correlations are obtained.

## Introduction

Nowadays, expanding sealing parts in automotive hollow body networks (HBN) are widely used. These parts are usually made up from expanding foams or an assembly of expanding foams and solid materials (see Fig. 1). The use of these sealing parts has demonstrated an influence on the noise inside the car [1]. These findings proved the necessity of optimizing both the position and choice of sealing parts to minimize/control sound propagation in the HBN. To do so, a numerical modelling of the HBN is required. Unfortunately, a complete three-dimensional numerical modelling would require significant computation time which goes against fast optimization in a short cycle time development. To get around this problem, this paper proposes a hybrid numerical method dedicated to the acoustical analysis of automotive hollow body networks. This hybrid method couples the transfer matrix (TM) with the finite element method (FEM).

## Modelling the fluid partitions

A simple HBN is shown in Fig. 2. It consists of two elongated fluid partitions, similar to waveguides, separated by a sealing part. Below the cut-off frequency ( $f_c$ ) of the waveguides, the elongated fluid partitions can be modelled with one-dimensional acoustical finite elements. In this case, only acoustic plane waves propagate in the HBN. At the nodes of the acoustical finite elements, only the acoustical pressure is defined [2].

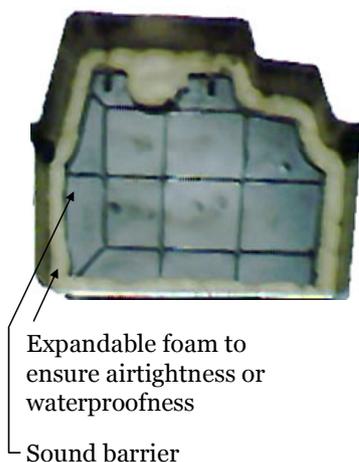


Fig. 1 Typical sealing part used in an automotive hollow body network.

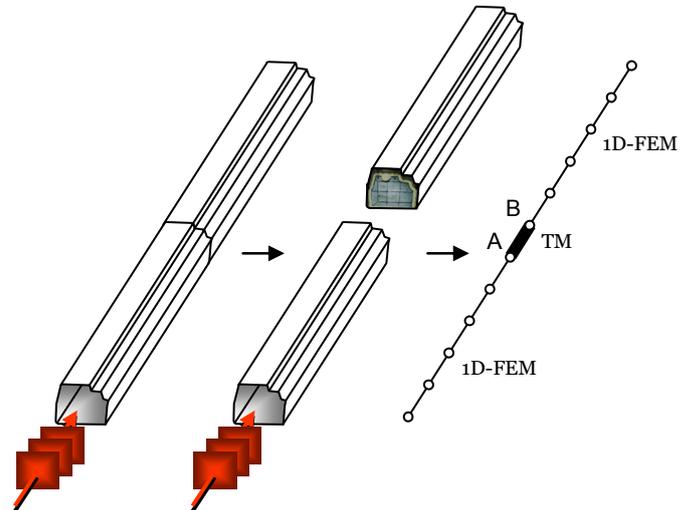


Fig. 2 Waveguide with a sealing part and its hybrid TM-FEM model.

## Modelling the sealing parts

Contrary to the fluid partitions, it is not possible to model the vibroacoustic behaviour of complex sealing parts using classical one-dimensional finite elements only. With a view to limit the numerical model to one-dimensional finite elements, a sealing part will be modelled using a transfer matrix.

Assuming normal incidence acoustic plane waves, one could characterize a sealing part by a transfer matrix of the form

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (1)$$

Coefficients  $T_{ij}$  are usually complex and frequency dependent. For simple acoustical parts (ex.: a single layer of open-cell foam or an expansion chamber), these coefficients can be analytically computed [3,4]. However, for complex parts, such as the sealing part shown in Fig. 1, these coefficients can be obtained experimentally [4] or from three-dimensional finite element simulations. It is worth mentioning that for complex parts, the plane wave assumption may not hold in the near field. Nevertheless, far from the parts, the plane wave assumption is still valid for frequencies lower than the cut off frequency of the waveguides. Consequently, during experimental or numerical evaluations of  $\mathbf{T}$ , one should not pick up the pressure in the near field of the parts. To prevent this, it is suggested that a plenum of air be added before and after a

complex part. The thickness of the plenum should be at least equal to the largest lateral dimension of the waveguide. In this case, the plenum of air and the original part form the characterized part on which  $\mathbf{T}$  applies.

The transfer matrix defined in Eq. (1) is used to link the acoustic pressure and velocity in front of the part (side A) to the pressure and velocity at its rear side (side B):

$$\begin{Bmatrix} p \\ u \end{Bmatrix}_A = \mathbf{T} \begin{Bmatrix} p \\ u \end{Bmatrix}_B \quad (2)$$

For the case shown in Fig. 2, the sealing part separates two fluid domains. Assuming both domains contain the same fluid (ex.: air), using Euler's equation, Eqs. (1) and (2) yield the following two equations:

$$p|_A = \left( T_{11} + j \frac{T_{12}}{\omega \rho_0} \frac{\partial}{\partial x} \right) p|_B \quad (3)$$

$$j \frac{1}{\omega \rho_0} \frac{\partial}{\partial x} p|_A = \left( T_{21} + j \frac{T_{22}}{\omega \rho_0} \frac{\partial}{\partial x} \right) p|_B$$

After some mathematics, the previous two equations can be rewritten as:

$$\frac{\partial p}{\partial x}|_A = \frac{j \omega \rho_0}{T_{12}} (p|_B - T_{22} p|_A) \quad (4)$$

$$\frac{\partial p}{\partial x}|_B = \frac{j \omega \rho_0}{T_{12}} (T_{11} p|_B - p|_A)$$

## Coupling fluid partitions and sealing parts

Using the classical Galerkin's procedure on the one-dimensional Helmholtz equation [2] for the fluid partitions together with Eq. (4), a symmetric weak integral formulation is obtained. After discretization of the weak formulation, the transfer matrix appears as a symmetric elementary matrix. This matrix is similar to an admittance matrix of the following form:

$$[A_{TM}] = \frac{j}{\omega T_{12}} \begin{bmatrix} T_{22} & -1 \\ -1 & T_{11} \end{bmatrix} \quad (5)$$

This corresponds to an elementary matrix that is inserted in the global matrix system. This method only uses the nodal pressures of the fluid partitions. Consequently, it does not add any additional degrees of freedom to the global system.

The schematic of this so-called TM-FEM model is shown in Fig. 2. One can note that only the nodal pressures in the fluid partitions are defined. The TM element is just seen as a relation between the nodal pressures just before and after the sealing part.

Finally, one can use the admittance matrix defined in Eq. (5) to determine the velocities on both faces of the sealing part characterized by  $\mathbf{T}$ . In this case, the following expression should be used:

$$\begin{Bmatrix} u_A \\ u_B \end{Bmatrix} = -j \omega [A_{TM}] \begin{Bmatrix} p_A \\ p_B \end{Bmatrix} \quad (6)$$

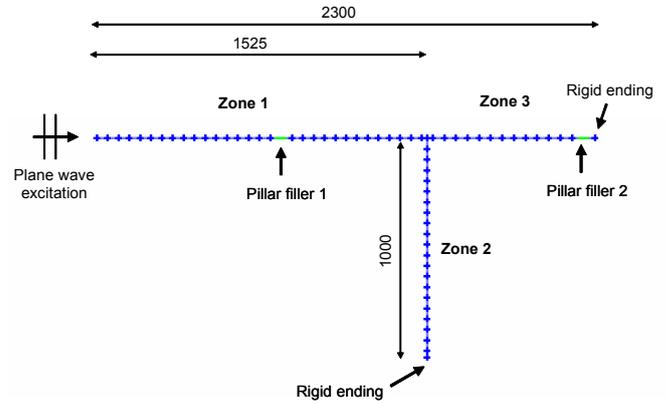


Fig. 3 Mesh of the T-shaped hollow body network

## Experimental validation

The TM-FEM is now tested on the simple T-shaped hollow body network shown in Fig. 3. Each hollow body segment is made up from a cylindrical tube of 49.15 mm in diameter ( $f_c \cong 4\,070$  Hz). Three analysis zones are defined on the network. The length of each zone is given in millimetres on Fig. 3. A reference microphone is located at the beginning of the first zone, where the acoustic excitation is applied. On the other ends, rigid walls are added. Two similar pillar fillers are placed in the HBN (one in zone 1 and one in zone 3). For the sake of simplicity in the evaluation of the transfer matrices, the pillar fillers are 50-mm thick open-cell melamine foams. On a numerical viewpoint, the HBN is meshed with linear one-dimensional finite elements. Each finite element node fits with a measurement point. Zone 1 contains 29 points, zone 2 contains 20 points, and zone 3 contains 14 points.

Figures 4 and 5 compare the quadratic pressure level (in dB-ref pressure at the reference microphone) for each analysis zone in the cases without and with the pillar fillers, respectively. For both cases, good correlations between measurements and simulations are obtained. However, in the case without pillar fillers, one can note that the pressure level is overestimated at the resonances. This difference might be due to the damping of air in narrow tubes (viscous and thermal losses). A damping loss factor of only 0.005 was used in the simulations. Moreover, and more importantly, damping due to the acoustic radiation of the walls exists. This phenomenon was not taken into account in the acoustic model, where the HBN was considered rigid. The overestimation of the pressure level is not visible when the pillar fillers are placed into the HBN, see Fig. 5. This is logical since the dissipation due to the pillar fillers dominates over the other types of dissipation in this particular HBN.

## Conclusion

This work has proposed a hybrid method for coupling transfer matrix and finite element method. The transfer matrix has been expressed in terms of a symmetric elementary matrix to be inserted in the global finite element matrix system. A correlation with experimentations has been successfully achieved for a simple T-shaped hollow body network. Future works should address a more

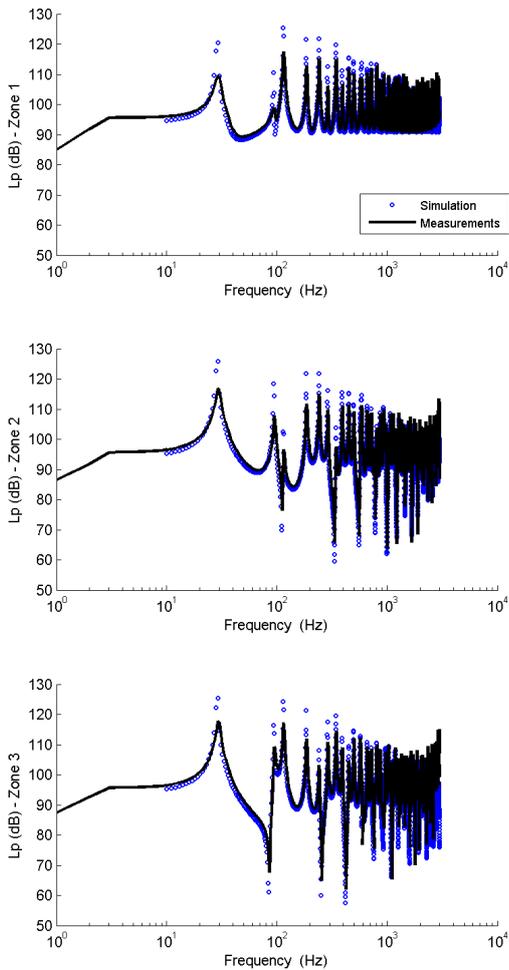


Fig. 4 Quadratic pressure level (dB – ref. pressure at input microphone) for each zone. Case without pillar fillers.

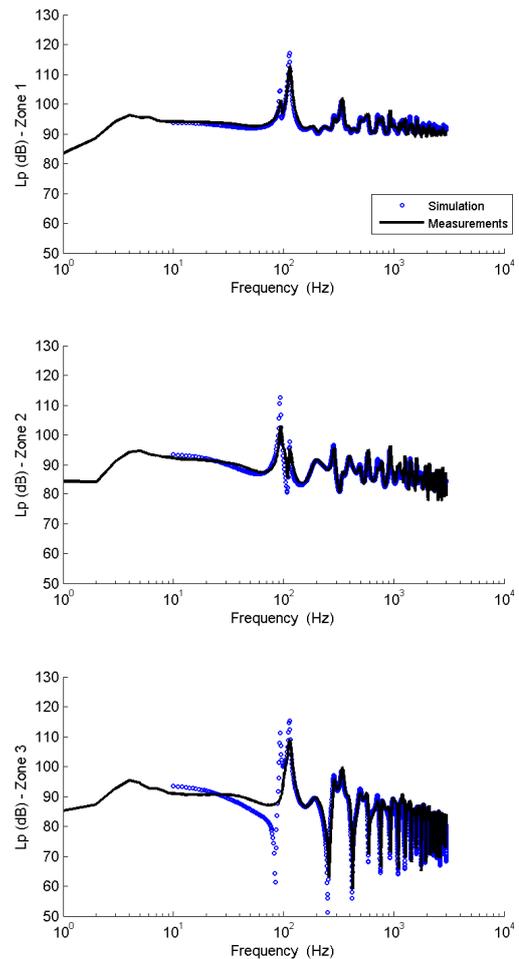


Fig. 5 Quadratic pressure level (dB – ref. pressure at input microphone) for each zone. Case with pillar fillers.

complex automotive hollow body network such as the one shown in Fig. 6. In this case, large air cavities would be modelled with three-dimensional fluid elements, waveguides with one-dimensional fluid elements, and the pillars filler and sealing parts with transfer matrix.

## References

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Fig. 6 Hybrid TM-FEM model of a real automotive hollow body network.