# Rheology and mobility in a sono-fluidized granular packing

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Dynamics near jamming (glassy phase, aging, memory effects and intermittency) shows amazing analogies among a variety of very different systems (colloids, dense suspensions, foams, granular materials). Recently, several proposals have emerged with the aim of describing in a general and unified way this behavior. With the purpose of testing experimentally some of these ideas, we performed several experimental studies on a dry granular materials under a weak level vibration generated by sound waves. We measure the rheology of an intruding object moving in the bulk as a function of the level of energy injected, the driving velocity and the shape of the moving object. We also present a simple model to account for the observed behaviour.

# 1 Introduction

Dynamics near jamming (glassy phase, aging, memory, intermittency) shows amazing analogies among variety of very different systems (colloids, dense suspensions, foams and granular materials). Recently, several proposals have emerged with the aim of describing in a general and unified way this phenomenology [1]. For a dissipative system like a granular packing, the mean kinetic energy due to the vibrations should play a role equivalent to the classical thermodynamic temperature for molecular systems. However, this simple vision has been challenged as novel relations between fluctuation and dissipation were postulated to account for the specific modes of relaxation in glassy out-of equilibrium phases [2]. Several attempts were made to identify such relations but it was found difficult experimentally to separate the measurements from the specific modes of energy input and thus, to obtain constitutive relations characterizing intrinsic material properties [3, 4]. So far, the fundamental constitutive behavior of a granular packing under vibration is a matter of debate and intense scientific activity. Recently, measurements were made on an horizontally vibrated granular layer and from the spreading dynamics, effective friction coefficients were obtained showing a hardening behavior at low vibration followed by a weakening regime at higher vibration [5]. Here, we propose to use sound waves to input in a specified and controlled way, weak vibrations in a granular packing. Sound wave propagation in granular matter was investigated for granular packing under large confinement pressure [6] and more recently, for a vanishing confining pressure close to a free surface[9, 10, 11]. Furthermore, it has been shown that the non-linear dynamics of the sound waves could be an essential piece to understand the onset and the long range memory effects for earth-quakes [7, 8]. Hence, we have built an experimental set-up that allows to investigates in details the rheology when a granular packing confined under gravity, is submitted to a weak level of vibration : this is the sono-fluidization effect[12]. To this purpose, we drag an intruder in the bulk of the packing and we record the force needed to move it at a constant speed. In the absence of vibration, similar experiments were already undertaken either in 3D [13, 14] or in a 2D granular packing model[15]. Note that in this report, the dragging velocities are quite low. We actually work in a regime where the so-called "inertia number" that characterizes the rheology of a granular packing under shear is orders of magnitudes smaller than the values were rheo-hardening effects due to shear, can be observed (see [16] and refs inside). Consequently, the rheology of the packing is essentially due to a coupling between packing elastic modes (driven externally or internally) and shear, created by the moving object.

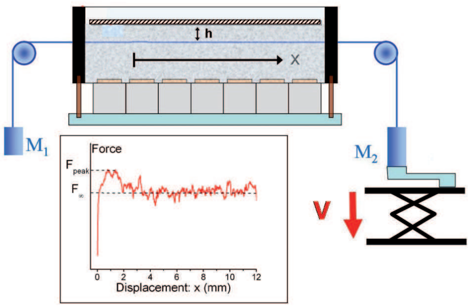


Figure 1: Sketch of the experimental setup. The driving is at a constant velocity $V$, The masses are such that $M_2 > M_1$. In inset, typical resistance forces $F(x)$ as a function of the horizontal displacement $x$

# 2 Experimental Setup

The rheomeric device that we use to explore the effective material strength is sketched on fig.1. It consists of a glass container closed at its bottom by seven piezoelectric transducers. The container horizontal section is a rectangle of length is $L = 19.00(\pm 0.05)cm$ and width $W = 2.30(\pm 0.05)$. The container is filled with a bi disperse mixture of glass beads of $1.0(\pm 0.1)mm$ and $1.5(\pm 0.2)mm$ mixed in equal masses (i.e. a mean bead diameter of $d = 1.2mm$ and the glass density is $\rho = 2.610^3 Kg/m^3$). This bidispersity is chosen to avoid crystallization. The beads are in direct contact with the transducers. Each piezoelectric transducer is excited by a square tension at a frequency$f = 400Hz$ that is set out of phase by $\pi$ compared to that of its neighbors. The external vibration driving is set externally via the electrical tension applied to the piezo transducers. We benchmarked the bulk agitation by monitoring the rms acceleration of a buried accelerometer that indicates a value $\gamma = 0.35g$ ($g$ is the gravity acceleration) for the highest driving voltage. From the acceration signal, integrated over time, we obtain the kinetic energy density: $\epsilon_k = \frac{\rho_{acc}}{2T} \int_0^T V^2(t)dt$. The accelerometer density $\rho_{acc}$ is roughly equal to the packing bulk density $\rho_b$. Note, importantly, that this energy is quite small when compared to the potential energy at the scale of one grains (at maximal vibration : $\epsilon_k \simeq 10^{-4} \rho_b g d$ ). On the packing top, we have placed a thin metallic lid to probe via an induction detector, the surface position. The pressure applied by the lid corresponds to 5 granular layers:

$P_0 = 5\rho g d = 150\ Pa$. This weak energy input does not induce any detectable motion on our current time scale. Note importantly that we have followed during 10 days, at maximal input energy, an horizontal line of colored beads and we did not evidence any convection in the bulk. The wire is set in tension by two masses $M_1 < M_2$. The masses values are chosen such that the mass difference will always exceed the dragging force. Thus the masses $M_2$ will always be in contact with the force probe placed just below. The force probe which is a spring of stiffness $k = 10^5 N/m$, is fixed to a vertical translation stage driven at a constant downwards speed: $V$, by a brush less motor. We use two different types of intruders which are dragged horizontally and at a constant depth $h$ below the surface. First, we use a metallic thread of diameter $\delta = 0.1\ mm$, spanning the whole horizontal size of the cell and second, the same thread to which a plastic bead of diameter $D > d$ is attached. The intruders are driven at constant velocities between $V = 10^{-6}\ m/s$ and $V = 5\ 10^{-3}\ m/s$. The experiments are conducted using the following protocol. First, the packing and the intruding piece are vibrated at the maximal vibration energy during 4 hours (we checked that using 10 hours would not yield any detectable difference). At the end of this preparation phase, a packing fraction of $\phi = 0.6$ is reached and then, the dragging experiment may start at a constant vibration energy characterized by a kinetic energy fluctuation set externally by the chosen voltage on the piezzo transducers. A typical force signal is presented in inset of the set-up figure. Note that at the beginning of the pulling, we observe an overshoot of the resisting force but after few millimeters of displacement, a steady-state is reached.

## 3 Drag force on the thread

The thread is placed at a height $h = 1.3 cm$ below the surface. The average force necessary to drag the thread $< F_t >$ is rescaled by the thread outer surface ($S_t = 6\ 10^{-5} m^2$) in order to obtain an average friction stress on the intruder: $\sigma_t =< F_t > /S_t$. Then, the mean stress is rescaled by the confining pressure $P(h) = P_0 + \phi \rho_s g h$ to form an effective friction coefficient: $\mu_{eff} = \sigma_t / P(h)$. This effective friction is displayed on fig.2(a) as a function of the dragging velocity. We use two representations, a normal and in inset, a normal-log scale. The error bar corresponds to an average over 5 independent experiments. The data show several salient and robust features. First, at a constant driving velocity and for a small level of vibration, the pulling resistance decreases strongly with the vibration energy. Second, with or without vibration, the stress-strain relations are of the hardening type: we indeed observe an increasing resistance to pulling with the dragging velocity. The third striking feature is that for the highest vibration energy and the lower driving velocities, we reach a linear regime of the type: $\mu_{eff} \approx V/V*$ with $V* \simeq 910^{-4} m/s$. Finally, above this regime, the increase of friction with the driving velocity is weak, typically of the type $\mu = \mu_0 + \beta \ln(V/V_0)$, where $V_0$ is a reference velocity. This is also true in the absence of vibration. In [17], we put forward a simple heuristic model based on the coupling between internal elasticity driven harmonically and a solid friction force. We show that under vibrations with a r.m.s. kinetic velocity $\overline{V}$, a linear force-velocity regime is obtained at low driving and high vibration. The rheology is characterized by an effective friction coefficient: $\mu_{eff} = \mu \frac{\sqrt{2}}{\pi} \frac{V}{\overline{V}}$. In our situation since we have $\overline{V} \simeq 1.2^{-3} m/s$, the magnitude of the linear relation is consistent with the theoretical picture.

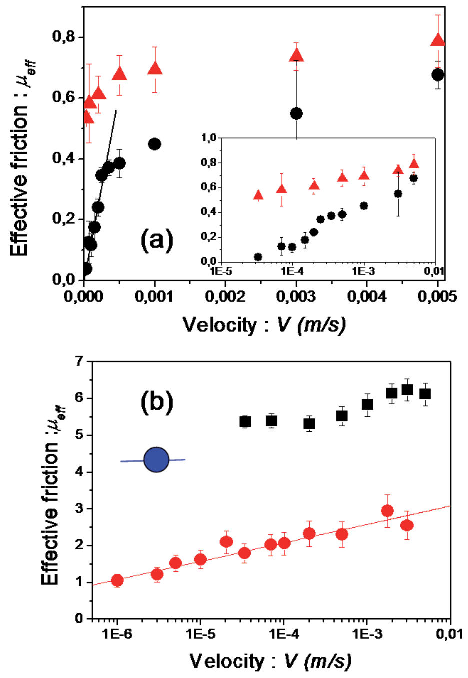


Figure 2: Effective friction of a moving intruder $\mu_{eff}$ as a function of the pulling velocity $V$; (a) effective friction on the thread only ($\triangle$) no vibration, ($\circ$) maximal vibration In inset is the same graph but is a lin-log scale; (b) Effective friction for an intruder of diameter diameter $D = 2\ mm$ as a function of the driving velocity for no vibration ($\triangle$) and at the largest vibration energy ($\circ$)

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## 4 Drag force on an intruding bead

Pulling a bead at a constant velocity in the bulk of a granular packing presents many practical difficulties. Here, we try to override this problem by considering the resistance force on an intruder consisting of a bead attached to the metallic thread. The thread (with the bead attached) is put originally at an horizontal position $h = 1.3\ cm$ below the surface and the resisting force on the system is measured. To extract the contribution of the bead $F_b$, we also measure the drag force on the thread in an independent experiment at the same velocity and remove the corresponding mean resisting force from the drag force on the thread+bead intruder. Then, to obtain an effective restitution coefficient that characterizes the bead-only contribution, we divide $F_b$ by the bead surface and by the confining pressure $P$ : $\mu_{eff} = F_b / \pi D^2 / (\phi \rho g h + P_0)$. This procedure may be questionable as, in principle, the presence of a bead may modify the contribution of the thread. We nevertheless expect that this contribution will be marginal as the thread extension is much larger than the perturbation due to the intruding bead. The corresponding effective frictions are displayed on fig. 2(b) for a intruding bead of diameter $d = 2\ mm$. The error bars correspond to a typical dispersion of the mean on 3 independent experiments.

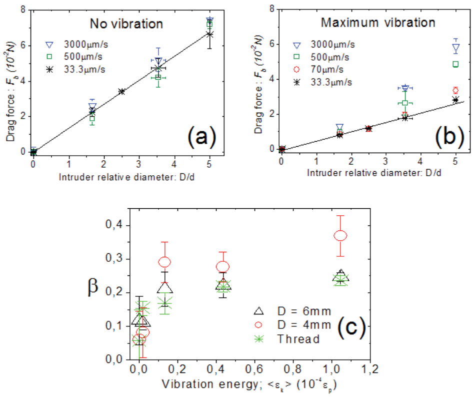


Figure 3: Dependence of the drag force with the intruding grain size ratio $D/d$; (a) Drag force $F_b$ as a function of the size ratio in the absence of vibration; (b) Drag force $F_b$ as a function of the size ratio for the maximal vibration; (c) logarithmic slope $\beta = \partial\mu_{eff}/\partial \ln V$ as a function of the vibration kinetic energy $\epsilon_k$ for various size ratios $D/d$ and for the thread only.

Note that for a sphere dragged in a medium for which the confining pressure is hydrostatic, if the friction force exerted on the surface is characterized by a dynamical friction $\mu$, the effective friction is $\mu_{eff} = \frac{\pi}{2}\mu$. We noticed, that before reaching a steady state, typically by pulling the intruder over a distance corresponding to a mean grain size $d$ (this, almost independently of the intruder size), we go through a phase where an overshoot of the dragging force is systematically evidenced. Also, similarly to the thread only case, the rheology is of the hardening type (may be marginally without vibration) and the higher is the vibration, the lower is the resistance to pulling. In the no vibration case, we apparently reach, at low driving velocities, a dynamical threshold value different from its limiting value at higher velocities. Interestingly and contrarily to the previous case, we never evidenced a linear force-velocity regime and this, for any of the intruders we tested. We found essentially a slow logarithmic increase of friction for the highest vibration with $D = 2mm$ on 4 decades in velocities (see fig.2(b)). On fig. 3, we display for different dragging velocities, the resisting forces without vibration (see fig.3(a)) and for the maximal vibration (see fig.3(b)) at different dragging velocities. Strikingly, we observe that we have a linear variation of the dragging force with the intruding grains diameter $D$. It recalls amazingly a typical Stoke's drag behavior, though we are NOT here in a viscous linear regime. We have deviations from this behavior for the higher drag velocities. This linear $F_b \propto D$ relation is even more surprising is we choose to represent it as an effective friction coefficient. Then, the drag force would be divided by the bead intruder surface ($\propto D^2$)and then, we would observe what we may call a striking "geometrical hardening" effect for small intruder size. The effective friction is the largest (everything else beeing constant) for smaller grain size ratio $D/d$. We also noticed that the actual values for the effective friction coefficients are quite large if one compares them to the usual values found for cohesionless granular packing either by triaxial tests or by angles of repose measurements. Here, in the absence of vibration $\mu_{eff}(d = 2\ mm) = 6$ and $\mu_{eff}(d = 6\ mm) = 2$. This means that this effective friction must take into account a flow process in the bulk. We indeed found evidences of a counter flow on the outer-boundaries as we observed displacement of the grains in relation with the intruder motion. It is clear, that in this case, the lateral confinement could also play a role in the determination of the effective rheology.

Finally, we estimate the dependence of the logarithmic hardening effect with the vibration energy. Hence, we tried to measure the logarithmic slope of the effective rheology : $\beta = \partial\mu_{eff}/\partial \ln V$. Note that the whole protocol to determine a full rheological curve as in fig. 2(b), is quite cumbersome and lengthy. It took typically three months worth work. This explains why these results may look fragmented as we have not performed yet extensive and systematic measurements for a large range of driving velocities and vibration intensities. We rather choose to estimate the value of $\beta$ and the impact of vibration on its value. We actually performed two series of experiments at well separated driving velocities $V = 3.3\ 10^{-5} m/s$ and $V = 5\ 10^{-4} m/s$. From those two points which average is determined over 5 independent realizations, we estimate the logarithmic slope $\beta$. The results are displayed on fig.3(c). We see that, indeed, the increase of the vibration energy by a factor 5 does not influence very much the velocity hardenning effect characterized by $\beta$.

## 4.1 Summary and conclusions

We presented series of experimental results demonstrating that even at a weak level of vibration, significant mechanical changes can be evidenced for the yield stress and the rheology of a granular packing. By injecting sound waves in the bulk, we obtain drag reductions as large as a factor 5 or more and this, for two types of solid intruders pulled at constant velocity. The rheology curves hence obtained are of the hardenning type, i.e. the resisting force increases with the pulling velocity. Interestingly, when the wire alone is driven very slowly, a linear force-velocity regime is found. However this is not a viscosity regime! The values measured for the effective friction coefficients are consistent with a simple heuristic model coupling an internal elastic mode of vibration with a solid friction on the thread surface [17]. We also obtain in the absence of external vibration a hardening regime. Along those lines, this could be due to an effect of vibrations generated fy the friction mechanism internally. We also measured the drag force on a spherical intruder. We observe a logarithmic force-velocity relation (over 4 decades in velocity). The surprise comes from the linear increase of the resisting force with the intruder size. In an effective friction picture this would mean that the smaller is the intruding grain, the larger is the effective friction that resists motion. This "geometrical hardening" effect must be rooted in properties of the bulk flow generated by the passage of the intruder. This is observed both in the absence of external vibrations and under a maximal level of vibrations. Its understanding remains an open problem. Finally, we estimate that the logarithmic velocity hardening, is only marginally influenced by the level of kinetic energy.

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