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Exact and approximate moments for dispersive pulse propagation

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We derive exact moments for pulse propagation in a dispersive medium. These moments are not only inherently interesting but clarify the validity of a recently proposed approximation scheme for wave propagation. The approximation method for pulse propagation is based on the Wigner position-wave/number representation and is very accurate, easy to apply, and moreover is physically illuminating. In particular one obtains the evolved approximate Wigner distribution from the initial Wigner distribution by a simple linear translation in phase space. Propagation with damping is also taken into account. We will show that the reason for the high accuracy of the approximation is that the important low order moments are exactly given by the approximation and that these low order moments preserve very well the basic shape of the pulse. Moreover, now that we understand why the approximation method works well, the approximation can be systematically improved. We give a number of specific examples of exactly calculable moments to illustrate the method and we compare exact and approximate moments.
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