

Equivalence of the waveguide invariant and two path ray theory methods for range prediction based on Lloyd's mirror patterns

Daphne Kapolka

Naval Postgraduate School, 13693 Tierra Spur, Salinas, CA 93908, USA
dkapolka@nps.edu

Previous work discusses the use of normal mode theory as a means of determining the range of a contact from a Lloyd's mirror interference pattern. This method relies on the long-range interference pattern between different modes. For shallow, range-independent environments where the sound is dominated by low-order modes, the waveguide invariant constant which characterizes the modal interference pattern is approximately equal to one. The speed of a contact which maintains a constant course and speed as it passes through its closest point of approach (CPA) can be determined from the asymptotic behavior of tonal frequencies from the Doppler shift. This information can be used along with changes in broadband striation frequencies in a Lloyd's mirror pattern over time to extract the range of the contact as it transits through CPA. If instead of using normal mode theory, the Lloyd's mirror pattern is derived as the coherent interference between straight-line direct and surface-reflected paths, a relationship between the striation frequencies and time of a crossing contact can also be derived. This relationship can be shown to be identical to the result obtained from the normal mode approach when the value of the waveguide invariant is equal to one.

1 Introduction

Interest in autonomous sensor systems has increased in recent years. In underwater acoustics, this interest has been fueled by decreasing detection ranges in the ocean due to quieter contacts of interest and noisier ocean environments. At the same time, the decreasing cost of microcontrollers, computer memory, processors and other computer hardware required to manufacture autonomous systems makes them more feasible.

A parallel interest in underwater acoustic communication systems led to the question of how an acoustic modem could be used to fulfill a dual use as an autonomous passive acoustic sensor. This paper focuses on the information which can be extracted from a single, omni-directional hydrophone. At this point, no effort is made to automate the detection and information extraction. Instead, it is shown how the time/frequency (spectrogram) information from a contact showing both a Doppler shift and a Lloyd's mirror pattern can be used to derive the contact's range at its closest point of approach (CPA). The novel aspect of this work lies in the proof that, under certain conditions, the waveguide invariance approach to calculating the range at CPA yields the same result as straight-line ray theory.

2 Background

The term Lloyd's mirror originated in the early 1800's to describe the interference patterns observed between the direct and reflected paths of light rays and was extended to the interference patterns later seen in acoustic signals in the ocean. Shortly after the original Lloyd's mirror work, Doppler and Fizeau independently showed that the apparent frequency of a wave changes as a result of the relative motion between the source and receiver.

These two effects, i.e., the Lloyd's mirror or "bathtub" pattern and the Doppler shift, were used by Hudson [1] to show how the velocity, depth, and range at CPA could be computed for a submerged contact passing by an acoustic sensor at constant course and speed. In order to successfully apply this method, the contact needs to emit both broadband and narrowband sound. Hudson's analysis assumed simple straight-line propagation in which sound traveling along a direct path from the contact to the receiver interfered with sound reflecting off the ocean surface. The existence of this interference pattern requires a relatively

smooth surface and short ranges to preserve the coherence of the two paths.

Originally introduced by the Russian scientists Chuprov, Grachev, Brekhovskikh and Lysanov [2], the waveguide invariant approach provides an alternative model for the striations in Lloyd's mirror patterns observed when a contact passes through CPA. Following the earlier Russian work, D'Spain and Kuperman [3] describe how the striations can also arise from the interference between normal modes, thus explaining why such interference patterns occur well beyond the ranges at which the two-path Lloyd's mirror model is valid. The waveguide invariant theory provides a convenient framework for the analysis of these interference patterns. This theory shows how a single scalar parameter, β , the waveguide invariant, summarizes the dispersive characteristics of the acoustic field in a waveguide.

3 Theory

3.1 Straight-line ray theory

In order to get an interference pattern between paths, the sound reaching the receiver along these different paths must be coherent, i.e., there must be a constant phase difference between them. Such interference is characterized by patterns of constructive reinforcement and destructive cancellation of spectral energy. If the sound received from different paths is incoherent, the received intensity is the sum of the intensity coming from each separate path and the deep nulls which characterize destructive interference are absent. The depth of the nulls also depends on the amplitude of the sound coming along the different paths. To the extent that the sound from two coherent paths is of equal amplitude, the pressure at the nulls will be zero. In general, this requirement for coherence is met most frequently at short ranges where wave action and turbulence are minimal. It is also more likely to be met by surface reflections due to the fact that virtually all incident sound energy is reflected off the water-air interface thus making the amplitudes of the direct and reflected paths approximately equal. These interference patterns can be realized from paths reflecting off the ocean bottom as well; however, the bottom characteristics, frequency, and angle of incidence strongly influence the amount of energy reflected back into the water column.

The following section presents a mathematical explanation of the Lloyd's mirror pattern seen in spectrograms and provides a means of determining contact range and speed at CPA based on the Lloyd's mirror pattern and the Doppler shift of narrowband tonals. The analysis in this work assumes that the contact maintains a constant course and speed as it passes through CPA. Since Lloyd's mirror patterns are commonly observed at fairly short ranges due to the interference between a surface-reflected and direct path, refraction is ignored in the following discussion. In most cases this should be a reasonable approximation. Even with pronounced sound speed gradients the radius of curvature of the sound rays tends to be quite large. However, the method could certainly be extended to include refraction if required. It should also be pointed out that at long ranges, many more multi-path interactions may need to be considered.

Figure 1 provides a diagram of the basic geometry for analyzing surface interference on the basis of simple straight-line propagation for both a direct and surface reflected path. If the surface is smooth enough, sound energy incident on the surface will reflect back at an angle equal to the angle of incidence. The contact, B, is the source of acoustic energy. It is at a depth of d meters below the surface and radiating both broadband and narrowband energy. The receiver, D, is h meters below the surface. The sound emanating from the contact follows a direct path to the receiver, annotated by r_d , and a surface reflected path of length r_s . The horizontal distance between the contact and the receiver is denoted by r .

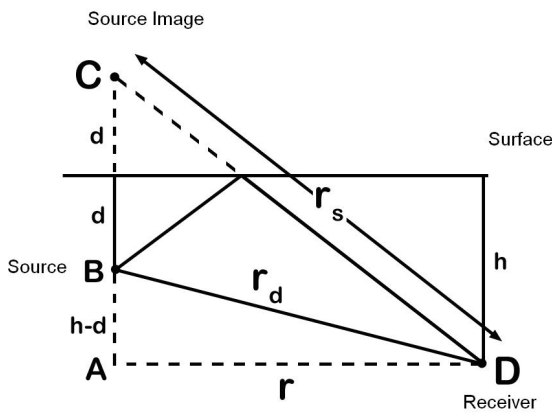


Figure 1. Geometry for analyzing surface interference. From [4].

It is clear from the picture that r_s and r_d are unequal, with $r_s > r_d$. This difference in path length will result in an interference pattern if the sound propagating along the two paths is coherent. Assuming that $r \gg h+d$, the path-length difference is approximately equal to

$$\Delta r = \frac{2hd}{r}. \quad (1)$$

Because of the 180° phase shift which occurs upon surface reflection, pressure minima, or nulls, occur where the path length difference between the direct and reflected paths is equal to an integral number of wavelengths. This gives the ranges to the nulls as

$$r_n = \frac{2hd}{n\lambda} = \frac{2hd}{nc} f. \quad (2)$$

Rearranging this expression yields the nulled frequencies as a function of range

$$f_n = \frac{nc}{2hd} r. \quad (3)$$

These equations give the ranges at which certain frequencies will be nulled, or, conversely, which frequencies can be expected to be nulled as a function of range. In either case, this means that as the range between source and receiver changes, the frequencies which have nulls at the receiver also change. This treatment is generally considered to be valid for fairly short ranges where the decorrelating effects of ocean turbulence, wave action, and multipath structure have not destroyed the observed coherence of the direct and surface-reflected paths.

A broadband spectrogram is shown in Figure 2. The light (unshaded) hyperbolic striations centered about $t=0$ are the nulls from the Lloyd's mirror interference pattern for the broadband noise. The frequency values f_1 , f_2 etc. represent the minimum frequency, f_n of each hyperbola. The actual frequency from the sound source that is being nulled by the surface reflection varies with time as the hyperbolae are traced out. These families of hyperbolae are often described as a "bathtub" pattern appearing in the spectrogram.

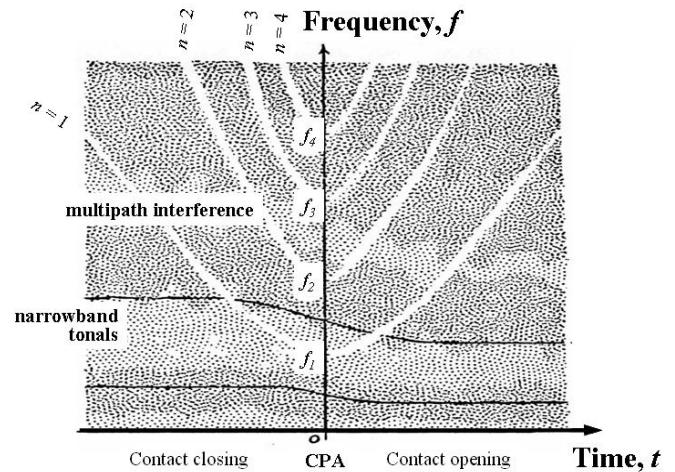


Figure 2. Frequency vs. time display of Lloyd's mirror interference pattern. After [1].

An analysis of the geometry shown in Figure 3 explains why the striations have the shape of hyperbolae in the spectrogram. In this figure, a contact is traveling along a constant course with speed v . If $t=0$ is taken as the time of CPA, the range to CPA is given by vt . The instantaneous range of the contact from the receiver is given by r , and range at CPA is R_o .

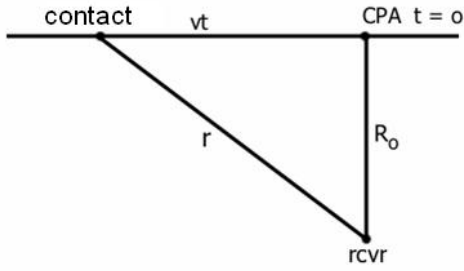


Figure 3. Geometry for analyzing a spectrogram pattern for a contact passing through CPA. From [4].

By the Pythagorean theorem, the distance to the contact at any time, t , is given by

$$r = \sqrt{R_o^2 + vt^2} . \quad (4)$$

Substituting the expression for the frequencies which are nulled as a function of the range, Eq. (3), into this expression and rearranging yields

$$R_o^2 = \left(\frac{2hd}{nc} \right)^2 f_n^2 - v^2 t^2 . \quad (5)$$

This shows that the nulled frequencies trace out the pattern of nested hyperbolae with respect to time.

The combination of the hyperbolic Lloyd's mirror pattern produced by the emission of broadband energy from a contact and the contact's discrete tonals can be exploited to produce a tracking solution. If a contact has an observable tonal its frequency undergoes an apparent frequency change, or Doppler shift, as the contact closes range, passes CPA and then opens range. The Doppler shift of narrowband tonals is also depicted in Figure 2. The center frequency, f_o , can be calculated as

$$f_o = \frac{f_u + f_l}{2} , \quad (6)$$

where f_u is the maximum closing frequency and f_l is the minimum opening frequency. The contact's tonal approaches this maximum and minimum frequency asymptotically as its range becomes large relative to the CPA range, because the component of velocity along the line of sound approaches the contact's velocity. The maximum Doppler shift is then given by

$$\Delta f = f_u - f_o = f_o \frac{v}{c} . \quad (7)$$

Rearranging for the velocity, v , gives

$$v = c \frac{\Delta f}{f_o} . \quad (8)$$

Therefore, if the contact has a discernable tonal as it passes through CPA, its velocity can be estimated from the Doppler shift. By examining the hyperbolic Lloyd's mirror pattern of the contact, an estimate of its range at CPA can also be developed. First, a determination of how the frequency of the sound is changing in time as the contact approaches CPA must be recovered. This is determined by measuring the slope of a regression line plotted along a striation. Taking the derivative of Eq. (5) with respect to time, the slope along a striation is given by

$$\frac{df_n}{dt} = \frac{ncv}{2hd} . \quad (9)$$

It is important to measure the slope of the striation line at long ranges where the slope is linear as it approaches its asymptotic value. Not only does this validate the assumption that $dr/dt = v$, but it also decreases the uncertainty by providing more points in the slope calculation.

Returning to Eq. (3), it is evident that the range r is given by

$$r = \frac{2hd}{nc} f_n . \quad (10)$$

Substituting Eq. (9) into Eq. (10) and assuming $dr/dt = v$ produces

$$r = \frac{v f_n}{df_n/dt} . \quad (11)$$

This expression is valid at ranges which are long compared to the CPA range since it depends on the assumption that all the contact's velocity is along the line of sound. Finally, by using the value of the contact's range, r , the velocity as determined by the Doppler shift, and the time over which the measurement was made, the contact's range at CPA can be determined from Eq. (5) as

$$R_o = \sqrt{\left(\frac{v f_n}{df_n/dt} \right)^2 - (vt)^2} . \quad (12)$$

If instead of differentiating Eq. (3) with respect to time it is differentiated with respect to distance, it yields

$$\frac{df_n}{dr} = \frac{nc}{2hd} . \quad (13)$$

Substituting $\frac{nc}{2hd} = \frac{f_n}{r}$ from Eq. (10) into this expression yields the more general result

$$\frac{df_n}{dr} = \frac{f_n}{r} . \quad (14)$$

This expression is valid at any range as long as the assumption $r \gg h+d$ holds. It is also interesting to note that Eq. (10) predicts that the ratio of nulled frequency to range should be a constant for each striation, i.e.,

$$\frac{nc}{2hd} = \frac{f_n}{r} = \text{constant} . \quad (15)$$

3.2 Waveguide invariant

The original equation for the waveguide invariant, β , as defined by Brekhovskikh and Lysanov [2], is

$$\beta = \frac{r}{\omega} \frac{d\omega}{dr} = - \frac{d(1/v)}{d(1/u)} , \quad (16)$$

where r is the range along the line of sight from contact to receiver and $d\omega/dr$ is the derivative or slope of the angular frequency with respect to range at which the

striation nulls occur. The quantities $d(1/v)$ and $d(1/u)$ are the derivatives of the phase slowness and group slowness respectively. The phase slowness and group slowness are simply the inverses of the phase and group velocities for that particular mode. As mentioned before, when the range between the source and receiver is much greater than the range at CPA, $r \gg R_o$, all of the motion is along the line of propagation. Therefore, $\delta\omega/\delta r$ may be expressed as $\delta\omega/v\delta t$. Putting Eq. (16) in terms of $\delta\omega/\delta t$ and velocity, v , yields

$$\frac{d\omega}{dt} = \beta \omega \frac{v}{r}. \quad (17)$$

This equation can be expressed in terms of frequency instead of angular frequency and rearranged to yield

$$r = \frac{\beta v f}{df/dt}. \quad (18)$$

Since the angular frequency in the above equations refers to the frequency at which striation nulls occur, this expression is identical to the two-path ray theory result presented in Eq. (11) as long as β is equal to one. Rearranging Brekhovskikh and Lysanov's expression in Eq. (16) and integrating in an environment where β is constant yields

$$\int \frac{d\omega}{\omega} = \beta \int \frac{dr}{r} \quad \text{or} \quad \ln \frac{\omega(t)}{\omega_o} = \beta \ln \frac{r(t)}{r_o}, \quad (19)$$

where ω_o and r_o are the nulled angular frequency and range at some arbitrary point in time along a striation. Kuperman and D'Spain exponentiate this term to express it in their 2001 paper [5] as

$$\omega(t) = \omega_o \left(\frac{r(t)}{r_o} \right)^\beta. \quad (20)$$

Again it should be noted that for $\beta=1$, this result is identical to the result of the two-path ray theory method since it predicts that the ratio of the nulled frequency to the range should be a constant.

4 Discussion

Problems arise for the waveguide invariant approach when β varies with range or azimuth. In these cases, β can be calculated if the bathymetry and other parameters of the ocean waveguide at the locations of the source and receiver are known. However, this can be problematic if the goal is to measure the source range. Fortunately, as long as sound energy does not refract significantly into the bottom and the bottom depth is constant, β is approximately equal to one [3]. Therefore, under most circumstances either theory can be used equally well to predict the range of a contact. These techniques were successfully used in [6] to determine the range of a contact at CPA to within 9% and its speed to within 4%. These experiments were carried out at the approach to San Diego Bay in shallow (10m) water and at fairly close range (185m) with a surface contact.

In cases where an array is available as opposed to a single hydrophone, it is possible to achieve range estimates without knowledge of the acoustic environment. Recent

work by Lee and Makris suggests a novel approach to the problem of source range estimation using the output of an array beamformer [7]. This "array invariant" method achieved range estimations within 25% out to several kilometers.

5 Conclusions

Waveguide invariant theory with $\beta=1$ reduces to the same equations as those predicted by an isovelocity two-path ray model for estimating the range of a contact traveling at constant course and speed through CPA from its observed Lloyd's mirror pattern. This fact supports the decision to assume a value of $\beta=1$ in situations where bathymetry and other ocean parameters are not known well enough to determine a more exact value.

Acknowledgments

This work was supported by ONR Ocean Sensing and Systems Applications Division (Code 321), Dr. Tom Swean (321OE) and Mr. Dana Hesse (321MS.)

References

- [1] Hudson, Richard F., *A Horizontal Range vs. Depth Solution of Sound Source Position under General Sound Velocity Conditions using the Lloyd's Mirror Interference Pattern*, Master's Thesis, Naval Postgraduate School (1983)
- [2] Brekhovskikh, L. M., and Lysanov, Y. P. *Fundamentals of Ocean Acoustics*, 3rd Edition. New York: Springer-Verlag, 140-146 (2003)
- [3] D'Spain, Gerald, and Kuperman, William A. "Application of waveguide invariants to analysis of spectrograms from shallow water environments that vary in range and azimuth." *J. Acoust. Soc. Am.*, 106, 4966-4999 (1999)
- [4] Kapolka, Daphne, *Underwater Acoustics for Naval Applications* (unpublished course notes), Naval Postgraduate School, Ch. 4, 2-7 (2007)
- [5] Kuperman, William A., D'Spain, Gerald, Song, Hee Chung and Thode, Aaron. "The Generalized Waveguide Invariant Concept with Application to Vertical Arrays in Shallow Water," *Ocean Acoustic Interference Phenomena and Signal Processing*, AIP Conference Proceedings 621, 33-37 (2001)
- [6] Wilson, Jason B., *Maritime Surveillance Using a Wideband Hydrophone*, Master's Thesis, Naval Postgraduate School (2007).
- [7] Lee, Sunwoong and Makris, Nicholas C., "The array invariant," *J. Acoust. Soc. Am.* 119, 336-351 (2006).