Modeling the modulated acoustic radiation force distribution in a viscoelastic medium driven by a spherically focused ultrasound transducer

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Acoustic radiation force created by focused ultrasound transducers is gaining great interest in medical diagnosis. This study aims to clarify the acoustic power delivery by means of a modulated focused transducer and to predict the performance of such systems. A spherical-cap shaped transducer, made of piezoelectric material, is used to create ultrasonic waves at a focal point. Different modulation methods, given in the literature and reviewed here, are available for creating a concentrated alternating force due to the acoustic radiation pressure. The relationship between the voltage input to the piezoelectric transducer and its resulting mechanical deformation is examined using a finite element model (FEM) for high frequency harmonic excitation (3MHz – 10MHz). The oscillating surface of the transducer drives the contacting media, which exerts an acoustic load on the transducer that is also considered in the FE analysis. Also, the motion of resulting acoustic waves in a lossy medium is studied for a more accurate estimation of the induced force distribution and energy dissipation within the medium. Ultimately, the intention is to relate the electrical power input to the transducer to the resulting dynamic force generated in the coupling medium.

1 Introduction

The shear elastic moduli of soft tissue, which can be used to distinguish between healthy and pathologic tissue, varies about four orders of magnitude. Techniques such as vibro-acoustography [1] or magnetic resonance elastography [2] can be used to differentiate tissue mechanical properties and, thus can be used as medical diagnostic tools. These techniques have employed the acoustic radiation force of focused ultrasound to provide dynamic excitation inside the tissue.

Vibro-acoustography aims to image an object in terms of variation in its mechanical properties. A highly localized force is used to oscillate the object; this force is created by a focused ultrasonic transducer. The acoustic radiation pressure of ultrasound is modulated to obtain the cyclic radiation force inside the object. The sound emitted by the object which is a function of object's mechanical properties and location, is captured using a hydrophone.

Magnetic Resonance Elastography (MRE) images the shear waves inside the tissue created by an external harmonic force. Focused ultrasound can be used as an external mechanical actuator to induce shear waves [3]. The propagation and attenuation of shear waves, which can be extracted from shear wave images, are functions of tissue viscoelastic properties, so they can be used to obtain the material properties.

The ultrasound transducer is a spherically-focused cap resonating in its thickness mode (Fig. 1). The acoustic radiation due to its motion is concentrated at the geometric center of the sphere which is also called the focal point of the transducer. Different modulation methods are discussed in the literature [4], such that the radiation force field creates a cyclic force. These include Amplitude Modulation (AM), confocal arrangement and X-Focal arrangement. These modulation methods with their corresponding radiation field simulations for the lossless media are also presented.

The radiation field of focused ultrasound in dissipative media is different than the radiation field in lossless media. The formulations for acoustic radiation force for linear and nonlinear ultrasound in a dissipative medium are also available in literature [5].

This study aims to clarify the acoustic power delivery by means of a modulated focused transducer and to predict the performance of such systems. The ultrasound transducer is a piezoelectric disk and the relationship between the voltage input to the piezoelectric transducer and its resulting mechanical deformation is examined in the transducer dynamics section for high frequency harmonic excitation (3MHz – 10MHz). The oscillating surface of the transducer drives the contacting media, which exerts an acoustic load on the transducer; this is also considered in the transducer dynamic analysis. Motion of the contact surface between the medium and transducer creates an acoustic field focused at the focal point. The acoustic radiation force field inside the dissipative medium is reviewed under the radiation force section. A relationship between the electrical power input to the transducer and the resulting dynamic force generated in the coupling medium is presented.

2 Theory

2.1 Transducer Dynamics

A cross sectional view of a spherically focused ultrasonic transducer showing the geometric parameters are given in Figure 1.

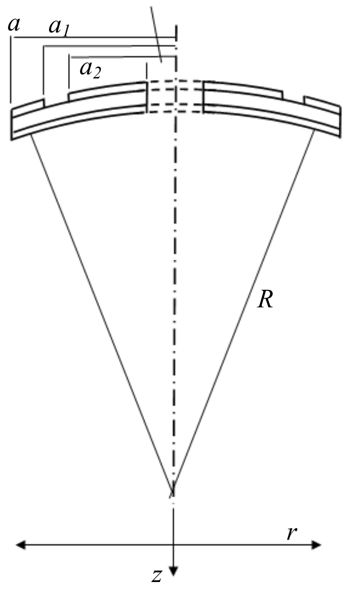

Figure 1. Spherically focused transducer. R is the focal length, a is the disk radius, a_1 and a_2 are radii defining concentric disks (confocal arrangement).

The focused cap transducer can be assumed to be a planar disk for smaller radius versus focal length ratios, *a/R*. Also, the effect of the transducer edges is neglected. These assumptions simplify the cap shaped transducer into an infinite plane, as shown in Figure 2. In this figure, *l* denotes the thickness of the transducer where *z* is the position vector from the center of the transducer.

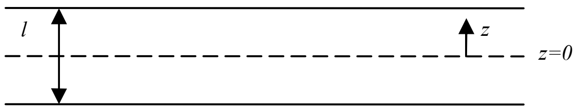

Figure2. For smaller disk radius versus focal length ratios the transducer is assumed to be a planar disk; also, effects at the edges are neglected.

Transducer particle velocity, *v*, is related to stress, *T*, and strain, *S*, as follows.

$$\frac{dT}{dz} = j\omega\rho v\,, \quad \frac{dv}{dz} = j\omega S \tag{1}$$

Here, ρ is the mean material density for the transducer, ω is the circular frequency of oscillations and *j* is the imaginary number.

Piezoelectric constitutive relations are

$$D = \varepsilon^S E - eS \tag{2}$$

$$T = c^E S - eE \tag{3}$$

where D denotes the electric displacement, E is the electric field, ε^S is the dielectric permittivity with zero or constant strain, c^E is the elastic constant in the presence of a constant or zero E field, and *e* is the piezoelectric stress constant.

Velocity v satisfies the wave equation

$$\frac{d^2 v}{dz^2} + \frac{\omega^2 \rho}{c^E} v = 0 \tag{4}$$

and has the solution

$$v(z) = A\sin(kz) + B\cos(kz) \tag{5}$$

where wave number k and wave speed c are defined as

$$k = \frac{\omega}{\tilde{c}}\,, \; \tilde{c} = \sqrt{\frac{c^E}{\rho}} \tag{6}$$

This leads to

$$\begin{aligned} T &= \frac{1}{j\omega} c^E \left[kA\cos(kz) - kB\sin(kz)\right] - eE \\ &= -j\sqrt{\rho . c^E}\left[A\cos(kz) - B\sin(kz)\right] - eE \end{aligned} \tag{7}$$

Applying the free-free boundary conditions to the both ends of the transducer, T(l/2)=T(-l/2)=0, the constants A and B of Eq (5) are found

$$-j\sqrt{\rho . c^E}\left[A\cos(k\frac{l}{2}) - B\sin(k\frac{l}{2})\right] - eE = 0$$

$$-j\sqrt{\rho . c^E}\left[A\cos(k\frac{l}{2}) + B\sin(k\frac{l}{2})\right] - eE = 0$$

$$A = \frac{eE}{-j\sqrt{\rho . c^E}\cos(k\frac{l}{2})}\,, \; B = 0 \tag{8}$$

2.2 Damping

Material damping for piezo materials are available in the literature as mechanical quality factors (Q_{mech}). Using the constants from Eq (8) magnitude response can be plotted for the undamped case, and the damping term can be added to these constants to obtain the desired damping.

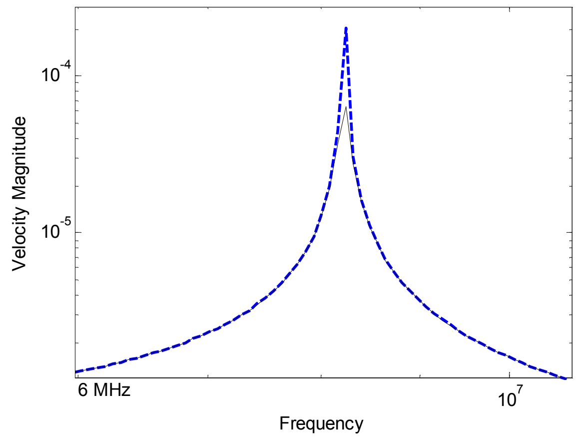

Figure 3. Velocity frequency response function of the transducer surface for unity electrical field (V/m) across the transducer, for free boundary conditions. - - undamped, – damped.

The piezo-electric material properties and transducer geometry is given in Table 1.

Transducer (PZT-5A)	Radius (*a*)	3.5 mm
	Focal Length (*R*)	7 mm
	Thickness (*l*)	0.23 mm
	Stress constant (*e*)	15.8 C/m^2
	Elastic constant (c^E)	11.1 GPa
	Density (ρ)	7750 kg/m^3

2.3 Acoustic Loading on the transducer

However the free-free boundary condition is not realistic; so, the loading on the surface due to acoustic pressure is also considered. Acoustic impedance of the medium is written as

$$Z_0 = \rho_m . \tilde{c}_m \tag{9}$$

where ρ_m is the medium density and c_m is the compression wave speed in the medium. Assuming a Voigt model for the medium, c_m can be calculated as

$$\tilde{c}_m = \sqrt{\frac{\lambda + 2\mu}{\rho_m}} \quad (10)\text{, so} \qquad Z_0 = \sqrt{\rho_m(\lambda + 2\mu)} \; (11)$$

The pressure on the transducer surface due to the acoustic radiation is,

$$p = z.v \tag{12}$$

$$p = \sqrt{\rho_m(\lambda+2\mu)}\left[A\sin(k\frac{l}{2}) + B\cos(k\frac{l}{2})\right] \tag{13}$$

This can be implemented as a new boundary condition, T(l/2)=p and T(-l/2)=0

$$\begin{aligned} &-j\sqrt{\rho.c^E}\left[A\cos(k\frac{l}{2}) - B\sin(k\frac{l}{2})\right] - eE = p \\ &= \sqrt{\rho_m.(\lambda+2\mu)}\left[A\sin(k\frac{l}{2}) + B\cos(k\frac{l}{2})\right] \\ &-j\sqrt{\rho.c^E}\left[A\cos(k\frac{l}{2}) + B\sin(k\frac{l}{2})\right] - eE = 0 \end{aligned} \tag{14a,b}$$

The solution for A and B (using Matlab):

$$A = eE\frac{2j\sqrt{\rho.c^E}\sin(k\frac{l}{2}) - \sqrt{\rho_m.(\lambda+2\mu)}\cos(k\frac{l}{2})}{\sqrt{\rho.c^E}\begin{bmatrix}2\sqrt{\rho.c^E}\sin(k\frac{l}{2})\cos(k\frac{l}{2}) - \sqrt{\rho_m.(\lambda+2\mu)}j \\ +2j\cos(k\frac{l}{2})^2\sqrt{\rho_m.(\lambda+2\mu)}\end{bmatrix}}$$

$$B = eE\frac{\sqrt{\rho_m.(\lambda+2\mu)}\sin(k\frac{l}{2})}{\sqrt{\rho.c^E}\begin{bmatrix}2\sqrt{\rho.c^E}\sin(k\frac{l}{2})\cos(k\frac{l}{2}) - \sqrt{\rho_m.(\lambda+2\mu)}j \\ +2j\cos(k\frac{l}{2})^2\sqrt{\rho_m.(\lambda+2\mu)}\end{bmatrix}} \tag{15a,b}$$

where A and B are constants from (5).

The frequency response for the transducer can be obtained using the constants A and B. The wave number k is related to the angular velocity ω and thus the transfer function (velocity/voltage) can be obtained for different frequencies.

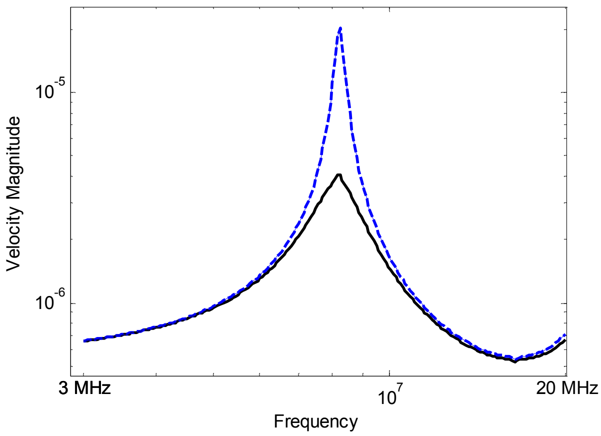

Figure 4. Velocity frequency response function of the transducer surface for acoustically loaded boundary condition, - - undamped, – damped.

The viscoelastic medium properties are given in Table 2.

Medium (0.25% Agar)		
	Density	1000 kg/m^3
	Volume Elasticity (λ_1)	2.6 GPa
	Volume Viscosity (λ_2)	0
	Shear Elasticity (μ_1)	4.89 kPa
	Shear Viscosity (μ_2)	0.32 Pa.s

Table 2. Transducer and coupled medium properties.

2.4 Radiation Force – amplitude modulation case.

The case of amplitude modulated (AM) of a single transducer is considered. The developed theory of Rudenko et al. (96) is utilized and the modulation term is added. The transducer, with resonating frequency ω is modulated at frequency $\Delta\omega/2$. The acoustic pressure field that is created is given by:

$$p(x,r,t) = \frac{p_0}{f(x)}\exp\left[-\alpha x - \frac{r^2}{a^2 f^2(x)}\right] \times\sin\left[\omega\left(t - \frac{x}{\tilde{c}_m}\right) + \phi(x,r)\right]\cos\left(\frac{\Delta\omega t}{2}\right) \tag{16}$$

where

$$f(x) = \sqrt{\left(1-\frac{x}{R}\right)^2 + \frac{x^2}{x_{dif}^2}} \tag{17}$$

$$\phi(x,r) = \arctan\left(\frac{x/R_{dif}}{1-x/R}\right) + \pi H(x-R) - \frac{r^2 x_{dif}}{a^2}\frac{d}{dx}\{\ln[f(x)]\} \tag{18}$$

$$\alpha = \frac{b\omega^2}{2\tilde{c}_m^3\rho_m} \quad (19),\quad b = \lambda_2 + \frac{4}{3}\mu_2 + \chi\left(\frac{1}{C_v} + \frac{1}{C_p}\right) \quad (20)$$

$$x_{dif} = \frac{\omega a^2}{2\tilde{c}_m} \tag{21}$$

Here, p_0 denotes the acoustic pressure at the transducer surface, c_m and ρ_m respectively denote the compression wave speed and density of the medium, H denotes the Heaviside step function, λ_2 and μ_2 respectively denote the medium volume and shear viscosities, χ is the thermal conductivity of the medium, and C_p and C_v are the heat capacities of the medium at constant pressure and volume, respectively. And, α is an absorption coefficient. 'b' is effective dissipation. 'x_{dif}' is Rayleigh Distance.

Modifying Rudenko et al. (96) for the case of a slowly modulated acoustic pressure and applying short term time averaging (given that $\Delta\omega << \omega$, the modulated radiation force under a linear approximation takes the following form:

$$F(x,r) = \alpha.\tilde{c}_m^{\,2}\left(\frac{p_0}{\tilde{c}_m^{\,2}\rho_m}\right)^2\frac{1}{f^2(x)}\exp\left[-2\alpha x - \frac{2r^2}{a^2 f^2(x)}\right] \times\frac{1}{2}\cos\left(\frac{\Delta\omega t}{2}\right) \tag{22}$$

3 Simulations

3.1 Radiation Force calculations

The radiation force field for a 3.5mm radius and 7 mm focal length transducer is calculated using Eq (22) in a dissipative medium assuming linear ultrasound. The force intensity significantly increases at the focal point, acting like a point force inside the medium.

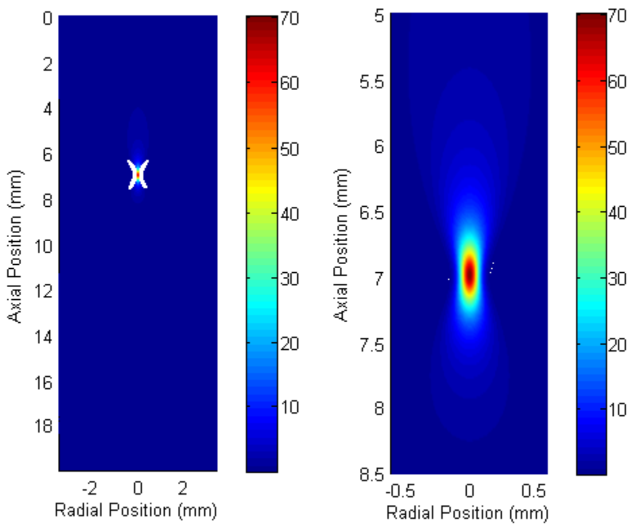

Figure 5. Normalized Radiation force calculations for 0.25% agar gel. Force values are normalized with respect to radiation force on the transducer surface

3.2 Displacement calculations using FEA

Amplitude modulation of the radiation force results an oscillating force between zero and its maximum value (not between positive maximum and negative maximum). The calculated force field is used as a harmonic input for finite element analysis.

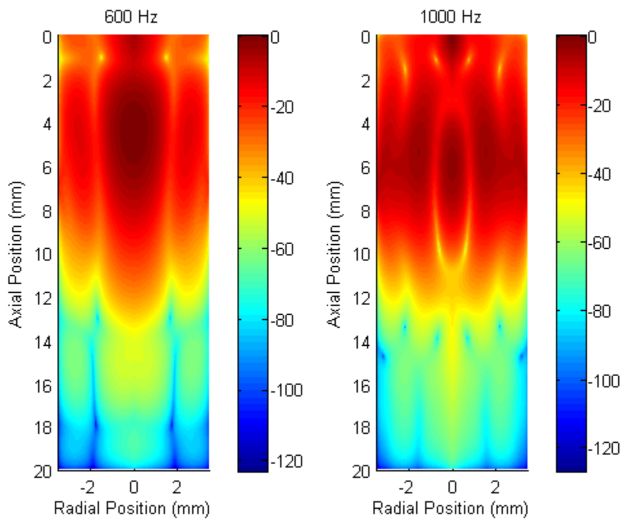

Figure 6. Displacement FEA results due to radiation force in 0.25 % agar gel in dB scale. Displacement values are normalized with respect to the maximum displacement (at the focal point).

Axisymmetric planar linear elements are used in the FEA. Utilization of linear elements is allowable since very small displacements are considered. Harmonic analysis is conducted in different frequencies. Figure 5 shows displacement results at two different frequencies.

4 Results and Discussion

An example of a transducer resonating at 10 MHz on a viscoelastic medium (properties given in Table 1) is considered, to calculate values of displacement inside the medium. For an input of unity electric field (1 V/m) across the piezo transducer's two poles, the magnitude of resulting velocity at resonance at the transducer surface is calculated as 4μm/s. The acoustic pressure at the transducer surface for the coupled medium is calculated using Eq (11) and found to be 6.5 Pa. The normalized radiation field is also recalculated to find the radiation force distribution inside 0.25% agar. The maximum radiation force obtained inside the gel is 0.23 μN observed at the focal point. The maximum values drops to smaller values as one moves away from the focal point. The area containing the force values greater than half maximum is calculated as $4.6\text{x}10^{-2}$ mm^2, and the average pressure acting on this smaller area is calculated as 1.08 kPa. When the ultrasound is modulated with a frequency equal to 1000 Hz, the maximum resulting displacement at the focal point is also calculated using the same FE model, and it is found to be around 2.6 nm (for 1 V/m input to the piezo). Note, the depolarization voltage for this transducer is higher than 100 V. If one assumes that this transducer could be driven at 50 V peak amplitude, the resulting maximum displacement in the radiation field is calculated to be 0.56 mm.

5 Conclusion

The radiation force of focused ultrasound, which is used to remotely create excitation inside viscoelastic media is considered. The aim of this study is to come up with an analysis method combining the different physical domains associated with this transduction problem. Governing piezo-electric equations are used to derive the response function for acoustically loaded and unloaded conditions. Damping, which plays an important role in mechanical response is also considered as mechanical quality factor. Transducer motion on the viscoelastic medium results in an acoustical pressure field inside the medium, which in turn loads the transducer. The force field due to acoustic radiation is calculated and resulting displacements inside the viscoelastic media are calculated using FEA. It is shown that for a given transducer, the resonance frequency and the response at this frequency can be approximately calculated, and the resulting pressure field and displacement field can be estimated. This research was initiated by the motivation of improving the use of focused ultrasound in MRE; results discussed here seem supportive for further MRE utilization of spherical ultrasound transducers.

Acknowledgments

This research is supported by U.S. NIH Grant # EB004885.

References

[1] M. Fatemi, J.F. Greenleaf, "Vibro-acoustography: An imaging modality based on ultrasound-stimulated acoustic emission", Proc. Natl. Acad. Sci. USA 96, 6603-6608 (1999)

[2] R. Muthupillia, P. J. Rossman, D. J. Lomas, J.F. Greenleaf, S. J. Riederer, R. L. Ehman, "Magnetic resonance imaging of transverse acoustic strain waves", Magnetic Resonance in Medicine 36:266-274 (1996)

[3] T. Wu, J.P. Felmlee, J,F, Greenleaf, S.J. Riederer, R.L. Ehman, "MR Imaging of Shear Waves Generated by focused Ultrasound", Magnetic Resonance in Medicine 43:111-115 (2000)

[4] S. Chen, M. Fatemi, R. Kinnick, J.F. Greenleaf, "Comparison of Stress Field Forming Methods for Vibro-acoustography", IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control 51:313-321 (2004)

[5] O.V. Rudenko, A.P. Sarvazyan, S.Y. Emelianov, "Acoustic radiation force and streaming induced by focused nonlinear ultrasound in a dissipative medium." J Acoust Soc. Am. 99,2791-2798 (1996)