

## Tracking of underwater acoustic tone sources with linear antenna arrays

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Institute of Applied Physics of the Russian Academy of Sciences, 46 Ulyanov Street, 603950 Nizhny Novgorod, Russian Federation denis@hydro.appl.sci-nnov.ru Track determination is a necessary stage for a variety of underwater measurement applications, such as measurement of acoustic fields of ships. The use of linear antenna arrays provides great advantages for measurements and tracking.

The investigated ship may be equipped by a special sound source of either pulse or tone signal utilized for tracking. The work considers two scenarios involving the case of a tone source. The scenarios are conditioned by parameters of the array and the source, which may vary depending on task. When a source is of relatively high frequency (several thousands Hz), tracking can be based on the dependence of the Doppler frequency shift and bearing on time; in this case, the parameters to be estimated are the ones of the whole track (velocity, time of closest approach, etc.). If the source frequency is relatively low (up to 300 Hz), the array is large (greater than a typical distance from the source to the array aperture), the array focusing ability allows to perform the determination of "instant" locations of the source at different time instants; estimating of parameters of the track as a whole is then based on the set of the obtained coordinates.

The developed methods have been successfully tested using a large amount of experimental data.

#### 1 Introduction

Antenna arrays are widely used for measurement of acoustic fields of ships (see, for example, [1]). Antenna arrays possess numerous advantages with respect to single receivers (hydrophones) traditionally used for this purpose. The corresponding methods of signal processing require precise knowledge of the investigated ship's track, whose determination represents a self-contained task. Usually an investigated ship is equipped with a special source, and its signal is used for measuring the track.

Techniques for source tracking have been developed and investigated for a wide range of measurement scenarios. For pulse sources, tracking methods are based, as a rule, on the analysis of signal time delays [2, 3]. For a tone source, in the case of free space, the current source position can be determined from inclination and curvature of the phase front (the focusing technique); in the case of an inhomogeneous medium, such as a real sea waveguide, these parameters are insufficient for tracking. In this case, matched-field processing [4, 5] is the most widely used technique. Matched-field processing allows to efficiently localize an underwater source by comparison of the received acoustic signal with the model signal obtained from a certain sound propagation model with various values of unknown parameters characterizing the source position. In some situations, a dependence of amplitude and/or Doppler frequency shift on time can be used as data containing information about the track [6, 7]. Under certain circumstances, alternative approaches to source tracking, for example, based on correlation analysis of energy angular spectrum, can be utilized [8].

In the present paper, methods of source tracking are proposed for the scenario when a source moves in the nearfield zone of a receiving antenna array. The specific scenarios are considered, differing in the direction the array is deployed (horizontal or vertical), its size and spacing, as well as the source frequency.

# 2 Tracking with the use of a horizontal or vertical antenna array with a small aperture

In this section, we will consider tracking techniques for the case when the source frequency is relatively high (several kilohertz), and a horizontal or vertical array is relatively short, i.e., characteristic distances between the source and each hydrophone are much greater than the array aperture. It is supposed that the source movement is uniform, the track is straight and lies in the horizontal plane (the depth of the source does not change); the array is supposed to be straight as well. The experiment geometry is illustrated by Fig.1 (for the case of a horizontal array) and Fig.2 (for the case of a vertical array). Note that, in the case of a horizontal array, its possible incline  $\beta$  with respect to the horizontal plane is taken into account (see Fig.1), whereas a vertical array is assumed to be strictly vertical.



Fig.1 Scheme of the experiment in the case of a horizontal array: (a) top view, (b) side view. CPA is the closest point of approach.



Fig.2 Scheme of the experiment in the case of a vertical array: (a) top view, (b) side view. CPA is the closest point of approach.

Under these assumptions, tracking can be performed in two stages. The first stage is determination of track parameters with respect to individual hydrophones; at the second stage, parameters of the track with respect to the array as a whole can be determined taking into account the information obtained at the first stage.

At the frequencies of several thousands of kilohertz, sound scattering from the sea surface waves is mainly incoherent [9], and its effect can be interpreted as a multiplicative noise. Under the assumption that the source is a point monopole, and the distances between the track and the hydrophones are not great (so the waveguide propagation effects are negligibly small), the model of the signal received by the *n*-th hydrophone can be expressed as

$$s_n(t) = \frac{\exp\left(ik_0r_n(t)\right)}{r_n(t)}\eta(t) + \xi(t) . \tag{1}$$

For each hydrophone, the current Doppler frequency shift is used for converting the signal model to the form with additive noise only. Under this approach, the track has to be straight and uniform. For each hydrophone, time dependence of the Doppler shift is used for determining the unknown parameters. The used model corresponds to a point source in free space:

$$\Delta(t) = \frac{f_0 V^2 (t - t_0)}{c \sqrt{R_0^2 + V^2 (t - t_0)^2}}.$$
 (2)

Here,  $f_0$  is the known source frequency, c is the sound speed, V is the source velocity,  $R_0$  is the distance from the source's closest point of approach (CPA) to the hydrophone (the CPA distance),  $t_0$  is the CPA time. To obtain empirical Doppler shift estimates  $\Delta_{e}(t_{i})$  (*j* is the snapshot number), overlapped time windows were used. The size of a window depends on the track instability and properties of the multiplicative noise. Thus, for each hydrophone, the three parameters  $R_0, t_0, V$  are obtained. As follows from the experimental results (see Section 4), the experimental dependence may include outliers caused by wrong frequency estimates. The outliers can be caused by several reasons, such as a decrease of the signal-to-noise ratio, as well as incomplete adequacy of the free space model, at large distances between the source and the hydrophone. In the presence of outliers, the performance of a least mean square estimator essentially degrades. To solve this problem, the blind estimation technique [10], which is not sensitive to the outliers, are used (this is the main particularity of the proposed approach in comparison with the existing methods [6, 7]). The blind technique involves minimization of mean logarithms of absolute residuals with respect to the unknown parameters:

$$\sum_{j} \ln \left| \Delta_{\mathrm{e}}(t_{j}) - \Delta(t_{j}, R_{0}, V, t_{0}) \right| \rightarrow \min_{R_{0}, V, t_{0}} .$$
(3)

As will be shown in section 4, such an approach yields good results when using for processing of experimental data. In the case when a sparse array is used, the parameters  $R_0, t_0$  have a distinguishable difference for different hydrophones. This allows us to determine the track parameters with respect to the array using the set of the above estimates for all the hydrophones. If a filled-aperture array is used, the current source bearing can be estimated as

well. For this case, it is proposed to obtain the estimates of velocity and the CPA time from the Doppler dependences, whereas all other parameters (e.g., the CPA distance) can be determined from the source bearing. For example, in the case of a vertical array, the track parameters L,h with respect to the array (see Fig.2) can be found by means of model approximation of time dependence of bearing:

$$\sin A(t) = \frac{L}{\sqrt{L^2 + h^2 + V^2 (t - t_0)^2}} \,. \tag{4}$$

The signal model (4) includes only the direct ray; the ray refraction because of the known sound speed profile can be taken into account. In the presence of outliers in experimental dependences, the blind estimation technique can also be used.

A numerical simulation has been performed for evaluating the estimation accuracy of the proposed method. It has been shown for typical experiment conditions (the CPA distance is from 30 to 60 m, the track is almost perpendicular to the line of the array) that the relative error of determination of the CPA distance does not exceed 4%.

### **3** Tracking with the use of a largeaperture array

Now consider the case when a horizontal antenna array possesses a large filled aperture (its size is comparable with a typical distance from the source to the array center, the array spacing is not greater than the half wavelength), and the source frequency is several hundreds (typically, from 200 to 300) Hz. At these frequencies, the Doppler shift is relatively small, so its dependence on time cannot be utilized for tracking. However, the sufficient size of aperture allows "instant" source localization. This may be made by means of the well known Bartlett estimator (see, for example, [4]):

$$\frac{\left|\mathbf{g}_{j}^{H}\mathbf{p}_{j}\right|^{2}}{\mathbf{g}_{j}^{H}\mathbf{g}_{j}} \to \max_{x_{j}, y_{j}}, \qquad (5)$$

where  $x_j$ ,  $y_j$  are the coordinates of the source in the horizontal plane at the *j*-th time instant,  $\mathbf{p}_{i}$  is the received signal after narrowband filtering at the source frequency,  $\mathbf{g}_i$  is the propagation transfer function containing the unknown coordinates of the source, the elements of the vectors  $\mathbf{p}_i$  and  $\mathbf{g}_i$  correspond to different receiving elements, H denotes the Hermitian transpose. All other parameters (the water depth, the depths of the array and the source, etc.) are considered to be known. Note that the function to be maximized in Eq.(5), after division by  $\mathbf{p}_{i}^{H}\mathbf{p}_{i}$ , represents the squared correlation coefficient between the distributions of the received signal and the model signal along the array. Accordingly, the algorithm (5) selects a position in the horizontal plane that provides the best correlation of the model signal corresponding to radiation from this point and the received signal at the current instant.

As an approximation for  $\mathbf{g}_j$  in Eq.(5), in the case of small (of the order of the water depth) distances between the source and the array, the most proper model is a ray model, where the *n*-th component of the vector  $\mathbf{g}_j$  is given by the expression

$$g_{n} = \sum_{l=1}^{L} V_{n}^{(l)} \exp(ik^{(l)}r_{n}^{(l)}) / r_{n}^{(l)}, \qquad (6)$$

where  $r_n^{(l)}$  is the distance from the *l*-th imaginary source to the *n*-th hydrophone,  $V_n^{(l)}$  is the product of the reflection coefficients for the *l*-th ray, *L* is the number of rays taken into account (*l* = 1 corresponds to the direct ray, *l* = 2, 3 correspond to the first surface and bottom reflections, respectively, etc.). Unlike the case considered in Section 2, the surface reflection at low frequencies is close to a mirror-like one, and the surface reflection coefficient can be assumed to be -1; for calculating the coefficients of bottom reflections, if the bottom parameters (density and sound velocity) are known, the Fresnel formula [11] can be utilized.

The number of rays L in Eq.(6) should be chosen according to the conditions of experiment. In a number of situations, especially if the depth is large, only the first two rays (the direct ray and the ray once reflected at the surface) are sufficient for Eq.(6). In most other situations, it is enough to take into account the first six rays, reflected from the bottom not more than once.

At the second stage, estimating of parameters of the track as a whole can be performed based on the set of the obtained coordinates  $(x_j, y_j)$ . In the case when the motion is close to straight and uniform, the method of least squares gives the following expressions for determination of the edge points of track:

$$\begin{aligned} x_{b} &= \frac{1}{A_{11}A_{22} - A_{12}^{2}} \sum_{j} x_{j} \left[ -A_{12} \left( \frac{1}{2} - \tau_{j} \right) + A_{11} \left( \frac{1}{2} + \tau_{j} \right) \right], \\ x_{e} &= \frac{1}{A_{11}A_{22} - A_{12}^{2}} \sum_{j} x_{j} \left[ A_{22} \left( \frac{1}{2} - \tau_{j} \right) - A_{12} \left( \frac{1}{2} + \tau_{j} \right) \right], \end{aligned}$$
(7)

where  $\tau_j = t_j / T$ ,  $t_j$  time of the *j*-th snapshot, *T* is the length of the track section taken into account (the starting instant  $t_1$  is assumed to be zero),

$$A_{11} = \sum_{j} \left(\frac{1}{2} + \tau_{j}\right)^{2}, \quad A_{22} = \sum_{j} \left(\frac{1}{2} - \tau_{j}\right)^{2}, \quad A_{12} = \sum_{j} \left(\frac{1}{4} - \tau_{j}^{2}\right).$$
(8)

The same expressions can be used to determine  $y_b$  and  $y_e$ , after replacing  $x_j$  by  $y_j$ . The edge coordinates are easy to transform into another set of parameters such as velocity, inclination with respect to the array, as well as the CPA time and distance. In the case when the number of outliers is great, and the method of least squares yields an inadmissibly high error, the method of "blind" estimates can be used.

Note that the method can be used in the case of inclined and/or curved array since its profile can be taken into account when calculating the model signal; so, the method only requires an exact knowledge of mutual arrangement of the hydrophones.

An error analysis performed experimentally and using numeric simulation has shown that when a source moves parallel to the array (under sea conditions, at the distances of  $\sim 100$  m), the relative error of the CPA distance determination does not exceed 5% for typical conditions of experiment.

#### **4** Experimental results

In this section, some results of application of the methods to experimental hydroacoustic data are presented.

First, we will present the results related to the case of an antenna array with a small aperture (section 2). In Fig.3, an example of experimental Doppler shift dependence on time, as well as its approximations by means of both the least-squares technique and the "blind" estimation technique (3), are shown.

As follows from Fig.3, there is a number of outliers at the experimental dependence, which cause an additional error when using the least-squares technique, whereas in the case of using the blind technique, outliers almost do not affect the model curve, so estimation of the track parameters is performed with a better accuracy.

In Fig.4, an example of experimental source bearing versus time, as well as its approximation with and without taking into account the ray refraction, is shown. As follows from Fig.4, taking into account the ray refraction improves the approximation accuracy.



Fig.3 Experimental Doppler shift versus time (points) and its approximations using the least-squares (LS) technique (dashed line) and the blind technique (solid line).



Fig.4 Experimental source bearing versus time (points) and its model approximation (solid line): (a) taking into account the ray refraction, (b) without taking into account the ray refraction.

In Figs.5, 6, some results of applying the method of tracking with the use of a large-aperture array considered in Section 3 are demonstrated. The arrays are shown in the lower parts of the plots, the "instant" positions of the source in the horizontal plane obtained in accordance with (5) are shown as dots. Fig.5 illustrates tracking under sea conditions when the movement is nearly uniform. "Gaps" between clusters of points are because the source periodically switched on and off during the motion. On the same plot, an approximation of the track as a whole obtained using Eqs.(7), (8) is shown as a line segment. The first three rays (L=3) were taken into account in Eq.(6). In Fig.6, the results of tracking of a non-straight track under lake conditions are shown. Since the reflection on the bottom was weak, the best results were obtained when taking into account the first two rays (L=2). To demonstrate the importance of taking the surface reflected ray into account, Fig.6(b) shows the results of tracking when using only the direct ray in the model: it is clear that the tracking accuracy greatly decreases.



Fig.5 A resulting track experimentally obtained in sea experiment. The source frequency was 235 Hz, the array with the spacing of 3 m consisted of 64 hydrophones.



Fig.6 Resulting tracks experimentally obtained in lake experiment using the model: (a) of half space (L=2), (b) of free space (L=1) for the same record. The source frequency was 1500 Hz, the array with the spacing of 0.19 m consisted of 64 hydrophones.

To demonstrate this fact, the experimental ambiguity functions obtained in accordance with (5), (6) are shown in Fig.7 for the cases of using the models of free space (L=1) and half space (L=2) for two positions of the source. The results are given for the same record as the one processed in Fig.6. As can be seen, the maximums at the ambiguity functions obtained using the model of free space are "extended" in the direction to the array center. Introducing the surface reflected ray makes the maximum much more localized improving the localization accuracy.



Fig.7 Ambiguity functions experimentally obtained in lake experiment in accordance with (5),(6) using the models: (a),(b) of half space (*L*=2), (c),(d) of free space (*L*=1).

#### 5 Conclusion

In the present paper, methods for tracking of a tone acoustic source moving in the near-field zone of a receiving antenna array are proposed. In the case of a relatively highfrequency (several kilohertz) source, the dependence of the Doppler frequency shift on time for different hydrophones, as well as the time dependence of bearing in the case of filled aperture, can be used. In this case, the track is supposed to be straight and uniform. In the case of a lowfrequency (several hundreds of hertz) source and a horizontal array, a purely spatial processing can be used, which allows to determine "instant" locations of the source at different time instants. In this case, the assumption that a track is straight and uniform is unnecessary. Testing of the methods using experimental data has shown their effectiveness.

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