Adaptive beamforming applied to underwater acoustic measurements

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In many applications, such as measuring the underwater noise level of moving ships, underwater acoustic measurements face serious difficulties related to several factors, including low signal-to-noise ratio, the influence of surface and bottom on a signal, as well as complicated spatial structure of the sea interference. The increase of the measurement interference resistance, compared to the case of a single receiver (hydrophone), can be provided by the use of spatially distributed receiving systems (antenna arrays), which are able to suppress the interference due to their spatial selectivity.
The present work is aimed at developing adaptive methods for underwater acoustic measurements with the use of vertical antenna arrays. The method must provide both the maximum reduction of external interference and the given measurement accuracy, i.e., the result must coincide with the output of a single receiver in the absence of interference. From the point of view of synthesis of array systems, the originality of the presented approach is mainly a combination of measurement functionality of the antenna array and the maximum interference suppression.
The results of numerical simulation and experimental testing under sea conditions show that the proposed adaptive methods provide high precision of measurements under strong and/or complex interference conditions.

1 Introduction

In recent years, there has been a growing tendency for strengthening the norms imposed on noise levels of various marine ships. This is related to several factors, such as ecological requirements [1], as well as defense against marine weapons such as acoustic mines and torpedoes [2–5]. Currently, taking into account a growing amount of local conflicts, acoustic mines are dangerous not only for warships, but also for civil ships [2]. The importance of such measurements is illustrated by the fact that the Acoustical Society of America has recently formed a special working group for development of an entirely new standard for underwater noise measurement of ships [6].

It should be noted that there is currently no universal standard or rule for measurement of underwater noise of moving ships, which would regulate the measurement distances, the procedure of the measurements, the reference distance etc. Most existing methods for measuring underwater noise levels of moving ships [2–4] require a uniform movement of a ship along a linear track at some distance from a single hydrophone located in the far field of a ship. As a rule, the physical parameter to be determined when measuring underwater noise level of a moving ship is maximum sound pressure level (SPL) in third-octave bands usually measured in dB re 1 μPa [3, 4], usually time-averaged and referred to the distance of 1 m [3, 7]. In some situations, when the signal level cannot be measured at a large distance, the measurements are performed in the near field, and the results may be extended to the far field using special techniques (see, for example, [8]).

The abilities of a single hydrophone to measure underwater noise level, which are bounded only by the background noise level, are practically exhausted. The increase of the measurement interference resistance is possible by means of spatially distributed receiving systems, such as antenna arrays (AAs), which can suppress the interference due to their spatial selectivity. The methods of interference suppression in antenna arrays are well known, first of all, this is the so-called adaptive technique of weight coefficient synthesis [9]. A more complicated problem is a combination of the requirements for maximum interference reduction and for the precision of the measurement results.

The present work is aimed at investigating a possibility of measurement of moving ships' underwater noise level with the use of vertical AAs and adaptive algorithms of spatial processing providing both maximum reduction of external interference and given measurement error. From the point of view of synthesis of array systems, the originality of our approach is mainly a combination of measurement functionality of an AA and the maximum interference suppression; as far as we know, this task has not been considered yet.

2 Problem formulation

Suppose that a receiving antenna array contains N hydrophones located at the positions z_n of the z axis which is in the same plane as the source track (see Fig.1).

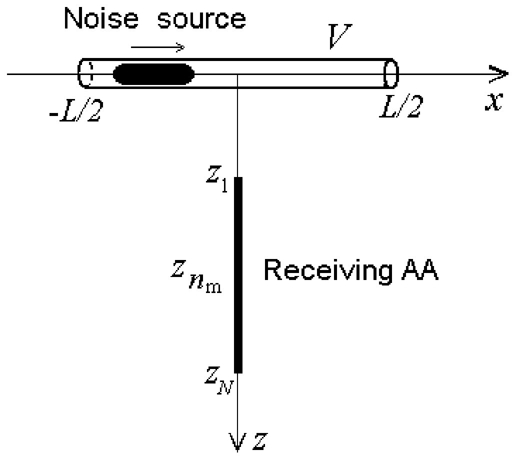

Fig.1 Scheme of measurement of underwater noise level with the use of a vertical AA.

Let us choose one of the hydrophones with the number n_m (hereafter referred to as the reference hydrophone) as the hydrophone performing conventional underwater noise level measurement. The sound pressure snapshots $p_n(t_j)$ at the hydrophone output will be considered in narrow bands neglecting the decorrelation of the signals in the AA. In this approximation, the signal processing in the AA is a summation of $p_n(t_j)$ with weight coefficients w_n^*:

$$p_A(t_j) = \sum_{n=1}^{N} w_n^* p_n(t_j) , \quad (1)$$

where $p_{\mathrm{A}}(t_j)$ is the AA output signal. Note that there always exists a "trivial" weight distribution w_n^* with the only nonzero element $w_{n_{\mathrm{m}}}=1$, and then $p_{\mathrm{A}}(t_j)=p_{n_{\mathrm{m}}}(t_j)$.

It can be supposed that the maximum of the sound level time dependence for a moving spatially distributed source is achieved in the neighborhood of the closest point of approach, i.e., when the source (in projection to the x axis) is completely located within some interval [–*L*/2, *L*/2]. Thus, some volume *V* bounded by a smooth surface can be introduced; the length of its projection onto the x axis has a value of *L* greater than the geometric size of the source, whereas the lengths of the projections onto other axes are conditioned by the "width" and the "height" of the source. The movement of the source within this volume is essential when measuring the underwater noise level. Suppose that the field of the source outside the volume *V* can be described as the result of radiation of exterior monopole sources distributed in the volume *V* with the density $\rho_j(\vec{r}_V)$, where $\vec{r}_V$ denotes the coordinates of a point inside *V*, and *j* is the index defining both the current location of the source within the volume *V* and the random instant magnitudes of monopole sources. Then the signal at the output of the AA can be written as

$$p_{\mathrm{A}}(t_j)=\sum_{n=1}^{N} w_n^* \int_V \rho_j(\vec{r}_V)G(z_n,\vec{r}_V)\,dV\ , \qquad (2)$$

where the Green's function for a monopole source is assumed to be known. Then the difference between the signal snapshots at the reference hydrophone output and at the AA output in the absence of interference can be expressed as

$$p_{\mathrm{A}}(t_j)-p_{n_{\mathrm{m}}}(t_j)=\int_V \rho_j(\vec{r}_V)\delta(\vec{r}_V)g(\vec{r}_V)\,dV\ , \qquad (3)$$

where the relative error

$$\delta(\vec{r}_V)=\frac{1}{g(\vec{r}_V)}\left[\sum\nolimits_{n=1}^{N} w_n^* G(z_n,\vec{r}_V)-g(\vec{r}_V)\right], \qquad (4)$$

$g(\vec{r}_V)=G(z_{n_{\mathrm{m}}},\vec{r}_V)$ and $g(\vec{r}_V)\neq 0$ for any $\vec{r}_V$. It is easy to show that when $|\delta(\vec{r}_V)|$ and its first derivatives are small enough, the relative error of underwater noise level measurement with the AA is bounded from above by $\delta_{\max}=\max|\delta(\vec{r}_V)|$. Together with the maximum value, the error may be also characterized by the root mean square error $\delta=\left(V^{-1}\int|\delta(\vec{r}_V)|^2 dV\right)^{1/2}$.

Assume, for simplicity of computation, that the source is one-dimensional; in this case, integration over volume is reduced to integration over *x* in the range from –*L*/2 to *L*/2.

The problem of synthesis of weight coefficients will be considered in vector-matrix notation. The snapshots $p_n(t)$ will be considered as elements of the $N\times 1$ column vector **p**. Analogously, introduce the $N\times 1$ weighting vector (WV) **w**; then the AA output signal (1) $p_A=\mathbf{w}^{\dagger}\mathbf{p}$, where † denotes the Hermitian transpose. Introduce the grid with the coordinates x_m, $m=1,\ldots,M$ on the interval $[-L/2,L/2]$. Then the Green's function transforms to the $N\times M$ matrix $\mathbf{G}=\|G(z_n,x_m)\|$, $\delta_{\max}=\max_m|\sum_n(G_{nm}^* c_n-g_m)/g_m|$, and the root mean square error is

$$\delta=[M^{-1}(\mathbf{G}^{\dagger}\mathbf{w}-\mathbf{g})^{\dagger}\mathbf{H}(\mathbf{G}^{\dagger}\mathbf{w}-\mathbf{g})]^{1/2}, \qquad (5)$$

where $\mathbf{H}=\mathrm{diag}\{|g_m|^{-2}\}$, g_m are the elements of the vector $\mathbf{g}=\mathbf{G}^{\dagger}\mathbf{w}_0$, $\mathbf{w}_0$ is a trivial WV, whose elements are equal to zero except the element with the number n_{m} which is equal to one.

Now take into account that the pressure at the hydrophones is an additive mixture of the sound pressure produced by a moving source and a stationary interference characterized by the covariance matrix **C**. The average interference power at the AA output, taking into account Eq.(1), can be expressed as a quadratic form $\mathbf{w}^{\dagger}\mathbf{C}\mathbf{w}$, and the interference power at the output of the reference hydrophone is $\mathbf{w}_0^{\dagger}\mathbf{C}\mathbf{w}_0$. Then the signal-to-noise (SNR) gain of the AA (in dB) with respect to a single hydrophone is natural to define as

$$Q=10\lg(\mathbf{w}_0^{\dagger}\mathbf{C}\mathbf{w}_0/\mathbf{w}^{\dagger}\mathbf{C}\mathbf{w})\,. \qquad (6)$$

As a result, the problem is reformulated as follows: it is required to determine the WV **w** assuring the maximum noise suppression for a given (allowable) error $\delta_{\max}$ or δ.

3 Non-adaptive and adaptive beamforming

For simplicity, we will limit our analysis to considering only the case of mean square error. In this case, the weight coefficients **w** can be found from the condition of the minimum of the functional

$$F(\mathbf{w})=(\mathbf{G}^{\dagger}\mathbf{w}-\mathbf{g})^{\dagger}\mathbf{H}(\mathbf{G}^{\dagger}\mathbf{w}-\mathbf{g})+\alpha\mathbf{w}^{\dagger}\mathbf{C}\mathbf{w}\,, \qquad (7)$$

which is achieved when

$$\mathbf{w}(\alpha)=(\mathbf{G}\mathbf{H}\mathbf{G}^{\dagger}+\alpha\mathbf{C})^{-1}\mathbf{G}\mathbf{H}\mathbf{g}\,, \qquad (8)$$

where $\alpha\geq 0$ is a numerical parameter obtained, given the error δ_0, after substituting Eq.(8) into Eq.(5) and solving the equation $\delta(\alpha)=\delta_0$.

Together with providing the given error, an important property of the solution (8) is its robustness to small changes of source parameters. We will consider the solution to be robust if an error calculated for slightly changed elements of the matrix **G**, WV elements, etc., does not markedly differ from the given value. Otherwise, the solution is non-robust, which is unallowable for practical applications. Note that the solution is, as a rule, non-robust if the matrix inverted in Eq.(8) is ill-conditioned; a large norm may also indicate that the solution is non-robust.

In the absence of information about the properties of interference, it may be supposed that $\mathbf{C}=\mathbf{I}$, where **I** is the identity matrix. The processing in this case will be referred to as a non-adaptive beamforming. In this case, the solution (8) is always robust due to the summand $\alpha\mathbf{I}$ in the inverted matrix; however, for the maximum interference resistance, the synthesis of WV should be performed for the observed noise covariance matrix, i.e., the beamforming should be adaptive. However, in this case the solution (8) will be, as a rule, non-robust because of poor conditioning of the matrices $\mathbf{GHG}^\dagger$ and $\mathbf{C}$; to avoid this, additional restrictions are needed to be included in the algorithm of synthesis.

A standard way to increase the robustness of WV in adaptive AAs is addition of the appropriately weighted identity matrix to $\mathbf{C}$ [10] or, equivalently, addition of the summand $\beta\mathbf{w}^\dagger\mathbf{w}$ to the right side of Eq.(7). In this case, the solution has the form

$$\mathbf{w} = (\mathbf{GHG}^\dagger + \alpha\mathbf{C} + \beta\mathbf{I})^{-1}\mathbf{GHg}\,, \qquad (9)$$

where $\alpha, \beta \geq 0$ are two numeric parameters requiring, accordingly, two conditions for their determining. The first one remains the same: $\delta(\alpha,\beta) = \delta_0$. The second condition must provide a reasonable compromise between interference resistance and robustness of the solution. It is obvious that, with the increase of β, the interference suppression decreases, so its robustness increases. Thus, there is an optimal value of β, when the solution is satisfactorily robust, but the interference resistance does not decrease too much. Several criteria may be proposed for selection of optimal value of β. A physically demonstrative one is to require a minimum of the product of the WV norm and the interference level at the output of the AA: $\beta = \arg\min[(\mathbf{w}^\dagger\mathbf{w})(\mathbf{w}^\dagger\mathbf{Cw})]$ provided that the parameter $\alpha(\beta)$ has been found from the equation $\delta(\alpha,\beta) = \delta_0$. An almost equivalent approach is to determine β from the condition

$$\beta = \arg\max_\beta[\alpha(\beta)\cdot\beta]\,. \qquad (10)$$

As an illustration, the results of statistical modeling of the measurement error (caused by random variations of elements of WV (9)) of the SPL of a monopole source are given in Fig.2 against the quantity $\beta/\beta_{\max}$, where the maximum value of the parameter $\beta_{\max}$ is achieved when $\alpha = 0$; the behavior of $\alpha\cdot\beta$ is also shown in Fig.2. Such a behavior of dependence of SNR gain and SPL error on the parameter $\beta/\beta_{\max}$ is typical for the case of an ill-conditioned matrix $\mathbf{GHG}^\dagger + \alpha\mathbf{C}$.

The properties of the proposed beamformer are convenient to investigate in terms of the array pattern (AP) in the vertical plane. Although the measurements are performed in the near zone of the AA, the AP allows, in particular, to estimate the directions where a suppression of the received signal takes place. A typical example of an AP corresponding to the considered beamformer is given in Fig.3.

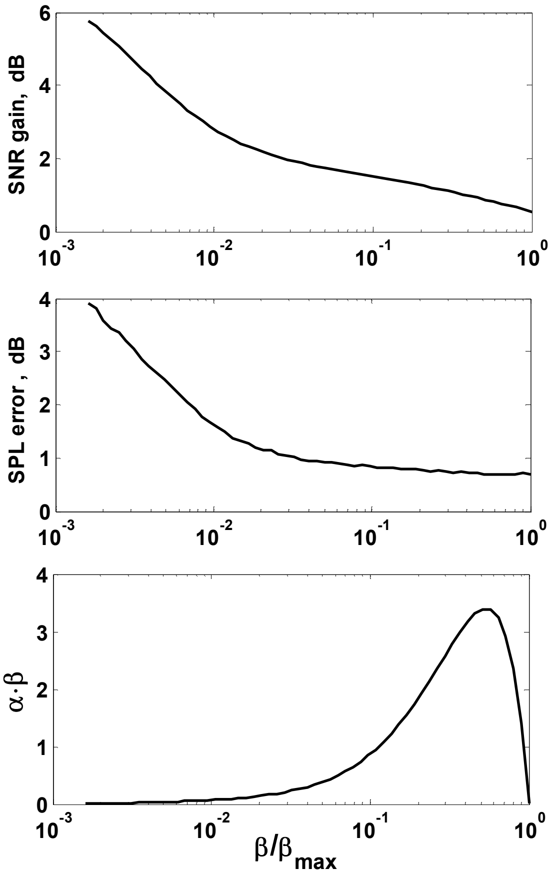

Fig.2 SNR gain, error of SPL measurement and the quantity $\alpha\cdot\beta$.

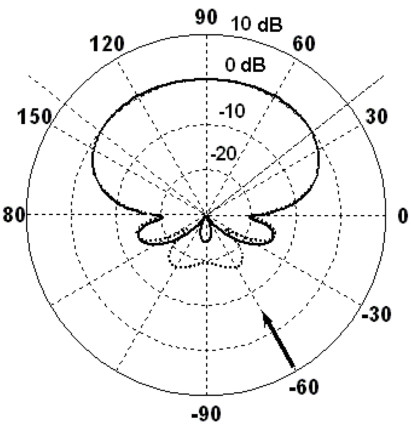

Fig.3 Examples of the array pattern in the cases of adaptive (solid line) and non-adaptive (dotted line) beamforming.

In the sector of angles corresponding to the interval $[-L/2, L/2]$, the AP is almost uniform; outside this sector, the AP tends to suppress the signal. In the adaptive case, when the interference was modeled as a plane wave propagating from the direction $\theta_0 = -60^\circ$ (shown by the arrow in Fig.3), the minimum of the AP is formed in the direction of the interference.

4 Experimental results

Below, we will demonstrate some results of experimental testing of the method. The source of noise was a motor boat moving with a velocity of ~ 4 knots above a vertical 11-element receiving AA with a length of 30 m. The WV synthesis with the mean square error δ_0 of 0.3 dB was performed in narrow bands (~ 0.32 Hz) for both the cases of non-adaptive and adaptive beamforming .

In Figs.4–6, examples of normalized dependences of signal levels (SL) on time at the output of a single (central) hydrophone (dotted line) and at the output of the AA (dashed line for the case of non-adaptive and solid line for the case of adaptive beamforming) are given for various third-octave bands.

As follows from Fig.4, using the AA substantially increases the interference resistance in the investigated bands in both the cases: non-adaptive and adaptive beamforming with the use of criterion (10). The SPL measurement error (more precisely, the difference between the maximum signal levels obtained with the use of various processing algorithms) does not exceed 0.3 dB.

It should be noted that the optimal ratios $\beta / \beta_{\max}$ obtained when using Eq.(10) lie in the range $0.2 \div 0.95$ in the whole frequency range investigated (from 30 Hz to 280 Hz).

An attempt to further increase the interference resistance by means of decreasing the ratio $\beta / \beta_{\max}$ (the parameter $\alpha(\beta)$ is obtained from the equation $\delta(\alpha, \beta) = \delta_0$) leads to a substantial increase of the error of SPL measurement. Fig.5 demonstrates normalized dependences of SL on time at the output of the central hydrophone and the AA for different values of $\beta / \beta_{\max}$ (in the adaptive case). It is clearly seen that, in this case, the increase of the interference resistance leads to an unacceptable increase of the SPL measurement error.

The results demonstrated in Figs.4, 5 correspond to the situation when the interference level was almost uniform along various hydrophones. Fig.6 demonstrates another situation, which took place in the third-octave band with the central frequency of 50 Hz, where the interference was non-uniform along the aperture: the maximum signal-to-noise ratio was relatively high (8 dB) only for two hydrophones, whereas for other hydrophones, on the average, it was less than 0 dB. In this case, the interference resistance of the non-adaptive algorithm is obviously insufficient (see Fig.6). At the same time, the adaptive beamforming with criterion (10) gives quite good results.

Finally, the SNR gain obtained when measuring the SPL of a moving ship by means of the AA is demonstrated in Fig.7 for the cases of non-adaptive and adaptive (with criterion (10)) beamforming. As it can be seen, a better SNR gain is achieved when using the adaptive beamforming.

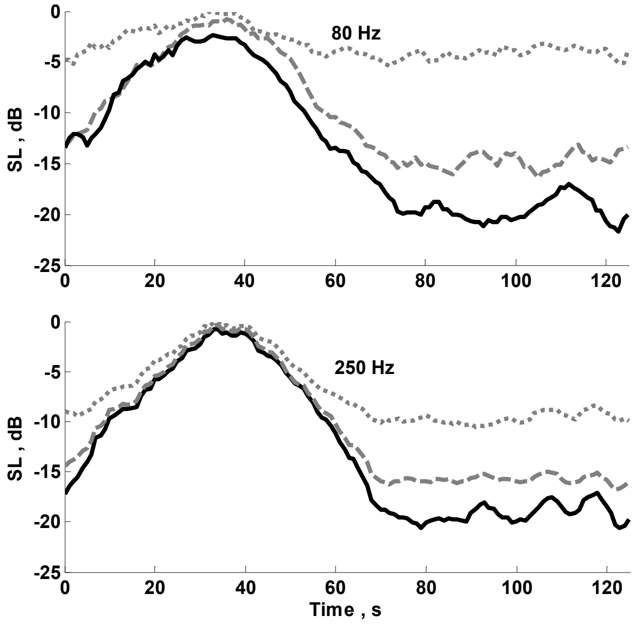

Fig.4 Normalized dependences of boat noise level on time.

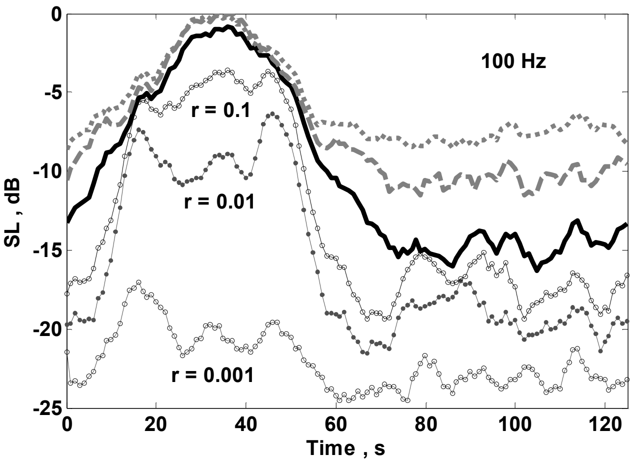

Fig.5 Normalized dependences of boat noise level on time; additionally, signal levels are given for various values of $r = \beta / \beta_{\max}$.

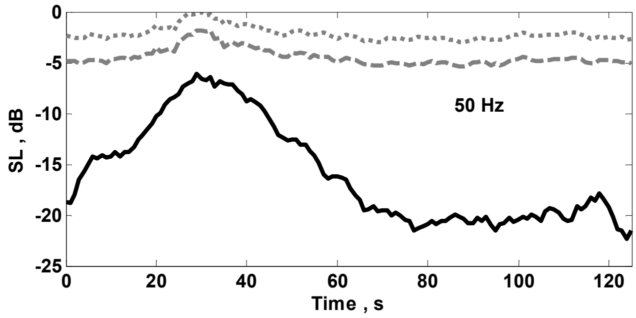

Fig.6 Normalized dependences of boat noise level on time (the case of spatially non-uniform interference).

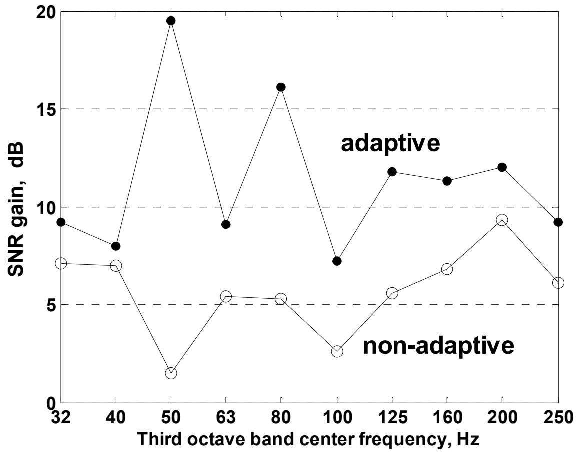

Fig.7 SNR gain when measuring the sound level with the use of the AA (averaged results).

5 Conclusion

It has been shown numerically and experimentally that an increase of the interference resistance when measuring the underwater noise level of moving ships may be achieved by using adaptive beamforming with the special criterion providing the robustness of the synthesized weight coefficients holding the given (allowed) underwater noise level measurement error.

Acknowledgment

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