Nondiffractive propagation of sound in sonic crystals

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We report the nondiffractive propagation of ultrasonic waves in sonic crystals, e.g., acoustic media with periodic modulation of the material parameters. Such materials have recently attracted a great interest, because of their potential applications in the control of sound propagation, used as reflectors, focusers or waveguides. All these properties are related with the dispersion introduced by the crystal anisotropy. In particular we consider the case of two-dimensional sonic crystals, consisting, e.g. in an array of steel cylinders in water. It is shown that, for given frequencies and directions of incidence, a narrow sonic beam can propagate without diffractive broadening. Such nondiffractive sonic beams exist in crystals with perfect symmetry, and do not require the presence of defects, differently from other waveguiding phenomena reported previously. The cancellation of diffraction occurs at frequencies and wavevectors for which dispersion curves (isofrequency lines) have zero curvature, i.e, are locally flat. By means of perturbative techniques, a simple analytical expression for the nondiffractive conditions has been obtained. The phenomenon is also demonstrated by numerical integration of the acoustic equations using the FDTD technique. We present experimental evidence of the nondiffractive propagation in a periodic array of steel cylinders in water.

1   Introduction

Sound crystals (SCs) are periodical structures of scatterers in a homogeneous medium, i.e. are media with periodical modulation of their acoustical properties. The study of SCs was stimulated by the previous results in other field like optics [1], that permitted to use the photonic crystals for designing materials with band gaps for the light propagation and to construct photonic crystal waveguides, and early similar results were obtained in the field of acoustics [2].

One of the most studied properties of the SCs is the existence of prohibited propagation acoustical frequency bands. These bands correspond to frequencies for which the acoustic wave does not propagate inside the crystal but it is reflected. Therefore the SCs are good candidates to manipulate band gaps and create waveguides and isolators.

Most of the studies about SCs are devoted to the case of one dimension structures, what permits some analytical treatment, in opposition to the multidimensional cases which properties are numerically investigated either through the plane wave decomposition or finite difference methods. Also the major part of them concern only about the temporal dispersion properties introduced by the crystal. We have focused our studies in the bidimensional case and demonstrated in a recent work [3] that the crystal modifies the spatial dispersion relation $\omega = \omega(k_z, k_\perp)$ with $k_\perp = (k_x, k_y)$. The group velocity of each component is determined by the gradient of the frequency in *k*-space, $v_g = \nabla_k \omega(k)$. As a consequence, for a given frequency the power propagates along the perpendicular direction of the spatial dispersion relation or equifrequency surfaces $k_z = f(k_\perp)$. During a finite distance *l* the phase accumulated by any component is $\phi = k_z(k_\perp)l$. In geometrical terms, the spatial dispersion relation presents in general some curvature, resulting in a diffractive broadening of the beam. As out in [3], a notable exception can be found in the case of sonic crystals, where the isofrequency curves in 2D) develop flat segments at a particular frequency for a given geometry of the crystal. In this case, waves with wavevectors lying on the flat segment do not dephase during propagation through the crystal, and the beam propagates without apparent diffraction keeping its original size. This fascinating effect, originally named self--collimation, has been also experimentally demonstrated to the date for different frequency ranges of electromagnetic waves, in particular in the optical [4], and microwave [5] regimes. We will show in this work the experimental demonstration for the acoustical case.

2   Theory and numerical simulation

The acoustical propagation is governed by the set of equations

$$\rho \frac{\partial v}{\partial t} = -\nabla p,$$
$$\frac{\partial p}{\partial t} = -B \nabla v,$$

(1)

where $\rho(r)$ and $B(r)$ are the medium density and bulk modulus (space dependent), $p(r,t)$ is the scalar pressure and $v(r,t)$ is the vector velocity. Assuming an harmonic dependence for the fields, the equation for a wave of given ω frequency is given by the eigenvalues equation

$$\frac{\omega^2}{\overline{B}(r)} p(r) + \nabla\left(\frac{1}{\overline{\rho}(r)} \nabla p(r)\right) = 0,$$

(2)

where the upper bar means that the quantity is normalised to that of the homogenous host medium. Being the lattice vectors $R = \{R = n_1 a + n_2 a;\; n_1, n_2 \in N\}$, with *a* the scatterers distance and the reciprocal lattice vectors $G = \{G : G \cdot R = 2\pi n;\; n \in N\}$, and developing the variables in that last vector base,

$$\overline{\rho}(r)^{-1} = \sum_G \rho_G^{-1} e^{i\mathbf{G}r}, \quad \overline{B}(r)^{-1} = \sum_G b_G^{-1} e^{i\mathbf{G}r} \quad \text{and}$$

$$p(r) = e^{ikr} \sum_G p_{k,G}\, e^{i\mathbf{G}r} \quad \text{(Bloch-Floquet theorem) we}$$

obtain the eigenvalues equation

$$\sum_{\mathbf{G'}} \left[\omega^2 b^{-1}_{\mathbf{G-G'}} - \rho^{-1}_{\mathbf{G-G'}} (\mathbf{k+G}) \cdot (\mathbf{k+G'}) \right] p_{\mathbf{G'}} = 0.$$

(3)

To solve this equation permits to know the crystal band structure, and therefore the possible existence of forbidden bandgaps together with the isofrequency contours that will allow noticing the nondiffractive zones existence.
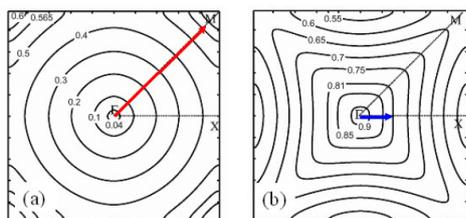
Figure 1.- Isofrequency lines, evaluated for *a*=5.25 mm and *r*=0.8 mm, forthe first (a) and second (b) bands, centered at Γ point, ascalculated by the plane wave expansion method. Numerals denote the reduced frequency $\Omega = \omega a / 2\pi c_h$.

By solving the eigenvalue problem one obtains the frequencies corresponding to each Bloch wave characterised by *k* (the two-dimensional Bloch vector restricted to the first Brillouin zone), resulting in the dispersion relation of the periodic medium. In Fig. 1 the equifrequency contours are plotted for the first [Fig.1(a)] and second [Fig. 1(b)] propagation bands, for the parameters corresponding to our experimental setup. The analysis of the families of equifrequency curves shows that there always exists a particular frequency corresponding to a flat segment, in each propagation band. The corresponding direction of the nondiffractively propagating waves in k-space depends on the number of the propagation band: e.g. is at 45˚ with respect to the crystal axes, i.e. in the < 1,1> direction, for the first band, or along the crystal axes, i.e. in the <1,0> and <0,1> directions, for the second band. The nondiffractive frequencies are slightly less than the central frequencies of the corresponding band gap (denoted by the points M and Γ in Figs. 1(a) and (b), respectively), by an amount depending on the filling factor. The asymptotic analysis in the limit of small filling factor, discussed in detail in [3], leads to the relation $\omega_{nd} = \omega_g \left(1 - f^{2/3}\right)$

where ω_{nd} is the frequency for a nondiffractive beam, and ω_g is the angular frequency corresponding to the bandgap.

The predictions of the previous analysis have been confirmed by the numerical simulation of Eq. (1) using the Finite Difference Time Domain (FDTD) technique, where an input beam with the above calculated nondiffractive frequency in the second band was propagated through the crystal. A typical result, obtained for medium parameters corresponding to our experimental setup and a source frequency of f=230 KHz, close to the nondiffractive propagation frequency, is shown in Fig.2, where the effect of self-collimation is convincing.
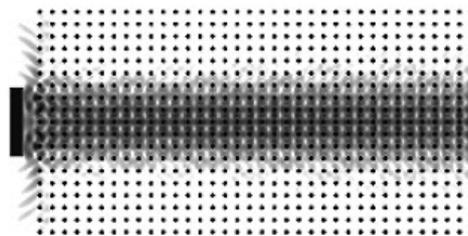
Figure 2.- . Sound beam propagation through a sonic crystal under subdiffractive conditions

3   Experimental demonstration

The first experimental evidence of ultrasound self--collimation has been obtained by measuring the two-dimensional pressure distribution across the transverse plane of the beam in three situations, as shown in Fig. 3: in case (a) close to the source (z=5 mm), in case (b) at z=10 cm from the source in the absence of crystal, and in case (c) at the exit plane of the crystal, located at the same distance from the source as in case (b). The transverse plane was scanned in steps of 1 mm, and the resulting distribution was later interpolated in order to get the smooth distributions shown in Fig. 3. Figure 3(b) illustrates the expected diffractive broadening after propagation in a homogeneous medium. In this case the width of the beam is roughly determined by the number of Rayleigh lengths, $l_R = R^2/2c_h$ the beam has propagated in the medium. For the considered case, l_R=7 cm. Since the beam width increases in a factor 2 every Rayleigh length, at l=10 cm the beam size is three times larger than the input beam, in agreement with the measurements (see also Fig. 4). In the presence of crystal, the beam evolved into a strongly elliptic one after the propagation through the periodic medium [Fig. 3(c)]. In this case, the beam was strongly diffracted in the vertical direction, along the direction of the steel cylinders, since in this direction any spatial modulation is absent. The diffraction in the horizontal direction, where the modulation was present, was strongly suppressed, and the final width of the beam remains nearly the same as at the entrance.
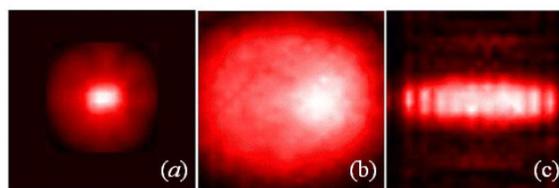
Figure 3.- Transverse distribution of the beam measured at the entrance (a) and at a distance z=10 cm without (b) and with (c) the crystal.

A better comparison can be can be obtained by inspecting the 1D transverse cross-section of the beam distribution, at the exit plane, with and without crystal. Figure 4(a) shows the measured beam cross-section at x=0 and z=10 cm from the source, both with (continuous line) and without (dashed line) the sonic

crystal. For comparison, we also show in Fig.4 (b) numerical results corresponding to the FDTD simulation of Eq. (2). Vertical axis corresponds, in both cases, to the pressure amplitude normalized to the maximum pressure in the case of free propagation. Note the energy redistribution, the decrease in size associated to self-collimation effect is accompanied by an increase in the maximum transmitted amplitude.
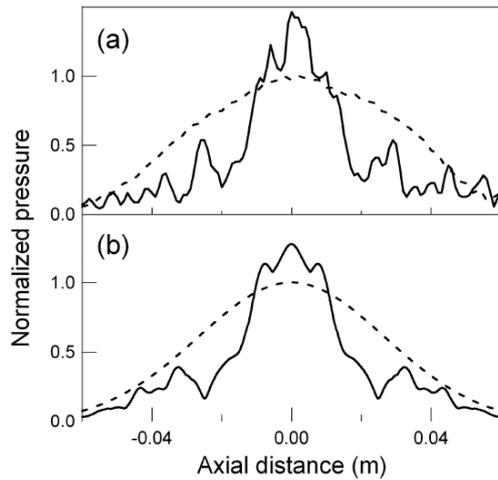
Figure 4.- Cross section of the beam at the exit plane of the crystal (continuous line), and at the same distance from the source without crystal (dashed line), as measured (a) and numerically evaluated (b). Parameters are the same as in Fig. 3.

The beam width has been evaluated at the exit plane as the corresponding to one half of the maximum pressure value, see Fig.5. The resulting width has been normalized to the width of the beam after propagating the same distance from the source without crystal. In this way, we explored the dependence of the beam width on the frequency in order to locate the optimum frequency for the self-collimation. The results are summarized in Fig. 6. The undesired effects of the frequency-dependent initial width, and the inhomogeneities in the initial distribution due to the excitation of different modes in the transducer, can be neglected. The experimental results are represented in Fig. 6 by symbols. It is clearly appreciated that the beam width presents a well defined minimum for 225 KHz. At this value, the width and the entrance and exit plane remains nearly the same (see also Fig. 4), evidencing the disappearance of diffraction. We note that the existence of a minimum is very significant, as it evidences the effect of self-collimation, and not that the sound propagates at the "geometrically transparent" directions of the crystal. In such a case, the effect would hold equally for all frequencies.
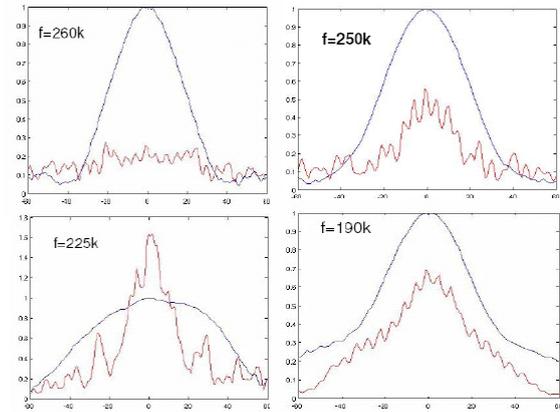
Figure 5.- Cross section of the beam at z=10 cm without crystal (blue line) and at the same distance, which corresponds with the crystal exit (red line) for different frequency values. The frequency value diminishes from the gap (260 kHz).

The frequency for which the beam presents the minimum width, however does not exactly coincide with the calculated zero-diffraction frequency (230 KHz), being slightly less. We interpret this discrepancy as a finite propagation distance effect. The spatial dispersion curve is never flat on a finite segment -- the curvature can become zero just on one or several points. Therefore the beam, even at the zero-diffraction point, weakly broadens in the propagation.

In Fig.6 the width of the beam after the crystal is given depending on the frequency, as measured experimentally (solid circles) and as evaluated analytically (lines). The analytic curve has been derived following and expanding the perturbative methods in [3] for the dispersion relation $\kappa_{\parallel}(\kappa_{\perp})$.
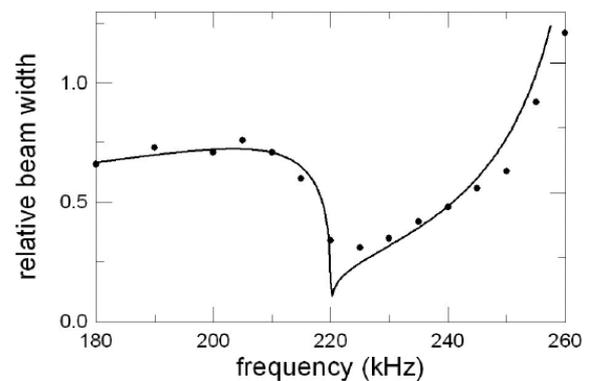
Figure 6.- Width depending on frequency. Experimental results at the crystal exit (circles) and comparison with theory (line).
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4   Conclusions

In conclusion, the subdiffractive propagation of acoustic waves in a sonic crystal has been demonstrated experimentally for the first time. Such novel materials have recently attracted a great interest [6], because of their potential applications in the control of sound propagation, used as reflectors,

focusers [7] or waveguides [8]. In particular we have considered the case of two-dimensional sonic crystals, but the phenomenon should be extendable also to the 3D case. It has been shown that, for given frequencies and directions of incidence, a narrow sonic beam can propagate with very small diffractive broadening. Such subdiffractive sonic beams exist in crystals with perfect symmetry, and do not require the presence of defects, differently from other waveguiding phenomena reported previously [8]. The cancellation of diffraction has been predicted using the plane-wave expansion method to evaluate the dispersion surfaces of the crystal and the spatial dispersion (isofrequency) curves. It occurs for frequencies and wavevectors for which dispersion curves have nearly zero curvature. Experimental evidence of the considered effect has been presented for a periodic array of steel cylinders in water with squared symmetry.

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